STABILITY MODELLING OF BOOM MOVER BY THE MAPLE PROGRAM

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Abstract

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This paper shows the usage of the Maplesoft 13 program to study the lateral stability of the tractor mulcher mounted on a hydraulic arm connected to the tractor using a 3 point hitch. The results of the tractor stability analysis are in a graphical form with respect to the torque forces caused by the mulcher support arms weight and other present forces during its work operation.

Keywords: tractor, mulcher, Maple, stability

INTRODUCTION

Technical and operating parameters of agricultural machines are verified by certified testing. This paper deals with a theoretical model of stability for general working and technical parameters. Parameters that correspond to the real machine were measured in laboratories of MeU and were used for specific calculations and charts.

The three-point hitch mounted boom mulchers are used for the ground's care for mowing and mulching of the slopes, hitches, pond dams and banks. The three-point hitch mounted boom mulcher is equipped with its own hydraulic power unit driven by the PTO drive shaft of the tractor. The working range of the machine is limited by the lifting capacity and side working outreach of the arm. This assembly including the weight of a mulcher greatly affects the stability of the tractor. This means the tractor needs to have sufficient weight. If it does not it is necessary to equip the tractor with an additional weight under the tractor or on the opposite side to the mulcher. This study deals with the relationship between the tractor stability, attached mulcher, and the maximal forces presented during the work.

The stability designation (stability point backup) and the work tool (mulcher) arm load designation were made using real data about the tractor with the hydraulic arm in the symbolic algebra environment in the Maple program (MAPLESOFT 04). Basic scheme of the tractor and the hydraulic arm with the mulcher showing the basic construction parameters and the important points for the calculations are in the Fig. 1. Corresponding values are in Tab. I. Construction parameters are written in ordinary typeface, the variables determining the arm geometry and the tool settings are in bold Greek symbols and the important points for the calculation are in bold latin.

MATERIALS AND METHODS

Calculations were made using the definition of static and dynamic stability (Grečenko 1994), which defines stability limit of the machine or vehicle so that the left tractor wheel force fit is interrupted (see Fig. 1). The reaction *fl* on the other side of an axle must be in terms of safety equal to a certain part of total weight of the machine. Grečenko gives coefficient 0.10–0.15 which means 10 to 15 % of the weight has to act on the left side of the machine to maintain a static stability. We can specify the arm length next to the closer axle or the value of the weight (work tool) attached to the arm by specifying the reaction to the second wheel.

RESULTS

Simplifying preconditions:

1. The tractor is always standing on a horizontal surface. The basis for calculation is a zero slope.
2. The mulcher (work tool) is oriented in a way its height h is always perpendicular to the ground plane. Eventually it is horizontal.
3. Centres of gravity of the 1st and the 2nd arm are located in the half of their length.
4. All individual weights of piston rods are centred in further operating point.
5. Mulcher centre of gravity is located in the distance h from the P2.

Points 3–5 allow for considerable simplification of the calculation. If the system is stable even in this case, then there is a real stability backup because the real machine has centres of gravity of the arms and the piston rods closer to axes of rotation hence closer to the edge of stability.

Maple calculations

Only the basic calculation solution procedure is described in red and most important Maple outputs are shown here in blue. Maple program commands are equal to mathematical notation and they are easily understandable. It is important to show them in the right order here to describe the calculation process well. Plotting commands are not shown here to save space.
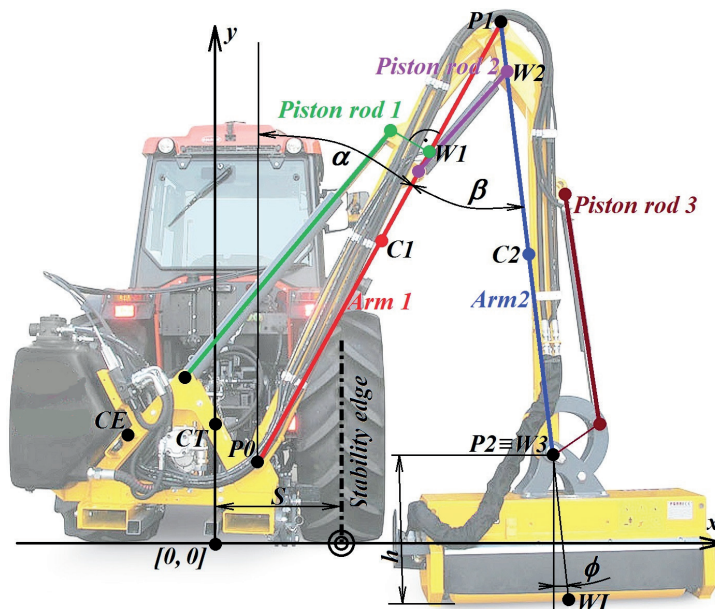

1: *Tractor and hydraulic arm with mulcher scheme – basic construction parameters, main variables and positions of points important for calculations*

I: *Construction parameters, work tool geometry and main auxiliary points*

Variable	Meaning	Value unit	Variable	Meaning	Value unit
Mt	Tractor weight	5130 kg	$CE=[CEx,CEy]$	Weight accessory midst	[–0.9,0.4] m
$M1$	1st arm weight	250 kg	$CT=[CTx,CTy]$	Weight tractor midst	[0,0.6] m
$M2$	2nd arm weight	200 kg	$P0=[P0x,P0y]$	Main hitch	[0.25,0.30] m
Mi	Work tool weight	250 kg	α	1st arm deviation from vertical	variable °
$Mp1$	1st piston rod weight	125 kg	β	Angle between 1st and 2nd arm	variable °
$Mp2$	2nd piston rod weight	80 kg	ϕ	Work tool deviation from vertical	variable °
$Mp3$	3rd piston rod weight	65 kg	$P1=[P1x,P1y]$	1st arm end point	calculation m
Me	Accessory weight	300 kg	$P2=[P2x,P2y]$	2nd arm end point	calculation m
S	Wheel track half	0.96 m	$C1=[C1x,C1y]$	1st arm mass centre	calculation m
$Lr1$	1st arm length	2.2 m	$C2=[C2x,C2y]$	2nd arm mass centre	calculation m
$Lr2$	2nd arm length	2.0 m	$W1=[W1x,X1y]$	1st piston rod point of weight	calculation m
h	Work tool height	0.5 m	$W2=[W2x,W2y]$	2nd piston rod point of weight	calculation m
$Lp1$	W1 to P0 distance	1.8 m	$W3=[W3x,W3y]$	3rd piston rod point of weight	calculation m
$Lp2$	W2 to P1 distance	0.25 m	$WI=[WIx,WIy]$	Work tool point of weight	calculation m
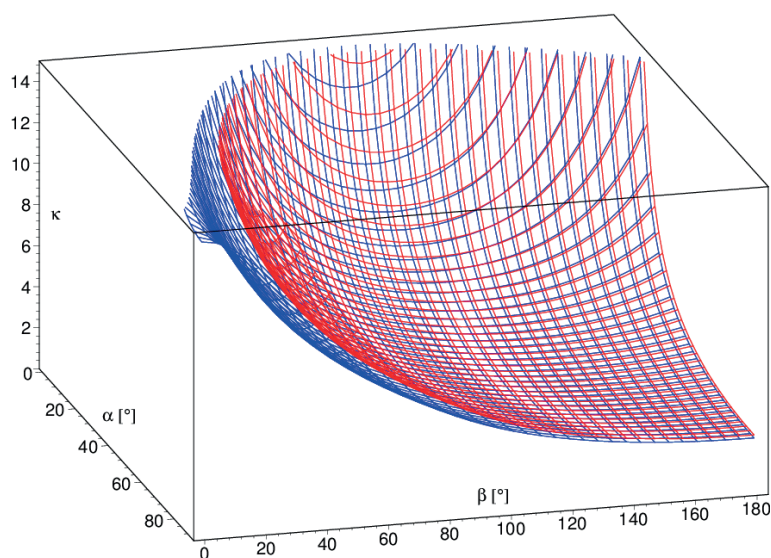
2: *Stability backup*
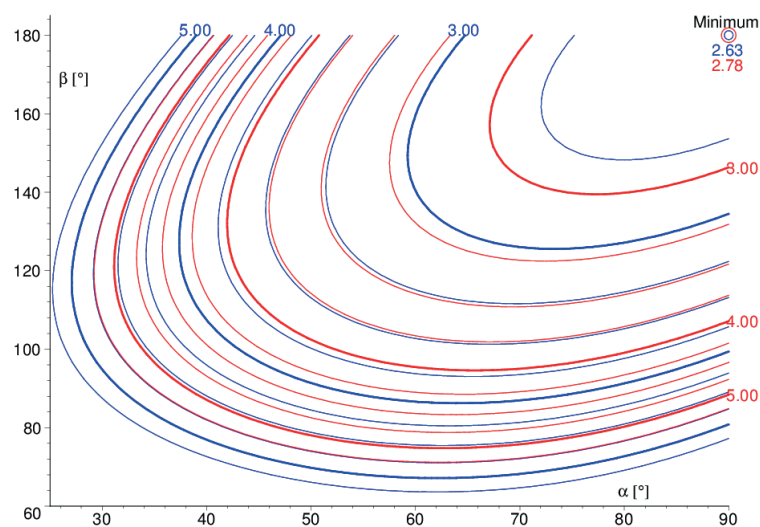
3: *Stability backup isolines near the minimum*

The first thing is setting of the construction parameters, the important points coordination calculation and determination of the distance of the operating point from the rotation point – edge of the stability.

restart; with(plots):

NV:= [Mt = 5130.,   M1 = 250.,   M2 = 200., Mp1 = 125.,   Mp2 = 80.,   Mp3 = 65.,   Me = 300., Mi = 250., S = 0.96, Lr1 = 2.2, Lr2 = 2., Lp1 = 1.8, Lp2 = 0.25, h = 0.5, CEx = –0.9, CEy = 0.4, CTx = 0., CTy = 0.6, P0x = 0.25, P0y = 0.3, phi = 0, g = 9.81]:
Values of construction parameters

CT:= [CTx,CTy]: CE:= [CEx,CEy]: P0:= [P0x,P0y]:
Important points

P1:= expand([P0x,P0y] + Lr1*[cos(Pi/2–alpha), sin(Pi/2–alpha)]):

P2:= expand(P1 + Lr2*[cos(3*Pi/2–alpha + beta), sin(3*Pi/2–alpha + beta)]): *End points of the 1st. and 2ⁿᵈ arm*

Pin:= P2–[h*sin(phi), h*cos(phi)]: *Work tool end point*

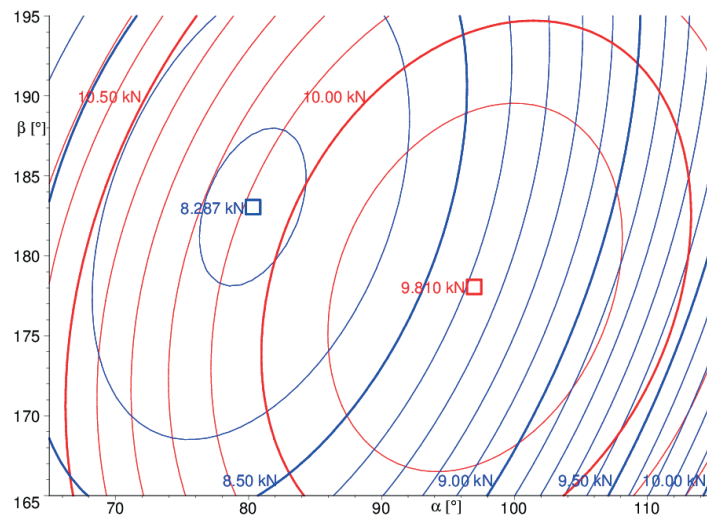**CL1:=(P0+P1)/2; CL2:= combine(P1+P2)/2:** *Gravity center of the 1st. and 2nd. arm*

W1:=combine(expand([P0x,P0y]+Lp1*[cos(Pi/2– alpha),sin(Pi/2–alpha)]))

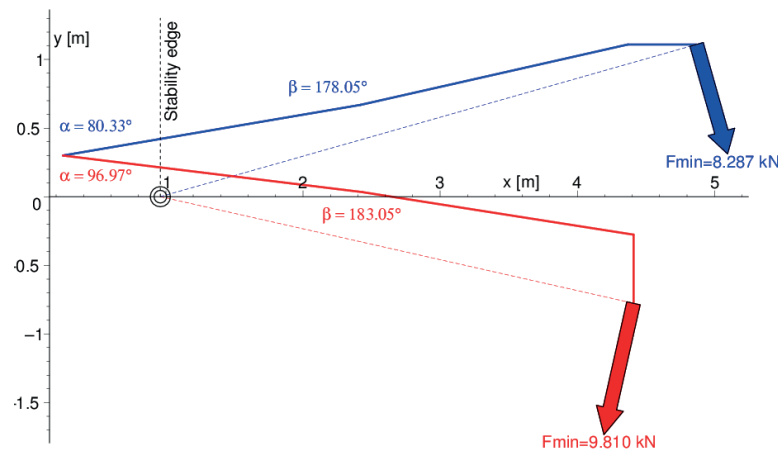**W2:=combine(expand(P1+Lp2*[cos(3*Pi/2– alpha+beta), sin(3*Pi/2–alpha+beta)])):**
Point of weight of the 1st. and 2nd. piston rod

R:=sqrt(add(w^2,w=Pin–[S,0])); *Distance of the work point from the point of stability*

$$R := \left(\begin{pmatrix} -S - h\sin(\phi) - \sin(\alpha)\cos(\beta)Lr2 + \\ +\cos(\alpha)\sin(\beta)Lr2 + \sin(\alpha)Lr1 + P0y \end{pmatrix}^2 + \begin{pmatrix} -h\cos(\phi) - \cos(\alpha)\cos(\beta)Lr2 - \\ -\sin(\alpha)\sin(\beta)Lr2 + \cos(\alpha)Lr1 + P0y \end{pmatrix}^2 \right)^{(1/2)} \quad (1)$$

4: *Maximal work force isolines near its minimum*

5: *System configuration for a minimal work force*

It is possible to determine torques rotating clockwise the system around the stability point.

MF1:= (CL1[1]-S)*M1*g: MF2:=(CL2[1]-S)*M2*g: *Weight torque of the 1st and 2nd arm*

M W 1 : = (W 1 [1] - S) * M p 1 * g : MW2:= (W2[1]-S)*Mp2*g: *Weight torque of the 1st and 2nd piston rod*

MW3:= (P2[1]-S)*Mp3*g: MFi:=(Pin[1]-S)*Mi*g: *Weight torque of the 3rd piston rod and tool weight torque*

These torques must be in balance with torques rotating the system counter clockwise around the stability point.

MFt:= (CT[1]+S)*Mt*g: MFe:= (-CE[1]+S)*Me*g: *Weight torc of the tractor and equipement*

It is possible to enter the stability condition. The heaviness moment rotating the system right must be less or equal to the heaviness moment rotating the system left. This is the reason why it is possible to multiply the torque rotating the system right with the safety coefficient κ that specifies how many times it is possible to raise the heaviness moment of the piston rod arms and work tool so that it is equal to the heaviness moment of the tractor

with its accessories. Using the condition of equality of both moments – Maple equation els it is possible to determine the safety coefficient κ.

e1:= (MF1 + MF2 + MW1 + MW2 + MW3 + MFi)* kappa=(MFt+MFe); e1s:=kappa=solve (e1,kappa); 4.

$$els := \kappa = -2\left(-Mt\ CTx - Mt\ S + Me\ CEx - Me\ S\right)/$$

$$\left(\begin{array}{l} -2\ Mi\ S + 2\ Mi\ P0x + 2\ Mp3\ P0x - 2\ Mp3\ S - \\ -2\ M1\ S + 2\ Mp2\ P0x - 2\ Mp1\ S + \\ +2\ Mp1\ P0x - 2\ Mp2\ S - 2\ M2\ S + \\ +2\ M1\ P0x - 2\ Mp3\ \sin(\alpha)\cos(\beta)Lr2 + \\ +2\ Mp3\ \cos(\alpha)\sin(\beta)Lr2 - 2Mi \\ \sin(\alpha)\cos(\beta)Lr2 + 2Mi\ \cos(\alpha)\sin(\beta)Lr2 + \\ M1\ \sin(\alpha)Lr1 - M2\ Lr2\ \sin(\alpha - \beta) + \\ +2\ M2\ \sin(\alpha)Lr1 + 2\ Mp1\ \sin(\alpha)Lp1 - \\ +2\ Mp2\ Lp2\ \sin(\alpha - \beta) + 2\ Mp2\ \sin(\alpha)Lr1 + \\ +2\ Mp3\ \sin(\alpha)Lr1 - 2\ Mi\ h\ \sin(\phi) + \\ +2\ Mi\ \sin(\alpha)Lr1 \end{array}\right) \qquad (2)$$

DISCUSSION

Stability backup

Only a few graphical outputs are presented here in the interest of brevity. The charts in the phase space are in red that responds to the tool vertical orientation – $\phi = 0°$ and blue colour that responds to the tool horizontal orientation – $\phi = -90°$. Fig. 2 shows κ – stability backup dependency on the ditch arm angle setting. Fig. 3 shows isolines for backup stability near the minimum.

It is very clear from the graphical data the backup stability is relatively high. The torque needed to turn over the tractor would have to be raised 2.5 times.

Determination of maximal possible force acting on the tool

The force F acting on the tool is crucial for the decisive stability calculation. The basic condition implies the force acts perpendicular to the arm – it is the lowest possible force able to turn over the tractor. The length of an arm is known in the example shown below. It is the distance of an operating point from the stability point that is represented by variable R. The calculation of the F (see equation e2s) is made using moment theorems and conditions (see equation e2).

e2:= (MF1 + MF2 + MW1 + MW2 + MW3 + MFi) + + R*F = (MFt + MFe):e2s:= F = solve(e2,F);

$$e2s := F = -\frac{1}{2}g(-2\,Mi\,S + 2\,Mi\,P0x +$$
$$+2\,Mp3\,P0x - 2\,Mp3\,S - 2\,M1\,S +$$
$$+2\,Mp2\,P0x - 2\,Mp1\,S + 2\,Mp1\,P0x +$$
$$+2\,M2\,P0x - 2\,Mp2\,S - 2\,M2\,S +$$
$$+2\,M1\,P0x - 2\,Mp3\,\sin(\alpha)\cos(\beta)Lr2 +$$
$$+2\,Mp3\cos(\alpha)\sin(\beta)Lr2 -$$
$$-2\,Mi\,\sin(\alpha)\cos(\beta)Lr2 +$$
$$+2\,Mi\,\cos(\alpha)\sin(\beta)Lr2 +$$
$$+M1\sin(\alpha)Lr1 - M2\,Lr2\,\sin(\alpha-\beta) +$$
$$+2\,M2\,\sin(\alpha)Lr1 + S^2 - 2\,S\,P0x -$$
$$-2\,\sin(\alpha)\sin(\beta)Lr2\,P0y +$$
$$+2\,S\,h\,\sin(\phi) - 2S\,\sin(\alpha)Lr1 -$$
$$-2\,h\,\sin(\phi)\,P0x + \sin(\alpha)^2\cos(\beta)^2\,Lr2^2 +$$
$$+\cos(\alpha)^2\sin(\beta)^2\,Lr2^2 + 2\,\sin(\alpha)Lr1\,P0x -$$
$$-2\,h\,\cos(\phi)P0y + \cos(\alpha)^2\cos(\beta)^2\,Lr2^2 +$$
$$+\sin(\alpha)^2\sin(\beta)^2\,Lr2^2 + 2\,\cos(\alpha)Lr1\,P0y +$$
$$+2\,h\,\cos(\phi)\cos(\alpha)\cos(\beta)Lr2 +$$
$$+2\,h\,\cos(\phi)\sin(\alpha)\sin(\beta)Lr2 +$$
$$+2\,Mp1\,\sin(\alpha)Lp1 - 2\,Mp2\,Lp2\,\sin(\alpha-\beta) +$$
$$+2\,Mp2\,\sin(\alpha)Lr1 + 2\,Mp3\sin(\alpha)Lr1 -$$
$$-2\,Mi\,h\,\sin(\phi) + 2\,Mi\,\sin(\alpha)Lr1 -$$
$$-2\,Mt\,CTx - 2\,Mt\,S + 2\,Me\,CEx - 2\,Me\,S)/$$
$$(-2\,S\cos(\alpha)\sin(\beta)Lr2 - 2\,h\,\sin(\phi)\sin(\alpha)Lr1 -$$

(3)

$$-2\,\sin(\alpha)^2\cos(\beta)Lr2\,Lr1 -$$
$$-2\,\sin(\alpha)\cos(\beta)Lr2\,P0x +$$
$$+2\,\cos(\alpha)\sin(\beta)Lr2\,P0x -$$
$$-2\,h\,\cos(\phi)\cos(\alpha)Lr1 -$$
$$-2\,\cos(\alpha)\cos(\beta)Lr2\,P0y +$$
$$+2\,h\,\sin(\phi)\sin(\alpha)\sin(\beta)Lr2 -$$
$$-2\,h\,\sin(\phi)\cos(\alpha)\sin(\beta)Lr2 + P0x^2 +$$
$$+P0y^2 + h^2\sin(\phi)^2 + \sin(\alpha)^2\,Lr1^2 + h^2\cos(\phi)^2 +$$
$$+\cos(\alpha)^2\,Lr1^2 - 2\,\cos(\alpha)^2\cos(\beta)Lr2\,Lr1 +$$
$$+2\,S\sin(\alpha)\cos(\beta)Lr2)^{\left(\frac{1}{2}\right)}$$

Maximum work force dependency on arm setting angles is shown in Fig. 4. There are also isolines for maximal work force near its minimum. Corresponding configurations are shown in Fig. 5.

The arm configurations and mulcher orientation for the maximal force load minimum are shown in Fig. 5. While another force is acting – if the force is not acting perpendicular to the effort arm these acting force can be bigger.

Assessment of the force acting on the left axle

The centre of gravity position of the whole system must be determined at first to calculate the force acting on the left axle. With regard to the known centre of gravity positions of each part of the system we can use the formula for point mass system centre of gravity position $\vec{T}$ calculation,

$$\vec{T} = \sum_{i=1}^{N}\vec{T}_i\,m_i \Big/ \sum_{i=1}^{N} m_i \;,$$

where $\vec{T}_i$ are centres of gravity position vectors, m_i are weights of each point masses and N is their count.

TM:= Mt + Me + M1 + Mp1 + Mp2 + Mp3 + + M): TTx:= (Mt*CTx + Me*CEx + M1*CL1[1] + + M2*CL1[2] + Mp1*W1[1] + Mp2*W2[1] + + Mp3*P2[1] + Mi*Pin[1])/TM;

$$TTx := \frac{\begin{pmatrix} Mt\,CTx + Me\,CEx + \\[4pt] +M1\left(\frac{1}{2}\sin(\alpha)Lr1 + P0x\right) + \\[4pt] +M2\left(\frac{1}{2}\cos(\alpha)Lr1 + P0y\right) + \\[4pt] +Mp1\left(\sin(\alpha)Lp1 + P0x\right) + \\[4pt] +Mp2\begin{pmatrix} -Lp2\,\sin(\alpha-\beta) + \\ +\sin(\alpha)Lr1 + P0x \end{pmatrix} + \\[4pt] +Mp3\begin{pmatrix} -\sin(\alpha)\cos(\beta)Lr2 + \\ +\cos(\alpha)\sin(\beta)Lr2 + \\ +\sin(\alpha)Lr1 + P0x \end{pmatrix} + \\[4pt] +Mi\begin{pmatrix} -h\sin(\phi) - \sin(\alpha)\cos(\beta)Lr2 + \\ +\cos(\alpha)\sin(\beta)Lr2 + \\ +\sin(\alpha)Lr1 + P0x \end{pmatrix} \end{pmatrix}}{(Mt + Me + M1 + Mp1 + Mp2 + Mp3 + Mi)}$$

(4)

The dependency of the whole system centre of gravity coordination *x* on the geometry of ditch arm and orientation of the work tool is in the Fig. 6. While we know the system's centre of gravity position TTx we can calculate the relative proportion of the system's weight acting on the left axle fl.
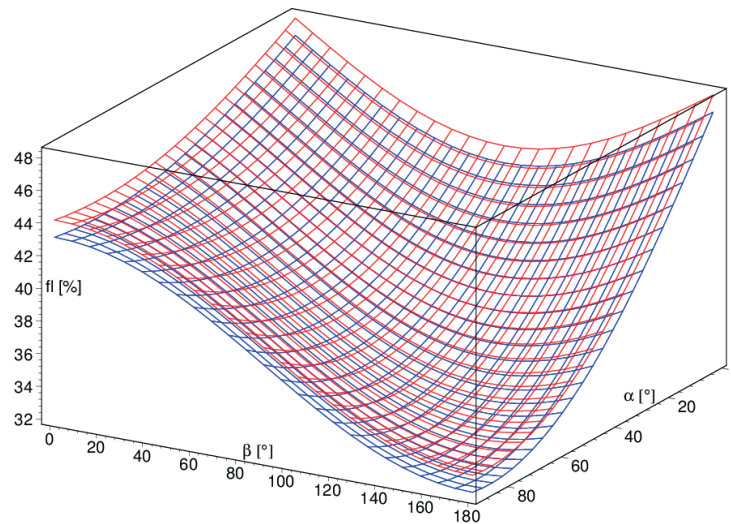
e3:= FL + FR = TM*g; e4:= FL*(−S) + FR*S = = TM*g*Ttx;

fl:= collect(subs(solve({e3,e4}, {FL,FR}), FL)/ TM/g, [Ttx,S]);

$$fl := \frac{1}{2} + \frac{(-Mi - Mt - Me - M1 - Mp1 - Mp2 - Mp3)\,Ttx}{2\,S\,(Mt + Me + M1 + Mp1 + Mp2 + Mp3 + Mi)} \quad (5)$$

Substitution $Ttx = TTx$ into the equation is needed. Dependency of fl on the angle setting of ditch arms and work tool setting is in Fig. 7.
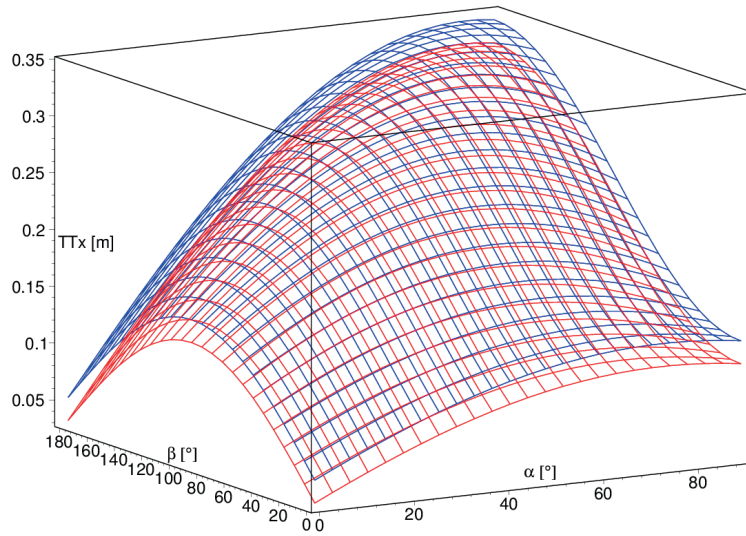
6: *System's centre of gravity position depending on its geometry*

7: *Relative load on the left axle depending on the geometry of the system*

CONCLUSION

The derived mathematical model presented here is verified from real data. Tractor set Z 16045 and mulcher ORSI mounted on the hydraulic ditch arm were used for input data. This system can be considered stable for every possible geometric configuration as it is evident from every presented chart. The derived mathematical model demonstrates sufficient stability backup even in the case of above-mentioned simplified preconditions. Therefore, it is possible to state the real system is safe and stable. This last statement can be substantiated with the calculation of a relative load on the left axle. That achieves a minimum of 32% which is two times the value needed by Grečenko.

The described mathematical model derived for stability solving was derived completely generally and it is independent on construction parameters. That is a reason why it is possible to use it for stability calculation of similar systems by only changing numerical values of used variables. Generalization of described model for situations when the tractor works in cross slope can be done but these situations occur sporadically in real life.

The advantages of a mathematical modelling in maple are mainly versatility, processing speed, accuracy and ability to visualize calculation results (2D and 3D) resulting from processing and calculation of static stability. Maple can be used for construction of function groups, strength calculations of individual elements and their sizing in agriculture. Calculations can reveal critical points of different mechanisms of already made machines. The correct function can be verified with computer animation.

Acknowledgement

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