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LEVERAGE EFFECT AND STOCHASTIC VOLATILITY IN THE AGRICULTURAL COMMODITY MARKET UNDER THE CEV MODEL

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Abstract

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The problem of price fluctuation is crucial to the concept of financial engineering nowadays. The aim of this paper is twofold; first to investigate the leverage effect of the main agricultural commodities – wheat and corn, i. e. the relationship between monetary returns and the volatility of commodity prices and, secondly to capture their stochastic volatility by forming an appropriate model.
The data are considered as 'post-crisis' data. That means the period after the biggest shock to the world economy. Thus, the Constant Elasticity of Variance (CEV) model is used calibrated to the Generalized Method of Moments (GMM). The paper is briefly based on the research of Geman and Shih (2009), who propose an extension in capturing the leverege effect in the commodity market.
Their results show a positive relationship between commodity price returns and the volatility in both the corn and wheat derivative market. According to these results, corn futures prices are characterized significantly under the CEV model. On the other side in the wheat futures market exists a driftless condition by using stochastic volatility models.

Keywords: leverage effect, stochastic volatility, commodity prices, generalized method of moments, constant elasticity of variance model, driftless process, post-crisis period, futures contracts

INTRODUCTION

Currently the commodity market is characterized by sharp increases connected to price movements. The Bouchet-Hourdon (2011) paper describes commodity price fluctuation as having changed rapidly during 2006–2009 with a significant downtrend. There are several differences in the agricultural products commodity markets. Prices of commodities are influenced by worldwide production and consumption. Prices of agricultural products also reflect the political or weather conditions in various countries. From a speculation aspect there is an assumption that prices of commodities are related to the dynamics of other fianancial instruments, for instance the values of financial derivatives. One of the main factors influencing agricultural products is seasonality. Overall production is derived from seasonal conditions, thus in the time series approach.

The leverage effect can be considered an important phenomena among financial practitioners. The first author to deal with the problem of the leverage effect is Black (1976). His paper documents the relationship between returns on assets and the level of volatility in the negative sense. In other words, in the case of stocks or company equity there exists a strongly negative relationship. This is contrary to the commodity markets, where commodity price returns and their volatility are inversely related, see Christie (1982). For the purposes of our investigation, both stochastic volatility and the leverage effect and constant

elasticity of the variance model are used. This model was introduced by Cox (1975). This model is derived from a stochastic differential equation. The work of Black and Scholes (1973) is the background for modelling the stochastic volalitity concept.

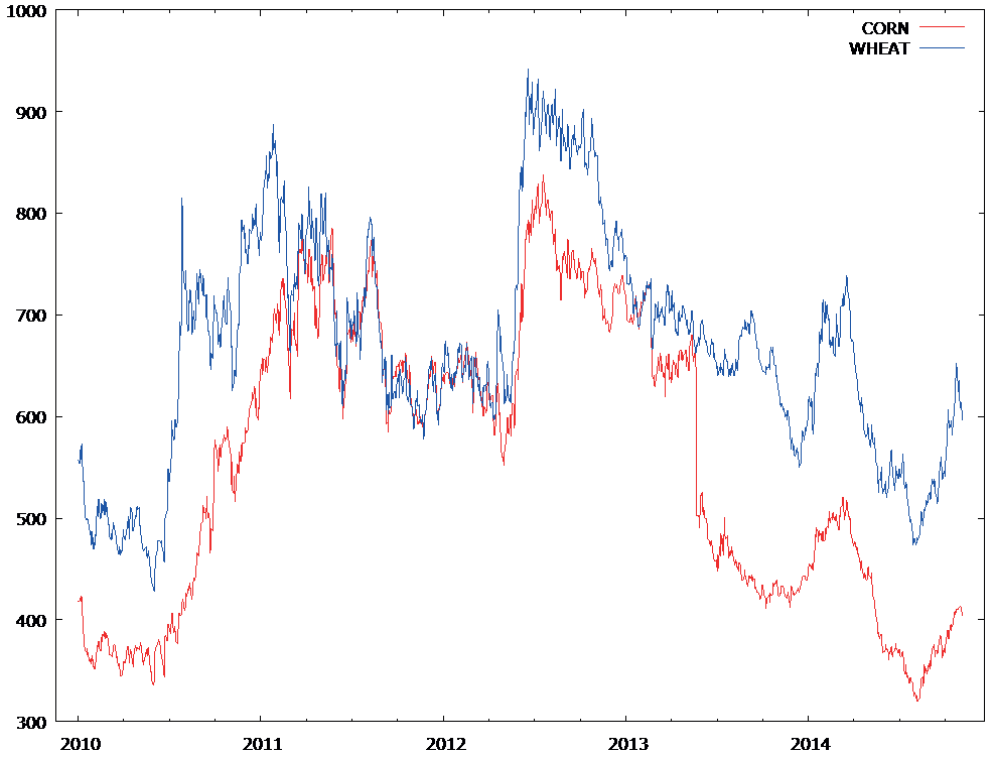
Many authors deal with the concept of capturing stochastic volatility, thus the capture and estimation of the leverage effect. Especially pertinent to the main research in the field of capturing leverage effect is the 2009 paper of Geman and Shih. The authors deal with coal, gold, cooper and crude oil commodities. These are the most important commodities in the energy market. On the other hand, there was no application of their research in the agricultural commodity market. The important aspect of their paper stresses the character of the 'post-crisis' data basement after the years 2008–2009. In general, the authors of this paper suggest the extension of the work of Geman and Shih (2009) using new data collected over the years 2010–2014 in the agricultural commodity markets. Authors suggest revisiting the findings because of new events or information.

The aim of this paper is twofold: first, to investigate the character of the leverage effect in the commodity market after the financial crisis and, second, to set up an appropriate stochastic volatility model. The paper addresses the research question of whether the leverage effect is also positive in the agricultural commodity markets. Its other question is connected to forming the parameters of stochastic volatility model under the CEV approach. The methodology of the paper is based on the volatility dynamics process originated from stochastic differential equations.

Literature review

Many authors have been concerned with price fluctuation in the commodity market over a long period of time. Since the sixties, modern financial theory has been focused on pricing, volatility and hedging.

By comparison to this paper there are several authors who considered the leverage effect in commodities market, including Kristoufek (2012). He investigates the relationship between the returns and volatility of energy commodities futures, including Brent, WTI crude oil and natural gas. Li *et al.* (2016) suggest significant inverse leverage effects for the corn commodity using GARCH regime-dependent models. The paper of Assa *et al.* (2016) deals with estimating the parameters of the CEV process, which are focused on estimating the leverage effect. He uses the maximum likelihood estimation for calibrating the CEV model. The Linetsky and Mendoza (2009) paper surveys the application of the CEV model in the fields of credit derivatives and bancruptcy. Chen (2015) shows that the CEV model exceeds other stochastic models in the case of derivatives. Carr (2011) proposes the application of the CEV stochastic volatility model in the field of financial leverage in the equity index.



1: *Price Trajectories of Corn and Wheat*
Source: Own Processing, Gretl

I: *Augmented Dicky Fuller test, Gretl software*

ADF test	CORN	WHEAT
p–value	0.410*	0.387*

*approval of the null hypothesis at 0.05 siginificant level

II: *Descriptive Statistic over Years 2010–2014, Own processing*

Variable	Mean	Skewness	Std. Dev.	Ex. kurtosis
CORN	–2.74753e–005	–1.77835	0.0198025	28.9718
WHEAT	5.33773e–005	0.293809	0.0206281	2.44043

For the purposes of this paper, the calibration of CEV stochastic volatility model assumes using Generalized Moments of Methods (GMM). This method has attracted many financial practitioners over the years. Authors Chan, Karolyi, Longstaff, Sanders (1992) use the GMM for capturing the long-term interest rate for testing of significance.

The major purposes of this paper require investigating the recent view on the relationship between commodity prices and volatility. Geman *et al.* (2005) research that relationship in the soybeans commodity market. According to the work of Wang, Wu and Yang (2014), there is a susceptibility of agricultural commodities to shocks caused by worldwide supply and demand.

The structure of this paper is as follows: The chapter on *Materials and Method* presents financial data with descriptive statistics and introduces the basic stochastic volatility model with its calibration. Next, the *Results* chapter displays the main findings of estimated parameters. The *Discussion* chapter shows a confrontation with its achieved results compared with other papers and future subjects for research. *Conclusion*, the last chapter presents our overall summary.

MATERIALS AND METHODS

The data base consists of corn futures prices (Instrument FCORN) and wheat futures prices (Instrument FWHEAT) traded in the international markets. Closing daily prices of these commodities are employed for research. The data are collected from January 2010 to December 2014. There are 1264 observations over that selected period. The time series focuses on the period after 2008–2009, which introduces the financial crisis. The length of high-frequency data is crucial for significant estimates.

Graphical analysis of the price trajectories shows the movement of selected commodities, see Fig. 1. The vertical axis shows the value of bushel units in terms of the USD. We see rapid increases after the financial crisis during the years 2008–2009. The chart of corn prices exhibits an incremental drop in 2013. According to a Bloomberg announcement, the decline was caused by the worst drought since 1930. The price trajectories for both commodities behave similarly over time. This is caused by their importance in food market. The price process of both commodities is considered to capture the high volatility period during the selected years. The price trajectories were influenced by different fundamental events in every point of time series. The variability of price process tends to stochastic behavior of the commodity prices. After the year 2014 the prices of selected commodities reverted to the values of beginning of observation.

The stationarity of the commodity prices is analyzed with the Augmented Dickey Fuller test. The lag of ADF test is choosen on the base of Akaike information criterion (AIC), see Fuller (1976). Tab. I shows the confirmation of non-stationarity in the data.

For further analysis the returns using logarithmic transformation are used. The formula is defined as:

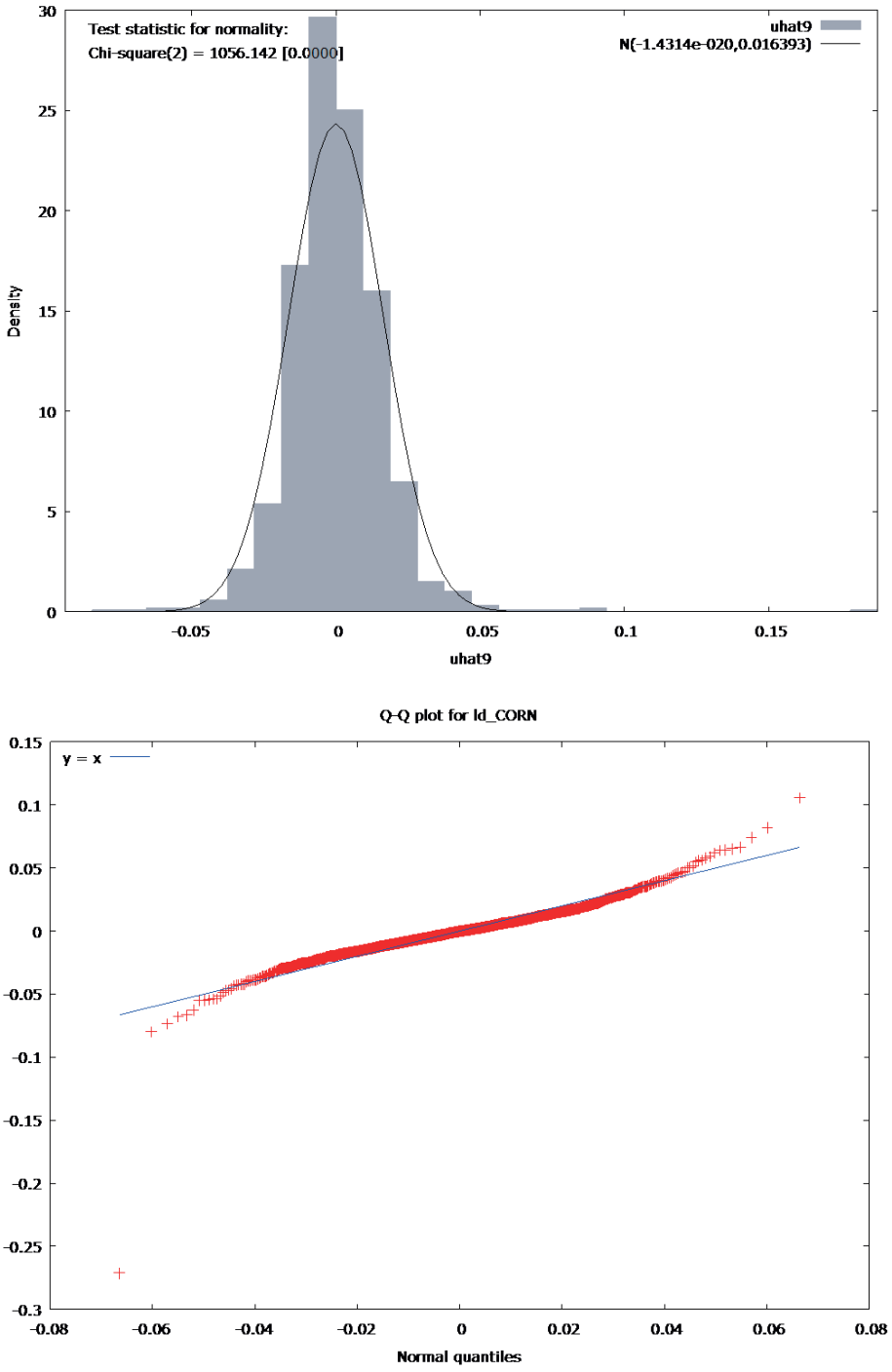
$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{1}$$

The time series are analyzed by descriptive stastistics using mean, skewness, standard deviation and kurtosis, see Fig. First, the standard deviation for corn is lower than that of wheat. These findings are also shown with graphical analysis in Fig. 1. The corn commodity displays a negative skew (–1.77835) with a longer body contrary to normal distribution. On the other hand, the wheat commodity exhibits a positive skew (0.293809) close to zero, so the distribution of observations is close to normal distribution. The kurtosis of wheat shows a value close to zero (2.44043). The recent observations of the wheat commodity are similar to their normal distribution.

Analysis of the Returns Histogram and Q-Q plot show the deviation from normal distrubution with a slow trend toward replication of normality. Adversely, the histogram of return in the case of wheat replicates the normal distribution with some distant observation detected in Fig. III. Onour and Bruno S. (2011) propose that in the special sample of time series – commodities are characterized by fattnes of tails with approximation to normal distribution.

Constant Elasticity of the Variance Model

This paper focuses on the CEV stochastic volatility model to capture certain charactersitics of selected commodities.



2: *Returns Histogram, Q–Q plot, CORN*

The origin of modeling stochastic volatility derives from the stochastic differential equation. Now consider the asset price S_t process as the conclusion of the differential equation:

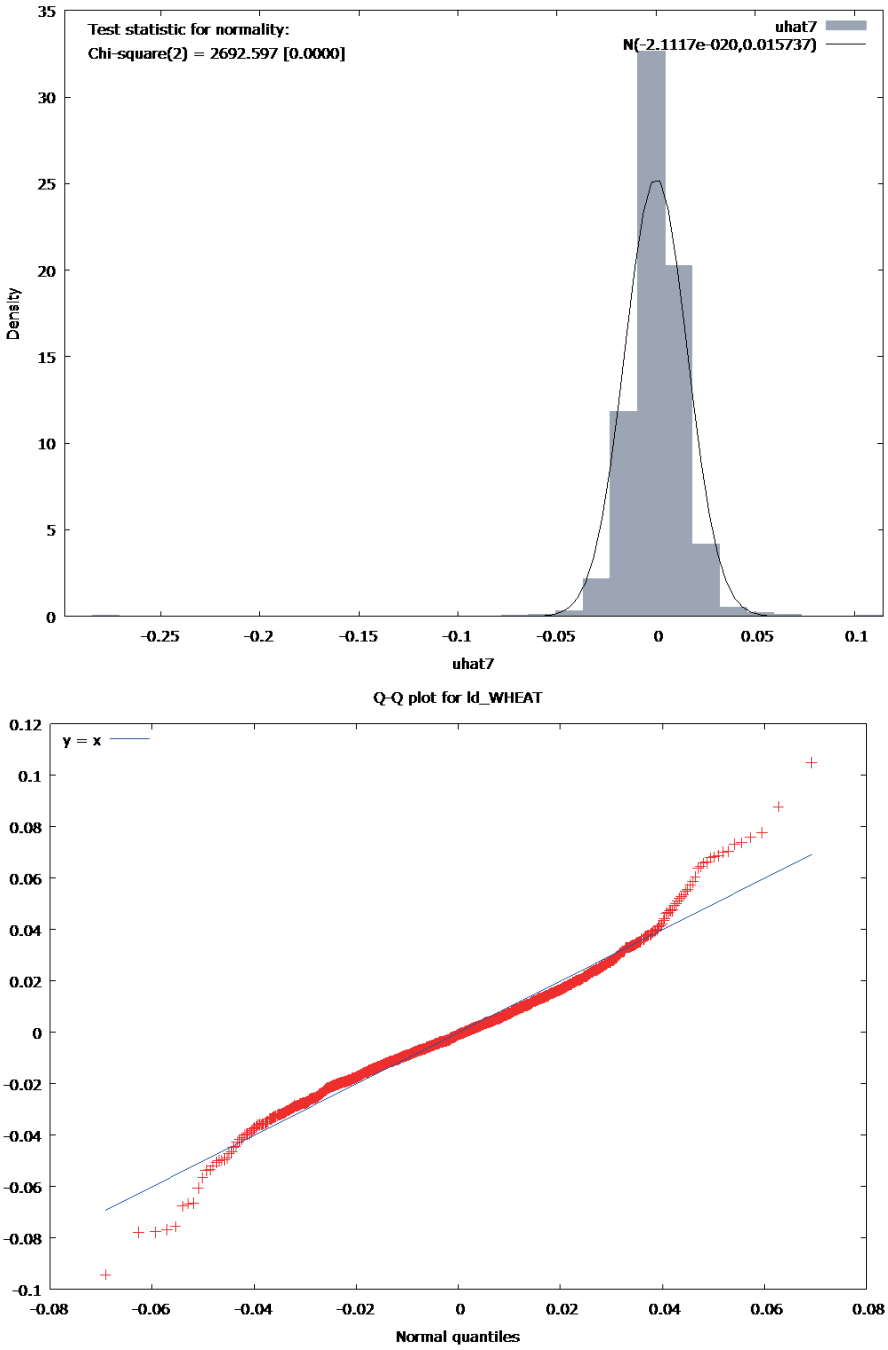
$$dS_t = \mu S_t\, dt + \sigma S_t\, dB_t, \tag{2}$$

where is the expected rare of return, σ we can defined as a standard deviation of stock price percentage change, i. e. volatility and thus B_t is a standard Wiener process, see Cox (1975).

From the assumption of formula (2) Cox (1975) proposes the CEV model, which is defined as:

$$dS_t = \mu S_t\, dt + \delta [S_t]^{\alpha} dW_t, \tag{3}$$

where μ and δ are defined as real parameters, α is an elasticity of S_t variance rate and W_t represents the Wiener process, see Cox and Ross (1976). The power of this model comes from the exponent α, which is also called the 'CEV exponent' and measures the leverage effect. This parameter moves



3: *Returns Histogram, Q–Q plot, WHEAT*

within an interval from 0 to 1. The measurement of volatility of the CEV model is $\delta[S_t]^{\alpha}$. From this approach we can see that volatility changes with respect to asset price S_t. This crucial point in the relationship is important in the field of pricing options.

According to Cox (1975) the value of the 'CEV exponent' at the level of 1 ($\alpha = 1$) following the Brownian motion of Black-Scholes model, see Black (1976). In the case of $\alpha < 1$, there is an inverse relationship between volatility and asset price. Cox, Ingersoll, Ross (1985) show if the value of α satisfies the conditions of $\alpha = 0.5$, they define that as 'square root.' For commodity markets, there is an assumption of positive relationships between the price of commodities and volatility in the case $\alpha > 1$, see Geman and Shih (2009), Emanuel and MacBeth (1982).

Geman and Shih (2009) also consider the mean-reverting models with different versions according to the model of CKLS, see Regland and Lindström (2012). For simplification and significant application, the Cox (1975) CEV model is used.

Calibration of the CEV Model

For the calibration of the Constant Elasticity of Variance (CEV) model, the Generalized Method of Moment (GMM) is used. The GMM method was introduced as a continuous process by Hansen (1982). As mentioned above, there is an important CKLS model using GMM to estimate various interest rate models through the stochastic process. For the estimation of parameters in the GMM procedure the R statistical software with graphical platform R Studio is employed.

Hansen (1982) deals with distribution of the random variable. Under the assumption of conditionally heteroskedasticity for the random variable, the process of estimation seems to be consistent. In the case of over-identifying model restrictions in the econometric model there is a stastic to test the minimized value of the criterion function. The assumption mentioned above is applicable to the CEV model due to the diffusion process.

According to econometric estimation, we can write the CEV model process as a discrete version in the form:

$$S_{t+1} - S_t = \mu S_t + \varepsilon_t. \quad (5)$$

Then the error variance is:

$$E\left[\epsilon_{t+1}^2\right] = \delta^2 S_t^{2\alpha}\ ,\ E\left[\epsilon_{t+1}^2\right] = \delta^2 S_t^2 \quad (6)$$

According to CEV model approach of Cox (1975) the model of CEV for estimation purposes is defined as:

$$dS = \mu S dt + \delta S^{\alpha} dZ^{p}. \quad (7)$$

CEV model is characterized by (L = 3) three parameters $\theta = (\mu, \delta, \alpha)$. The GMM model assumes the moment equations *K*.

The estimation method is focused on minimazing the quadratic function:

$$J_t\left(\hat{\theta}\right) = g_t'\left(\hat{\theta}\right) W\left(\hat{\theta}\right) g_t\left(\hat{\theta}\right), \quad (8)$$

where W is weight matrix to capture the possible over-estimation in the model.

Then we can define the estimator of GMM as:

$$\hat{\theta} = \arg min J_t(\theta) \quad (9)$$

The weight matrix is choosen by a covariance matrix with the smallest value, which is defined as:

$$W(\theta) = \left[\frac{1}{T}\sum_{t=1}^{T} f_t f'_t\right]^{-1} \quad (10)$$

The Sargan-Hansen test introduced by Sargan (1958) and further by Hansen (1982) creates the J-statistic criterion for evaluating the identification of estimation. If the statistic equals to zero, the estimation model is exactly identified. In the case of over-identification of coefficients, the J-statistic is a positive value.

RESULTS

According to the main aim of this paper, the stochastic volatility CEV model is employed. There are estimated results for both corn and wheat prices. First we focus on testing the J-statistics (see Statistic test in Tab. III and Tab. IV). From estimated results, the values of the J-statistics test are lower, close to zero in both corn and wheat commodity markets respectively. The stochastic volatility CEV model has a statistical significance in the case of both tested commodities. In other words, the models are acceptable for further analysis.

For estimation of the leverage effect the parameter α of the CEV model is estimated. During the observed period, the price and volatility relationship represents a significant role in the agricultural commodity market. The findings in Tab. III and Tab. IV show an alpha-parameter value higher than 1. This represents the positive relationship between price and volatility. Both in wheat and corn commodity prices the values are close to 1.2.

Other parameters of the CEV model exhibit both dynamics and drifts in the diffusion process. The value (0.2153) of parameter μ in the wheat market proposes the process without drifts. In the case of parameter σ of corn market the value 0.3521 exhibits standard deviation of commodity price percentage change.

DISCUSSION

This paper investigates an approach to financial phenomena – the leverage effect, which is the primary goal of many economists and financial practitioners. The concept of leverage effect originates from research by Black (1976) and Christie (1982). According to their results the authors provide empirical evidence of the negative relationship

III: *GMM Parameters Estimates for Corn (1.1.2010–31.12.2014)*

Model	μ	δ	α	Statistic test	d.f.
$dS = \mu S dt + \delta S^{\alpha} dZ^{p}$	0.5210	0.3521	1.2564	0.1102	1

IV: *GMM Parameters Estimates for Wheat (1.1.2010–31.12.2014)*

Model	μ	δ	α	Statistic test	d.f.
$dS = \mu S dt + \delta S^{\alpha} dZ^{p}$	0.2153	0.0080	1.2105	0.0025	1

between stock returns and volatility in the case of several firms.

This paper illustrates an investigation of the relationship between the price and volatility of agricultural commodities. There are several works (Cox, 1975, Emanuel and MacBeth, 1982 or Geman and Shih, 2009) that are only focused on metal or energy commodities. The other important aspect of the paper is the use of observations of commodity prices after the main influencing of the world economy in years 2008–2009.

The methodology emphasizes the concept of stochastic differential equations (Øksendal *et al.*, 2000) and its application in the field of diffusion stochastic models, such as the CEV model. The CEV model for the estimation of the leverage effect (Cox, 1975) can also be considered as a local volatility model under driftless conditions (Gatheral *et al.*, 2010). The character of stochastic price trajectory with driftless conditions is observed in the estimated results of this paper. The research is based on the work of Geman and Shih (2009) mentioned above. According to their conclusions they recommend the extension of work in the leverage effect approach with a new cover of commodities, for instance the agricultural commodities.

In the other paper of Chevallier and Ielpo (2013) there is an investigation of the leverage effect and skewness densities in the example of commodities in comparison to financial assets. The paper of Gao (2009) is focused upon forming an optimal investment strategy under the CEV model stochastic volatility process. From these results the investment purposes are becoming increasingly significant.

Cox (1975) focuses on the options market and implied volatility. His research findings support the comparison between price and volatility based on the elasticity of the implied volatility skew of volatility surface concepts. The status, in which the alpha-parameter is less than zero, can be used to capture the volatility smiles in options markets, see Rubinstein *et al.* (2013). We can discuss the acceptance or suitability of selected stochastic volatility models. Many researchers study the stochastic diffusion process in the example of options with appropriate paramaters such as vega, delta, theta etc. The example of this paper shows the power of structured futures contracts with higher degrees of liqudity.

There is an important work by Bekaert and Wu (2000) that is connected to the leverage effect with respect to the CAPM concept. According to their results, the paper shows the existence of a volatility feedback story at the firm level.

The further extension of research should consist of revisiting actual or adding other commodities, such as sugar, cocoa, rice or soybeans. For commodity producers as a retail subject, we conclude that additional research concerning modeling volatility smile with fatter tails to open opportunities to hedge or sell the production.

CONCLUSION

Both financial practitioners and economists are far more emphasize the dynamics of price behavior in both financial and commodity markets. The results of this reseach have implications for practitioners in financial engineering in the field of hedging. Investors are interested in protection of their investment funds by short-selling in option derivative markets.

The aim of this paper is to investigate the leverage effect of selected commodities and estimate the model parameters. The hypothesis is connected to the existence of the leverage effect affecting financial data covering the years 2010–2014. According to the results, there is a positive relationsip between commodity prices and volatility. This relationship has an implication for both corn and wheat commodities. In other words, it is obvious that with an increasing price of commodities, price volatility has a tendency to increase. The findings support the influence of "long-term volatility dynamics" on the future price of both commodities. The process of "long-term" price variability fluctuation is significant despite there are many fundamental shocks affecting the price trajectory. The leverage effect has an important role in the formation of "long-term volatility dynamics".

The wheat futures market is characterized more by driftless conditions than in the case of wheat futures prices. From the concept of calculating stochastic volatility models, there is a parameter σ, which is focused on the power of commodity pricing. The price of corn plays a more powerful (significant results) role in the CEV model. The statistic test shows that the time series have fatter tails with replication on normal distribution. The normality test supports the character of time series in the commodity markets. The results also support the significance of price volatility of selected commodities during the period by force of circumstance of economic crisis.

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