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THE PROBABILITY OF DEFAULT UNDER IFRS 9: MULTI-PERIOD ESTIMATION AND MACROECONOMIC FORECAST

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Abstract

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In this paper we propose a straightforward, flexible and intuitive computational framework for the multi-period probability of default estimation incorporating macroeconomic forecasts. The concept is based on Markov models, the estimated economic adjustment coefficient and the official economic forecasts of the Czech National Bank. The economic forecasts are taken into account in a separate step to better distinguish between idiosyncratic and systemic risk. This approach is also attractive from the interpretational point of view. The proposed framework can be used especially when calculating lifetime expected credit losses under IFRS 9.

Keywords: credit risk, economic forecast, IFRS 9, Markov chains, probability of default

INTRODUCTION

The probability of default is one of the most important risk parameters estimated in credit institutions, especially banks, and plays a major role in credit risk analysis and management. Given the fact that one of the fundamental activities of banks is granting loans, the banking industry places a great deal of emphasis on credit risk. Credit risk is commonly understood as the potential that a borrower or counterparty will fail to meet its contractual obligations (see Basel Committee for Banking Supervision, 2000). For a bank, it is crucial to evaluate the credit risk associated with potential clients (loan applicants), as well as with actual clients. Credit risk evaluation of (potential) clients is performed within credit scoring, which is a process (or statistical approach) for predicting the probability that a loan applicant or a client will default (Hand, Henley, 1997; Berger, Frame, 2007). Therefore, credit scoring is usually divided into two basic categories – application credit scoring (used for credit risk evaluation of loan applicants) and behavioural credit scoring (used for credit risk evaluation of actual clients).

Banks have historically focused mainly on application credit scoring since granting loans is a vital part of their business. A qualitative approach (based on the credit officer's judgement) dominated for these purposes up until the 1970s. However, this approach is associated with several obvious problems – in particular subjectivity, inconsistency, inefficiency and incomprehensiveness. Since the 1970s, with the developments in information technology, a quantitative approach has prevailed and statistical credit scoring models have been developed and enhanced. These models help to overcome the deficiencies of the qualitative approach based on the credit officer's judgement. Despite the fact that the quantitative approach may also be associated with several problems (development of models using historical data, the assumptions needed to apply certain statistical methods do not hold etc.), credit scoring models have become a standard technique for credit risk evaluation and estimation of the probability of default, and according to Bailey (2004) are now one of the most popular models used in finance in general. For further discussion on the history of credit scoring and associated issues see Chandler, Coffman (1979), Crook (1996), Thomas (2000), or Abdou, Pointon (2011).

Over the years, logistic regression has become the standard and the most-used approach in credit scoring (Crook *et al.*, 2007; Lessmann *et al.*, 2015; Nguyen, 2015). According to Rachev (2008), credit scoring is the most popular application of logistic regression. Due to its simple and intuitive character, as well as the relatively good results it provides, logistic regression has maintained its position as a standard tool even after more sophisticated models were developed, such as neural networks, support vector machines, genetic algorithms and various hybrid and ensemble models. For an overview see Abdou, Pointon (2011), Li, Zhong (2012), or Lessmann *et al.* (2015).

Behavioural scoring models are focussed on the prediction of the probability of default of actual clients. Particularly in the banking industry, prediction of the probability of default gained even greater importance with the introduction of the Basel II capital requirements framework in 2004 (see Basel Committee for Banking Supervision, 2004)[1].Within the Internal Ratings-Based Approach (IRBA), the Probability of Default (PD) constitutes one of the four fundamental parameters for the calculation of credit risk capital requirements, and, as it was mentioned in the beginning, one of the most important parameters in credit risk analysis and management. The other three parameters are Loss Given Default (LGD), Exposure at Default (EaD) and Maturity (M). In this regard, banks are required to retain an adequate level of capital, especially to cover potential unexpected losses. In 2013, these requirements were implemented in the framework of European Union law by the introduction of the Capital Requirements Directive 2013/36/EU (European Parliament and Council, 2013a) and the Capital Requirements Regulation 575/2013 (European Parliament and Council, 2013b).

The international financial reporting standard IFRS 9 *Financial instruments* (hereinafter referred to as the "IFRS 9") – which should be effective as of the 1st of January 2018 – will emphasize and deepen requirements in the area of credit risk analysis and management even more. In a sense, it will also create a stronger link between credit risk and accounting, and significantly impact the banks' economic results. Also for these reasons, IFRS 9 has been greatly debated in the banking industry.

The main objective of this paper is to propose a straightforward, flexible and intuitive computational framework to address some of the major issues of PD estimation under IFRS 9 associated with lifetime expected credit losses – especially PD estimation for more than one period ahead, incorporating the macroeconomic forecast.

The computational framework proposed in this paper is based on transition matrices and probabilities within the theory of Markov chains. Transition matrices have been used in the context of credit risk evaluation since the end of the 20th century. Popular applications included empirical studies of default risk and rating migrations of bonds (e.g. Altman, Kao, 1992; Carty, Fons, 1994), pricing of bonds and derivatives (Jarrow *et al.*, 1997; Kijima, Komoribayashi, 1998), and credit portfolio valuation (Gupton *et al.*, 1997). Since the beginning of the 21th century, the range of applications of transition matrices has become even wider and transition matrices have become an integrated part of modern credit risk management. After the introduction of the Basel II framework, transition matrices became popular also for estimating transition probabilities (including default probabilities) in various applications in the calculation of credit risk capital requirements. This paper does not deal with the initial transition matrix estimation – for possible estimation approaches, discussion of their advantages and disadvantages, and technical details, see Lando, Skødeberg (2002), Jafry, Schuermann (2004), or Engelmann, Ermakov (2011).

Researchers have also investigated and modelled the dependence of transition probabilities on several factors, such as industry, country, and especially macroeconomic variables and the business cycle – see Nickel *et al.* (2000), Bangia *et al.* (2002), Koopman, Lucas (2005), Duffie *et al.* (2007), Figliewski *et al.* (2012), or Gavalas, Syriopoulos (2014). The business cycle (measured with the development of gross domestic product) has proved to be a significant factor influencing transition probabilities, and thus should be taken into account when future transition and default probabilities are estimated. As, for example, the latter of the above-mentioned studies indicates, gross domestic product is considered to be a key macroeconomic variable in the context of studying the impact of macroeconomic variables or the business cycle on transition matrices, and hence will be used as a key variable in this paper as well. Gavalas, Syriopoulos (2014) provide a comprehensive list of relevant studies in this area of research.

This paper proceeds as follows: The second part of the Introduction describes the IFRS 9 standard and its requirements, the consequences of its adoption, a comparison with the Basel requirements, and the expected impacts on banks. In the second section, the data and methodology are presented, in particular the fundamentals of the theory of Markov chains and the regression model used to estimate the "economic adjustment coefficient" for subsequent incorporation of economic forecasts into the PD estimation process.

1 The consultative document was issued in 2001 (see Basel Committee for Banking Supervision, 2001).

The main part of the paper is the third section, which proposes a straightforward, flexible and intuitive computational framework for multi-period PD estimation taking macroeconomic forecasts into account. The fourth section concludes the paper.

IFRS 9 Financial instruments

IFRS 9 basically replaces the international accounting standard IAS 39 *Financial Instruments: Recognition and Measurement* (hereinafter referred to as the "IAS 39"). The replacement has taken place in three phases – Phase 1: Classification and measurement of financial assets and financial liabilities; Phase 2: Impairment methodology; and Phase 3: Hedge accounting (for details, see IFRS Foundation, 2015). The phase that will have the greatest impact on business processes and the most important reported characteristics of banking institutions is the impairment methodology phase. The new impairment methodology constitutes a framework for the calculation of expected credit losses (and thus loss allowances).

Even as a result of the global financial crisis, the biggest weakness of IAS 39 has proved to be the mechanism of calculating impairment (credit losses) associated with financial assets and their loss allowances accounting. The deficient impairment framework of IAS 39 was the strongest reason for it to be replaced by IFRS 9. According to IAS 39, the mentioned process took place based on the so-called Incurred Loss Model, which means loss allowances are recognized after a certain adverse event has already occurred, typically the default of a client who was granted a loan. Together with the introduction of IFRS 9, a change is taking place in this matter and the requirements for impairment calculation and accounting are based on the so-called Expected Credit Losses Model. Hence, credit losses and consequently loss allowances should be recognized based on expectations, which means before a certain adverse event (potentially) occurs – for further details see IFRS Foundation (2015) or e.g. KPMG (2014).

The Expected Credit Losses Model is applied to debt instruments recorded at amortized cost or fair value through other comprehensive income, such as loans, debt securities and trade receivables, lease receivables and most loan commitments and financial guarantee contracts. According to the IFRS 9 requirements, impairment of financial assets is measured as 12-month expected credit losses or lifetime expected credit losses, depending on whether there has been a significant increase in credit risk associated with the given asset since initial recognition. Therefore, the new impairment requirements for financial assets are expected to cause an increase in the overall level of loss allowances. The assessment mechanism for a significant increase in credit risk will play an important role in the calculation of expected credit losses. Focus should be placed on the change in risk of default from the initial recognition of a given asset.

Although using a 12-month time horizon for the calculation of expected credit losses does not have any deeper conceptual basis, the IASB considers this horizon as a suitable compromise between a reliable estimation of expected credit losses and the implementation and operational costs associated with the implementation of the described system. The reason is especially the fact that a 12-month horizon is used nowadays by many institutions within the IRBA for the calculation of credit risk capital requirements (IFRS Foundation, 2015; Ernst & Young, 2014).

The calculation of expected credit losses should be based on a weighted average of credit losses that can occur within various scenarios with the certain probability. Also, the time value of money should be taken into account. Calculated expected credit losses should be discounted with the effective interest rate (or its approximation). When estimating expected credit losses, all relevant and supportable information about current, historical, but also future conditions that can be obtained without undue cost and effort should be taken into account.

One of the most significant impacts of the implementation of the IFRS 9 requirements will be an increase in the overall level of loss allowances, and thus costs, which will affect banks' profit and loss accounts. This will also lead – through lowering the Tier 1 capital – to a decrease in capital adequacy ratios, which are one of the most important reported characteristics of a bank. Estimates of the increase in loss allowances due to IFRS 9 differ. For example, according to IASB loss allowances may increase by 25–60 % (see IFRS, 2013); Hans Hoogervorst (2015), the chairman of IASB, stated that an increase of roughly 35–50 % is expected; and according to Deloitte (2016) banks mostly expect an increase in loss allowances of "up to 25 %". In the recent EBA impact assessment of IFRS 9, the estimated increase of provisions is around 20 % on average (see European Banking Authority, 2016). In the case of adverse economic forecasts, the numbers may be even higher.

As, for example, KPMG (2013) states, the implementation of the IFRS 9 requirements should diminish the effect of the business cycle on profit and loss accounts of banks (reporting within the IFRS framework). The main reason is the forward-looking character of the IFRS 9 concept, taking macroeconomic forecasts into account, and thus calculating credit losses ahead (for example in the beginning of the expected crisis) in contrast to IAS 39. Therefore, profit and loss fluctuations due to the business cycle should be smaller. However, regarding the procyclicality of IFRS 9 loss allowances, opinions of researchers and practitioners may vary. Novotny-Farkas (2016) elaborates on this topic and summarizes, among other things, that a loan loss accounting model reflecting economic conditions is procyclical by its nature; nevertheless, IFRS 9 is likely to mitigate the effect of the features of IAS 39 that potentially amplified procyclicality. The extent of the mentioned mitigation effect and the procyclicality of IFRS 9 loss allowances will largely depend on how it is implemented.

Selected aspects of the Basel framework and IFRS 9 will now be compared. Expected losses under the Basel framework (IRBA) are generally (in most cases) greater than loss allowances under IAS 39. However, it is expected that this relationship will be turned over with the implementation of IFRS 9 – i.e. loss allowances (expected credit losses) under IFRS 9 will be greater than expected losses under the Basel framework (IRBA). Deloitte (2014) states that banks mostly expect this difference to be up to 20 %. However, Karen Stothers (2015) from OSFI (the Office of the Superintendent of Financial Institutions) mentioned that turning over the relationship between IFRS 9 loss allowances (expected credit losses) and Basel expected losses may not be so obvious as it may look at the first sight, since there are several methodological differences in the estimation of credit risk parameters under both of the mentioned areas (IFRS 9 and Basel).

The main differences are that there are two contradictory aspects associated with IFRS 9. It is especially calculating lifetime expected credit losses when a significant increase in credit risk is observed (i.e. increasing expected losses compared to the Basel framework, which uses one-year PDs), which is partially compensated by the "neutral" character of credit risk parameters (especially PD and LGD) compared to the Basel framework, which includes prudential measures like considering an economic downturn (so this provides a contradictory effect to the first of the two mentioned). Another methodological difference is the point-in-time (PiT) or through-the-cycle (TtC) nature of PD estimates. While under the Basel framework the PDs are generally estimated more as TtC (neutralising economic fluctuations) due to the desired low volatility of credit risk capital requirements, under IFRS 9 the PDs should be more "real-time" estimates, and thus PiT, including forward-looking information (especially macroeconomic forecasts). For a more detailed description of the differences, see Deloitte (2013).

As it was already mentioned above, the impact of IFRS 9 will be significant. It will likely cause an increase in loss allowances, and thus a decrease in Tier 1 capital and capital adequacy ratios. Given the fact that the modelling systems for the estimation of credit risk parameters under IRBA and IFRS 9 should be in a sense connected, the implementation of IFRS 9 may also have an effect on pricing (since the mentioned credit risk parameters are often inputs for the calculation of risk margins) and other internal processes such as sales/marketing, collection etc. From a technical point of view, the implementation of IFRS 9 will have a significant impact on internal IT systems, and will require coordinated cooperation especially between Risk, Finance and IT departments.

To complement the literature review, Dvořáková (2014) or Bragg (2016) can be referred to for a more comprehensive context of the IFRS system. Strouhal (2015) provides a glossary of definitions applied in IFRS standards. Albu *et al.* (2013) deal with perceptions of stakeholders involved in financial reporting in four emerging economies (the Czech Republic, Hungary, Romania, and Turkey) regarding the possible implementation of IFRS for SMEs, in terms of costs, benefits, and strategy of adoption. Albu *et al.* (2014) investigate translation and application of global accounting standards in a local context, with Romania as a country case study. Jindřichovská *et al.* (2014) give a thorough overview of the development of accounting and application of IFRS in the Czech Republic with further references. Regarding IFRS 9 in particular, Kněžević *et al.* (2015) provide some remarks on the new classification rules. Since IFRS 9 extends the use of fair value for financial instruments, Palea (2014) can be referred to for discussions on fair value accounting. Bernhardt *et al.* (2014) discuss the new rules for hedge accounting, especially from the risk management's perspective. Beerbaum (2015) gives some remarks on selected aspects of the new impairment requirements, especially a significant increase in credit risk. Novotny-Farkas (2016) discusses the IFRS 9 impairment requirements in the context of financial stability. Gebhardt (2016) compares the impairment models under IAS 39 and IFRS 9 using a case study of Greek government bonds.

MATERIALS AND METHODS

As it was mentioned above, one of the most important tasks regarding PD estimation under IFRS 9 is going to be estimation for more than one period (usually a year) ahead taking the economic forecast into account. This paper addresses these issues by introducing a straightforward, flexible and intuitive computational framework. As was mentioned in the first section, this text is not focused on estimating one-year PDs/initial transition matrices, but rather on working with these PDs/transition matrices in a sense of their extension over more future time periods and incorporation of the expected future economic development.

A multiple-period PD estimation will be performed with Markov models. Economic forecast will be taken into account via decomposition of the "economic adjustment coefficient" estimated by linear regression using the official Czech data (a share of non-performing loans and gross domestic product).

Markov models

Markov models are based on the theory of Markov chains. A Markov chain is a discrete random process that contains a finite set of states *S*, including probabilities governing this process between individual states from *S*. Thus, the process begins in a certain state *i* and in the next step it moves to the state *j* with the so-called transition probability $p_{i,j}$. This process is characterized by the Markovian property expressing that the probability of the process being in a certain state in time $t+1$ depends only on the state the process is in time *t*, and not on the previous states (see for example Brooks, 2008). Therefore, if the random variable is denoted by *X*, the described property can be written in the following manner (Corbae *et al.*, 2008):

$$P(X_{t+1} = j | X_0 = i_0, \ldots, X_{t-1} = i_{t-1}, X_t = i) = \\ = P(X_{t+1} = j | X_t = i) = p_{i,j}. \tag{1}$$

Time-homogeneous Markov chains are commonly applied, which means that $p_{i,j}$ does not depend on *t*. However, given the fact that IFRS 9 requires future economic conditions to be taken into account, this simplification without further adjustment would not be acceptable, especially in the case of potential economic crises. In which case, the PD would be underestimated. Therefore, a subsequent adjustment will be made using decomposition of the calculated "economic adjustment coefficient". In a sense, time-inhomogeneity will be introduced in the presented framework. For a time-inhomogeneous Markov Chain the Markovian property can generally be rewritten as (1) with $p_{i,j}(t)$ on the right side expressing the dependency of the transition probabilities on *t*.

The basis for estimating PD for more than one period ahead is a so-called transition matrix. The transition matrix consists of certain transition probabilities between individual states. To meet regulatory and internal credit risk requirements, many banks have frameworks for one-year PD estimation in place, especially those that use the IRB approach for calculating credit risk capital requirements. Thus, it is expected that banks will leverage these existing systems and will try to adapt them to be compatible with the coming requirements of IFRS 9. Therefore, it is natural to work with a time horizon of one year even in this paper.

If a particular rating system has *r* rating grades – typically internal rating systems within IRBA or rating systems used by the "The Big Three" credit rating agencies[2] – the transition matrix **P** can be written in the following way (the last state is assumed to represent the default):

$$\mathbf{P} = \begin{pmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,r} \\ \vdots & \vdots & \ddots & \vdots \\ p_{r-1,1} & p_{r-1,2} & \cdots & p_{r-1,r} \\ 0 & 0 & \cdots & 1 \end{pmatrix}. \tag{2}$$

The last row demonstrates that the rating grade *r* (default) is the so-called absorbing state, meaning that once this state is achieved, it is not possible to move back to the previous states. Thus, the transition matrix **P** can be described by the following four characteristics (Engelman, Ermakov, 2011):

- The entries in **P** are transition probabilities, therefore $0 \leq p_{ij} \leq 1$ applies for $i,j = 1,\ldots,r$;
- the sum of probabilities in each row must be equal to one (given their nature), i.e. $\Sigma_{j=1}^{r} p_{i,j} = 1$ for a given *i*;
- the entries in the *r*-th column ($p_{i,r}$) represent the PD related to the rating grade *i*;
- the rating grade *r* (default) represents the absorbing state, i.e. $p_{j,r} = 0$ for $j < r$ and $p_{r,r} = 1$.

It can be summarized that at the beginning (at time *t*) the process is in a certain state *i* that is represented by the $(1 \times r)$ state vector $\boldsymbol{s}_t$. If, for example, there are five rating grades and the process is initially in the second one, this can be written as $\boldsymbol{s}_t = (0\ 1\ 0\ 0\ 0)$. It is desired to calculate the state vector $\boldsymbol{s}_{t+n}$ (containing probabilities of the process being in the individual possible grades) using the $(r \times r)$ transition matrix **P**. If a one-year time horizon is considered, then the more precise denotation is $\mathbf{P}_{t+1}$. The general formula for calculation of the state vector $\boldsymbol{s}_{t+n}$ can be written as

$$\boldsymbol{s}_{t+n} = \boldsymbol{s}_t \cdot \mathbf{P}_{t+1}^{n}. \tag{3}$$

It is necessary to repeat that homogeneity is assumed. For a more detailed mathematical description and extensions see Wassermann (2004) or Corbae *et al.* (2008).

Economic adjustment coefficient

A simple linear regression model (with heteroscedasticity and autocorrelation robust standard errors) will be used to capture the impact of the economic development on the probability of default. Based on findings of Vaněk (2016), a share of non-performing loans (as a proxy for the probability of default) and the gross domestic product were chosen as the input variables. Thus, the impact of a change in the gross domestic product on a change in the share of non-performing loans will be investigated. Specifically, the model will take the following form

$$\mathrm{NPL} = \alpha + \beta \cdot \mathrm{GDP}, \tag{4}$$

2 "The Big Three" credit rating agencies consist of Moody's, Standard & Poor's and Fitch Ratings. These ratings (especially of corporate or government bonds) are often mapped to internal rating systems within the various calculations mentioned above.

where NPL represents a share of non-performing loans and GDP represents the gross domestic product. Since interest lies in the interactive dynamics of these time series, variables will enter the model in the form of differences in the case of NPL and growth rates in the case of GDP. The parameters will be estimated using the standard ordinary least squares (OLS) method. The value of β will be understood as the economic adjustment coefficient that will be used together with the official GDP growth forecast to adjust the default probabilities.

Data

The source of the GDP data (chain linked volumes, index 2010 = 100) is the Eurostat database, and the source of the NPL data is the World Bank database and the Czech National Bank's ARAD database. Since NPL can be represented in several ways, depending on the definition of "non-performing loans" and different methodologies of how to measure this variable, three types of variables are used:

1. NPL_WB: a share of bank non-performing loans to total gross loans (source: World Bank),
2. NPL_CNB_1: a share of defaulted loans to total gross loans, calculated as a ratio of the sum of substandard, doubtful and loss loans and the sum of standard and watch loans (source: Czech National Bank),[3]
3. NPL_CNB_2: a share of residents' and non-residents' non-performing loans to gross loans (source: Czech National Bank).

In the subsequent case studies in the next section, the official economic forecast of the Czech National Bank (CNB) is utilized – specifically the quarterly GDP growth in baseline and adverse scenarios adopted from the CNB financial stability report 2015/2016 (see Czech National Bank, 2016).

RESULTS

The proposed computational framework builds on the work of Vaněk (2016) and consists of the three steps. The first one is to estimate the one-year PD/initial transition matrix (not within the scope of this paper). The second step is to estimate the economic adjustment coefficient that captures the impact of the expected future economic development on the probability of default. The thirds step is to adjust the original one-year PDs/transition matrices for the future periods of time by the decomposition of the effect quantified in the second step, and calculation of the multi-period PDs within the concept of the Markov models.

This logic ensures that impacts of idiosyncratic and systemic risks are more separated. Idiosyncratic risk is a risk specific to individual clients or a group of clients. Systemic risk is a risk that influences clients as a whole (typically economic development). Similar logic is also followed for example by Sousa *et al.* (2013). This methodology allows us to identify and quantify a share of the final PD that is attributed to the incorporation of the future economic development, which is attractive also from a managerial point of view.

Introductory case study

For illustrative purposes, only two states will be considered in this part – the "non-default state" and the "default state". This can be thought of as a rating system with two grades. However, as was described above, an extension to the general case with an arbitrary number of rating grades can be done in a straightforward way. Also, it will be assumed that a client at time t is assigned the probability of default of 4 % for a one-year time horizon. In other words, the transition probability of a transfer from the non-default state to the default state during the next year is 4 %. Thus, the state vector $\boldsymbol{s}_t$ can be written as

$$\boldsymbol{s}_t = (1\ 0) \tag{5}$$

and transition matrix $\mathbf{P}_{t+1}$ takes the form

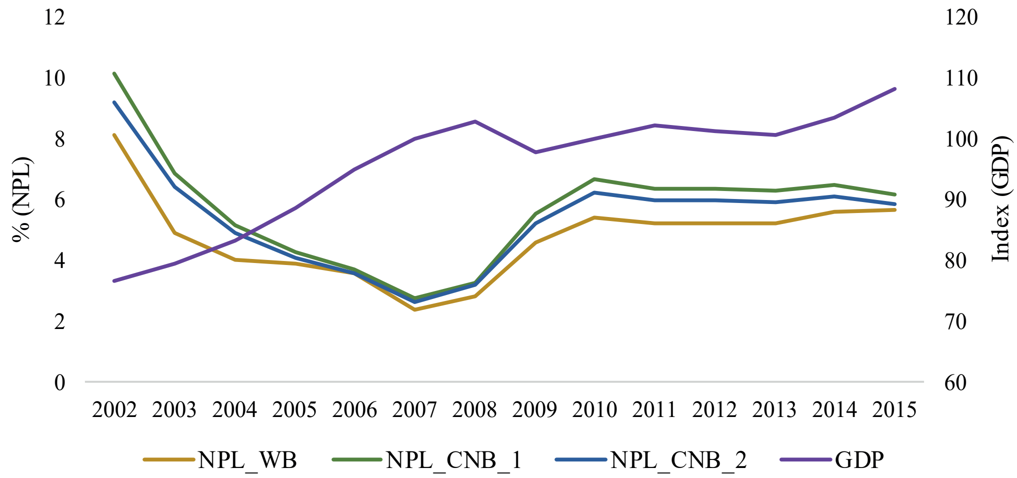
$$\mathbf{P}_{t+1} = \begin{pmatrix} 0.96 & 0.04 \\ 0 & 1 \end{pmatrix}. \tag{6}$$

It can also be assumed that the three-year probability of default is desired to be estimated. Thus, based on (3), the calculation can be made in the following way:

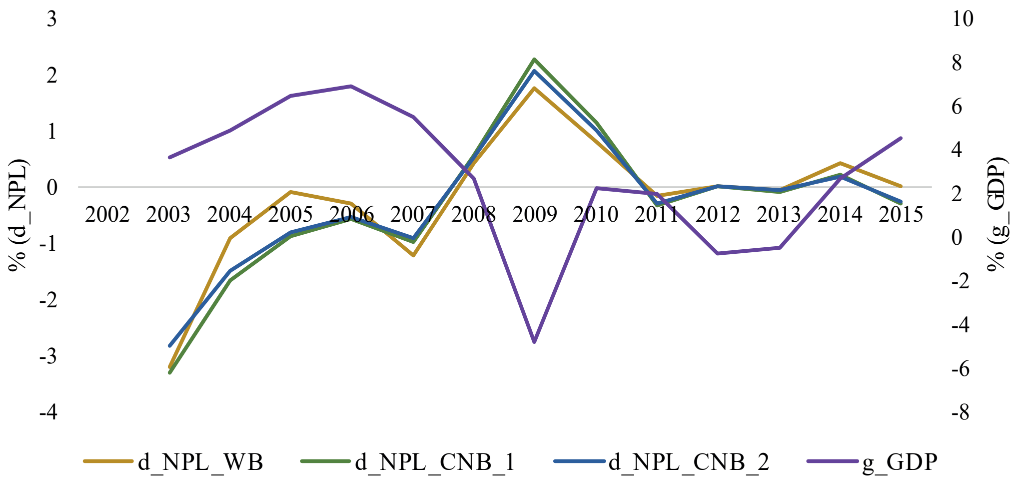
$$\begin{aligned} \boldsymbol{s}_{t+3} = \boldsymbol{s}_t \cdot \mathbf{P}_{t+1}^3 = (1\ 0) \cdot \begin{pmatrix} 0.96 & 0.04 \\ 0 & 1 \end{pmatrix}^3 = \\ = (0.8847\ 0.1153). \end{aligned} \tag{7}$$

The estimated 3-year PD is 11.53 %. However, this estimate is based on a homogeneous Markov model and does not take into account the economic forecast. Therefore, the following task is to calculate the economic adjustment coefficient and adjust this estimate based on the expected economic conditions.

3 This calculation follows the methodology set in Decree No. 163/2014 Coll. (see Czech National Bank, 2014), where the Czech National Bank introduces the classification of receivables by quality. There are five categories: standard, watch, substandard, doubtful and loss, whereby the first two are considered as non-default categories and the remaining three as default categories.



1: *The development of NPL and GDP in 2002–2015*



2: *The development of NPL differences and GDP growth rates in 2003–2015*

I: *Results of the regressions*

	d_NPL_WB		d_NPL_CNB_1		d_NPL_CNB_2	
	Coefficient	***p*-value (*t*)**	**Coefficient**	***p*-value (*t*)**	**Coefficient**	***p*-value (*t*)**
constant	0.312	0.312	0.394	0.280	0.375	0.253
g_GDP	–0.184	0.011	–0.256	0.002	–0.233	0.002
R^2	0.262		0.388		0.412	

Economic adjustment coefficient estimation

To investigate the impact of economic development (represented by the GDP growth) on the probability of default (represented by a share of non-performing loans in differences), the regression (4) will be run. Before doing so, the relationship between these two variables will be investigated graphically. This analysis can be performed in various ways.

In this paper, the relationship will be analysed on the aggregate level using the Czech data described in the second section of this paper. The development of the NPL variables and GDP is illustrated in Fig. 1.

As it could be expected, the development of the three variables representing NPL is very similar. Also, an antagonistic development of NPL and GDP can be seen. This pattern makes sense and can also be expected – when the economy grows, the share of non-performing loans is expected to go down.[4]

4 The share of non-performing loans in the Czech Republic is relatively very low, even in the context of the European Union – see Mesnard *et al.* (2016).

As it was mentioned above, the focus lies rather on the dynamics of these time series; hence, the differences of NPL (d_NPL) and growth rates of GDP (g_GDP) are depicted in Fig. 2.

The results of the regression model (4), more precisely written as $d\_NPL = \alpha + \beta \cdot g\_GDP$, are presented for all three NPL variables in Tab. I (as a common practice, the statistically insignificant constant term is left in the model for technical reasons).

Even though the numbers of all three regressions do not differ distinctly, from a statistical point of view the regression using d_NPL_CNB_2 provides the best results (the coefficient of determination R^2 indicates the relatively best goodness of fit for this model). Therefore, the following subsections will work with this case.

The results show that if GDP growth increases by one unit (in this case 1 percentage point), the NPL share decreases by 0.233 percentage points. Thus, the economic adjustment coefficient (hereinafter referred to as the "EAC") is –0.233. The quantitative results correspond to the graphic analysis performed above. This EAC will then be used to adjust the estimated PD to capture the expected economic development.

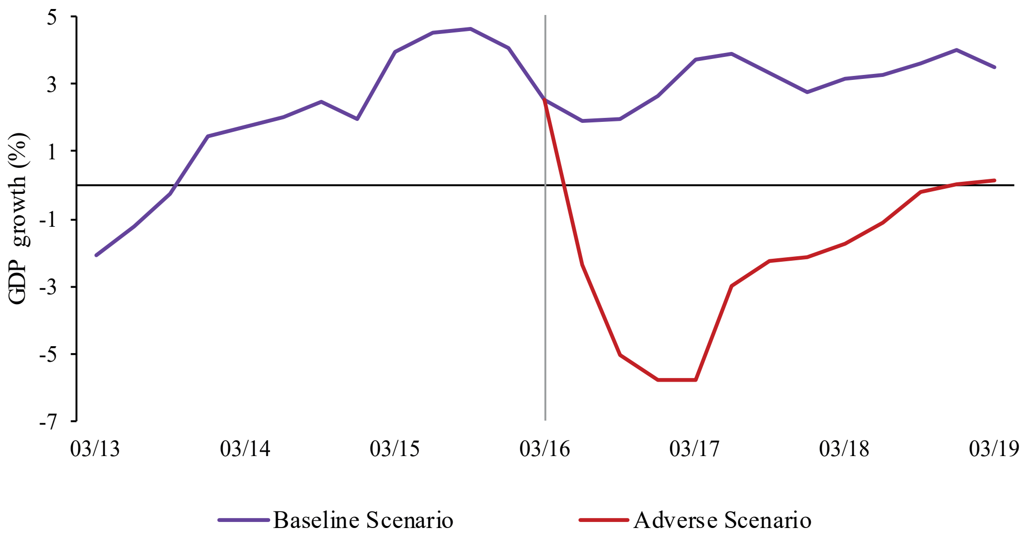
Incorporation of economic forecast

As it was mentioned in the second section, this paper will work with the official economic forecast of the Czech National Bank. Fig. 3 depicts the baseline and adverse scenario forecasts of the Czech GDP growth. The adverse scenario forecasts can be used especially for simulating an economic crisis and therefore also for various stress testing exercises.

Based on the presented information, a practical exercise of incorporating economic forecasts into multi-period PD estimation can be performed. Suppose that at the end of the year 2015 (t), an estimation of the one-year PD of 4 % was performed (see the introductory case study), and it is desired to estimate the three-year PD taking economic forecast into account. Thus, the basis is the transition matrix (6) as before.

This estimate abstracts from the economic environment. When addressing this issue, it would be better to summarize the expected economic development in a table (see Tab. II), where the annual averages are calculated from year-on-year quarterly GDP growth rates. Given the fact that the value in 2016 Q1 is known and the adverse scenario begins in 2016 Q2, the annual average for 2016 is obtained as an average of 2016 Q2–Q4.

When adjusting PD estimates, the future and present economic conditions are compared. Thus, the right side of the table presents differences (Δ)



3: *The GDP growth forecast of the Czech National Bank*

II: *Summary of the forecasted GDP growth (in %) for 2016–2018*

	GDP growth rate		Δ (base = 2015)		
	baseline	adverse	baseline	adverse	
2015	4.30		–		–
2016	2.28	−4.39	−2.02	−8.69	Δ_{t+1}
2017	3.42	−3.28	−0.88	−7.58	Δ_{t+2}
2018	3.51	−0.74	−0.80	−5.04	Δ_{t+3}

between the forecasted growth rates and the growth rate actually observed in 2015 (t). The goal is to estimate the three-year PD, thus PD (t + 3). Estimation of the PD (t + 3) will be performed within the framework of Markov models, therefore it is necessary to estimate $\boldsymbol{s}_{t+3}$. However, in this case, it is no longer possible to calculate this state vector using formula $\boldsymbol{s}_{t+3} = \boldsymbol{s}_t \cdot \mathbf{P}_{t+1}^3$. Instead, a transition matrix in every time period needs to be adjusted by the expected economic conditions. For this purpose, the formula will be decomposed into

$$\boldsymbol{s}_{t+3} = \boldsymbol{s}_t \cdot \mathbf{P}_{t+1} \cdot \mathbf{P}_{t+2} \cdot \mathbf{P}_{t+3}, \tag{8}$$

or in general into

$$\boldsymbol{s}_{t+n} = \boldsymbol{s}_t \cdot \prod_{k=1}^{n} \mathbf{P}_{t+k}, \tag{9}$$

where $\mathbf{P}_{t+k}$ represents a transition matrix for a given time period, specifically adjusted by the forecasted economic conditions using the decomposition of the EAC. Thus, in a sense, time-heterogeneity (or time-inhomogeneity) is introduced in the presented framework via the EAC decomposition to capture the future economic development. This can be done in a straightforward manner. If we desire that half of the estimated effect should be attributed to increasing the transition probability to the default grade and half of the effect should be attributed to decreasing the transition probability to the non-default grade (so the difference in the absolute value corresponds to the effect as a whole), this adjustment can be formulated in the following way (for two rating grades):

$$\begin{aligned}\mathbf{s}_{t+n} &= \mathbf{s}_t \cdot \prod_{k=1}^{n} \mathbf{P}_{t+k} = \\ &= \mathbf{s}_t \cdot \prod_{k=1}^{n} \begin{pmatrix} p_{1,1} - \dfrac{\Delta_{t+k} \cdot EAC}{2} & p_{1,2} + \dfrac{\Delta_{t+k} \cdot EAC}{2} \\ 0 & 1 \end{pmatrix}.\end{aligned} \tag{10}$$

Naturally, it is desired that transition probabilities fall into the interval ⟨0, 1⟩. However, the proposed adjustment does not ensure this property, so some eventual correction is necessary. Moreover, it might be desirable to set a certain threshold for transition probabilities. The floor of 0.03 % that is set for PD in European Parliament and Council (2013b) within the framework of the credit risk capital requirements calculation will be considered here. To maintain the dynamics of the model and to avoid obtaining the extreme transition probabilities solely due to macroeconomic development, the mentioned floor will be used for all transition probabilities. Therefore, in this case, the allowed interval will not be ⟨0, 1⟩ but ⟨0.0003, 0.9997⟩.

In this simple example, where only two rating grades are considered, the correction (taking the mentioned floor into account) is straightforward and takes the form

$$p_{1,1}^{corr} = \min\left(\max\left(p_{1,1} - \frac{\Delta_{t+k} \cdot EAC}{2},\ \tau \right), 1 - \tau \right), \tag{11}$$

$$p_{1,2}^{corr} = \min\left(\max\left(p_{1,2} + \frac{\Delta_{t+k} \cdot EAC}{2},\ \tau \right), 1 - \tau \right), \tag{12}$$

where τ denotes the threshold (floor) – in this case $\tau = 0.0003$. A generalization of this computational framework is presented in the next subsection. Continuing with the example this paper works with, the calculation of the "economically adjusted" 3-year PD will be as follows:

$$\begin{aligned}\boldsymbol{s}_{t+3} &= (1\ 0) \cdot \begin{pmatrix} 0.9576 & 0.0424 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0.9590 & 0.0410 \\ 0 & 1 \end{pmatrix} \cdot \\ &\cdot \begin{pmatrix} 0.9591 & 0.0409 \\ 0 & 1 \end{pmatrix},\end{aligned}$$

$$\boldsymbol{s}_{t+3} = (0.8808\ 0.1192). \tag{13}$$

The original three-year PD was 11.53 %, and the adjusted value is 11.92 %. So, after the adjustment, the estimated PD is slightly higher. This is caused by the fact that the original PD was estimated in a relatively optimistic year when a 4.30 % GDP growth rate was observed. In the following three years the growth decreased roughly by 1–2 percentage points. It can be said that during optimistic times with the expected slight downturn in GDP growth, this adjustment helps to "filter" this optimism out and make the PDs more "real-time" in a sense that they take the future economic development into account. If a higher GDP growth rate than the present one is expected, then the adjustment would work in the opposite way and, naturally, the resulting PD would be lower.

As it was noted above, this concept can also be used for simulating an economic crisis and various stress testing exercises. For example, one might be interested in how the estimated three-year PD would look if a strong economic downturn was expected. For this purpose, the adverse forecast scenario of GDP growth can be utilized. In which case, the result would be $\boldsymbol{s}_{t+3} = (0.8620\ 0.1380)$. So the adjusted three-year PD would be 13.80 %. In this case, this adjustment works in compliance with the requirements of IFRS 9. It helps to estimate higher PDs; hence, higher expected credit losses will be calculated and higher loss allowances will be recognized. Therefore, banks will create higher "reserves" and will be better prepared to cover potential losses coming from the expected crisis. In the case of expected high economic growth, the logic would of course be reversed.

In a stricter case, where the whole estimated effect is attributed to increasing the transition probability to the default grade (thus PD), the formula (10) without division by two would be applied. The results (in %) are summarized in Tab. III.

III: *Summary of multi-period PD estimates (in %) under different assumptions*

time period	PD with no adjustment	PD (EAC: half effect)		PD (EAC: whole effect)	
		baseline	adverse	baseline	adverse
t+1	**4.00**	4.24	5.01	4.47	6.02
t+2	7.84	8.16	9.65	8.49	11.44
t+3	**11.53**	**11.92**	**13.80**	**12.32**	**16.03**
difference	–	0.39	2.27	0.79	4.50

The results correspond to what has been stated above. If the whole effect is attributed only to increasing the transition probability to the default state, the difference compared to the original PD would be significantly higher. In the case of two rating grades, it makes sense to also work with the whole effect; however, in the case of more rating grades (for example 6 as used below), this setting may be excessively strict since all transition probabilities to higher rating grades (not only the default grade) are increased. Therefore, the generalization below considers the first of the presented versions of the EAC decomposition.

It should also be noted that various macroeconomic scenarios can be defined and the "overall" effect would then be calculated as a probability-weighted average of the effects of individual scenarios.

Framework generalization

As an illustration, only two "rating grades" were considered in the example. However, the presented framework is flexible and can be extended to consider a virtually arbitrary number of rating grades. It would naturally depend on the character of the rating grades and their interpretation. Nonetheless, the underlying idea of the EAC could remain very similar. This section will present a more generalized framework for multi-period PD estimation incorporating the economic forecast, with examples using 6 rating grades (the lower the rating grade, the better) where the last one represents the default (D).

In the previous subsection, the effect of the EAC was decomposed into two parts – the first part (half) was used to adjust the non-default rating grade, the second part (half) was used to adjust the default rating grade. This logic will also be followed in the more generalized framework. However, it depends on what direction of the adjustment is desired to be emphasized. Therefore, to demonstrate the flexibility of the proposed framework, four possible alternatives that may be of interest will be illustrated.

Alternative I

The first half of the EAC effect is used for non-default grades adjustment, the second half of the EAC effect is used for a default rating grade adjustment. It is desired that the higher (worse) the rating grade, the bigger adjustment towards the default grade should be made. In this case, a uniform decomposition within both left (to the better rating grades) and right (to the worse rating grades) directions is considered (within non-default grades). This setting can be achieved by the adaptations below (notation as in (2) is maintained). Adjustment

$$p_{i,r} + \frac{\Delta_{t+k} \cdot EAC}{2(r-1)} \cdot \frac{2i-1}{r-1} \text{ for } i = 1,\ldots,r-1 \tag{14}$$

of the entries in the default column will ensure that the half of the EAC effect will be decomposed among them. Since this operation will form the basis for further operations, for the simplicity of notation the right side of (14) will be referred to as γ, thus (14) could be rewritten as $p_{i,r} + \gamma$. Formula

$$p_{i,j} + \frac{\gamma}{r-i-1} \text{ for } i,j < r-1 \text{ and } j > i \tag{15}$$

adjusts the entries above the diagonal (excluding those on the diagonal) in a sense that transition probabilities from the initial state to higher rating grades are increased by the same amount (uniformly). Adaptation

$$\begin{aligned} &p_{i,j} - 2 \cdot \frac{\gamma}{i} \text{ for } i,j < r-1 \text{ and } j \le i \\ &p_{i,j} - \frac{\gamma}{i} \text{ for } i = r-1 \text{ and } j \le i \end{aligned} \tag{16}$$

adjusts entries under and on the diagonal in the analogically opposite way as in the case of adjustment (15).

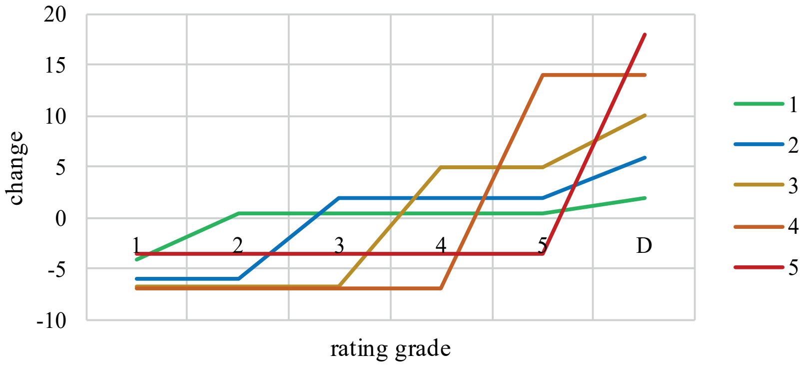
As it was mentioned above, for the sake of clarity, all of the considered alternatives will be illustrated using an example with 6 rating grades. Also, the forecasted downturn will be considered and it is assumed that EAC is negative. For illustrative purposes, in the examples it is assumed that $\Delta_{t+k} \cdot \text{EAC} = 100$ (for $k = 1,\ldots,n$). The focus here lies in proportional changes, not in absolute values. However, any value can be considered.

With the above-mentioned assumptions, changes in the transition matrix in the case of Alternative I can be summarized in Tab. IV.[5] Changes are

5 Since the default grade is absorbing, the "D" row is omitted.

IV: *Summary of changes in the transition matrix (Alternative I)*

	1	2	3	4	5	D
1	–4	0.5	0.5	0.5	0.5	2
2	–6	–6	2	2	2	6
3	–6.67	–6.67	–6.67	5	5	10
4	–7	–7	–7	–7	14	14
5	–3.6	–3.6	–3.6	–3.6	–3.6	18



4: *Changes in the transition matrix per individual rating grade (Alternative I)*

calculated according to equations (14), (15) and (16) with $p_{i,j}$ omitted.

From Tab. IV it can be seen that the adjustments follow the description from the beginning. It can also be observed that the sum of the changes in the default grade (column D, red box) is 50 (half of the total effect of 100) and the sum of the changes in non-default grades (the rest, blue box) is also 50 (with a negative sign). Thus, the total of the row changes remains 0 (which is of course required, otherwise the sum of probabilities in each row would not be equal to 1) and the total EAC effect is equally decomposed into non-default and default grades (in absolute values). The changes are also depicted in Fig. 4.

Within the generalized framework, it is also necessary to impose a certain correction mechanism to ensure that all transition probabilities fall into the desired interval. The correction mechanism presented here is generally applicable, and thus will not be repeated below. This mechanism can be summarized in the following three steps:

Step 1: Setting the interval the transition probabilities are allowed to fall into. Considering the certain threshold (floor) τ for all transition probabilities (for the same reason as in the previous section), this interval would be generally calculated as follows:

$$\langle 0 + \tau, 1 - (r - 1) \cdot \tau \rangle. \tag{17}$$

The reason for this method of calculation is that at most $r - 1$ transition probabilities may be equal to the floor but the remaining value must be a complement of 1. Therefore, in the considered example with the above-mentioned floor of 0.03 %, the interval would be $\langle 0 + 0.0003, 1 - (6 - 1) \cdot 0.0003 \rangle = \langle 0.0003, 0.9985 \rangle$.

Step 2: Replacement of all transition probabilities $p_{i,j}$ $(i,j = 1,\dots,r)$ that are lower than the floor with the floor value:

$$p_{i,j}^{corr} = \tau \text{ for } p_{i,j} < \tau \left(p_{i,j} \text{ otherwise}\right). \tag{18}$$

Step 3: Adjustment of all transition probabilities other than those adjusted in step 2 (i.e. all $p_{i,j} > \tau$) by the correction term

$$p_{i,j}^{corr} = p_{i,j} \cdot \frac{1 - \varphi \cdot \tau}{\sum_j p_{i,j} - \varphi \cdot \tau} \text{ for a given } i \text{ and } j = 1,\dots,r, \tag{19}$$

where φ is the number of transition probabilities corrected in Step 2 (the number of transition probabilities equal to the floor). This correction is made row by row, separately.

Alternative II

The first half of the EAC effect is used for non-default grades adjustment, the second half of the EAC effect is used for default rating grade adjustment. It is desired that the higher (worse) the rating grade, the bigger the adjustment towards the default grade should be made. In this case, the decreasing decomposition from the current grade in both directions is considered (within non-default grades).

Adjustment of the entries in the default column follows (14). The other adaptations are presented below. Formula

$$p_{i,j}+\frac{\gamma}{r-i-1}\cdot\frac{2(r-j-1)+1}{r-i-1}\ \text{for}\ i,j<r\ \text{and}\ j>i \qquad (20)$$

adjusts the entries above the diagonal (excluding those on the diagonal) in a sense that transition probabilities to higher rating grades from the initial state are increased in a decreasing manner. Adaptation

$$\begin{aligned} &p_{i,j}-2\cdot\frac{\gamma}{i}\cdot\frac{2(i-j)+1}{i}\ \text{for}\ i,j<r-1\ \text{and}\ j\le i \\ &p_{i,j}-\frac{\gamma}{i}\cdot\frac{2(i-j)+1}{i}\quad \text{for}\ i=r-1\ \text{and}\ j\le i \end{aligned} \qquad (21)$$

adjusts the entries under and on the diagonal in an analogically opposite way as in the case of adjustment (20). With the above-mentioned assumptions, changes in the transition matrix in the case of Alternative II can be summarized in Tab. V. Changes are calculated according to equations (14), (20) and (21) with $p_{i,j}$ omitted.

A similar description to Alternative I could be written. The difference is that the changes within the individual rows are not uniformly but decreasingly distributed (from the current grade). The changes are also depicted in Fig. 5.

Alternative III

In this alternative, the EAC effect is not divided between non-default and default grades, but between increasing transition probabilities to higher (worse) grades, and decreasing transition probabilities to lower (better) grades – both directions with a decreasing trend (from the current grade). In this alternative, it does not apply that half of the effect is applied to the default rating grade. Instead, the sum of "plus" changes (above the diagonal, red box in Tab. VI) equals the sum of "minus" changes (under and including the diagonal, blue box in Tab. VI). The sum of row changes is zero, so naturally the sum of transition probabilities is one.

First of all, new notation will be introduced. The following takes the transition matrix from Alternative II as a basis:

$$\sigma_i=\sum_{j=i+1}^{r}\delta_{i,j}\ , \qquad (22)$$

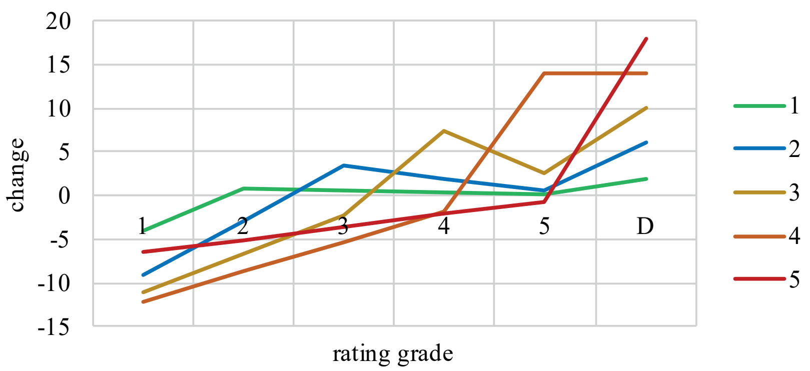
where $\delta_{i,j}$ represents "plus" changes associated with probabilities $p_{i,j}$, and thus σ_i denotes the sum of these changes in each individual row. The adaptations of transition matrix entries in Alternative III can then be written as follows. Formula

$$p_{i,j}+\frac{\sigma_i}{r-i}\cdot\frac{2(r-j)+1}{r-i}\ \text{for}\ i,j\le r\ \text{and}\ j>i \qquad (23)$$

adjusts the entries above the diagonal (excluding those on the diagonal) in a sense that changes in transition probabilities to higher rating grades from the initial state continuously decrease, including in the default grade. The entries under and on the diagonal remain the same as in Alternative II. With the above-mentioned assumptions, changes in

V: *Summary of changes in the transition matrix (Alternative II)*

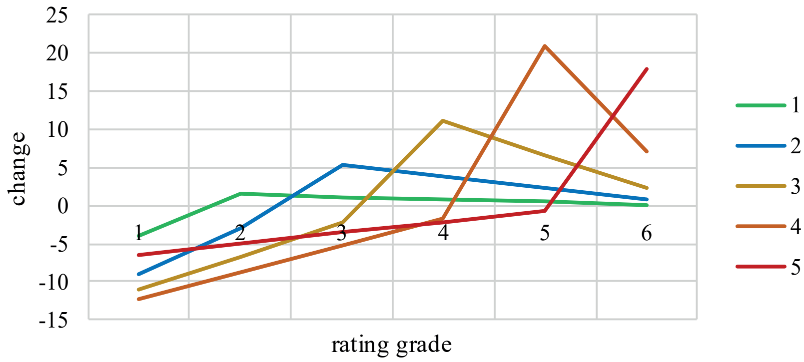
	1	2	3	4	5	D
1	-4	0.875	0.625	0.375	0.125	2
2	-9	-3	3.333	2	0.667	6
3	-11.11	-6.667	-2.222	7.5	2.5	10
4	-12.25	-8.75	-5.25	-1.75	14	14
5	-6.48	-5.04	-3.6	-2.16	-0.72	18



5: *Changes in the transition matrix per individual rating grade (Alternative II)*

VI: *Summary of changes in the transition matrix (Alternative III)*

	1	2	3	4	5	D
1	–4	1.44	1.12	0.8	0.48	0.16
2	–9	–3	5.25	3.75	2.25	0.75
3	–11.11	–6.667	–2.222	11.11	6.667	2.222
4	–12.25	–8.75	–5.25	–1.75	21	7
5	–6.48	–5.04	–3.6	–2.16	–0.72	18



6: *Changes in the transition matrix per individual rating grade (Alternative III)*

the transition matrix in the case of Alternative III can be summarized in Tab. VI. Changes are calculated according to equations (21) and (23) with $p_{i,j}$ omitted.

It can be seen that changes in transition probabilities decrease continuously in both directions further from the current state. The difference here is that also the default grade is included in these mechanics, so the EAC effect is not divided between non-default and default grades, but on increasing transition probabilities to higher grades (including the default grade) and decreasing transition probabilities to lower grades. The changes are also depicted in Fig. VI.

Alternative IV

The first half of the EAC effect is used for non-default grades adjustment, the second half of the EAC effect is used for default rating grade adjustment. It is desired that the higher (worse) the rating grade, the bigger adjustment towards the default grade should be made. In this case, an increasing decomposition from the current grade in both directions is considered (within non-default grades). Therefore, this alternative is the opposite of Alternative II.

Adjustment of the entries in the default column follows (14). The other adaptations are presented below. Formula

$$p_{i,j} + \frac{\gamma}{r-i-1} \cdot \frac{2(j-i-1)+1}{r-i-1} \text{ for } i,j<r \text{ and } j>i \qquad (24)$$

adjusts entries above the diagonal (excluding those on the diagonal) in a sense that transition probabilities to higher rating grades from the initial state continuously increase. For entries under and on the diagonal, adaptation (21) applies. With the above-mentioned assumptions, changes in the transition matrix in the case of Alternative IV can be summarized in Tab. VII. Changes are calculated according to equations (14), (21) and (24) with $p_{i,j}$ omitted.

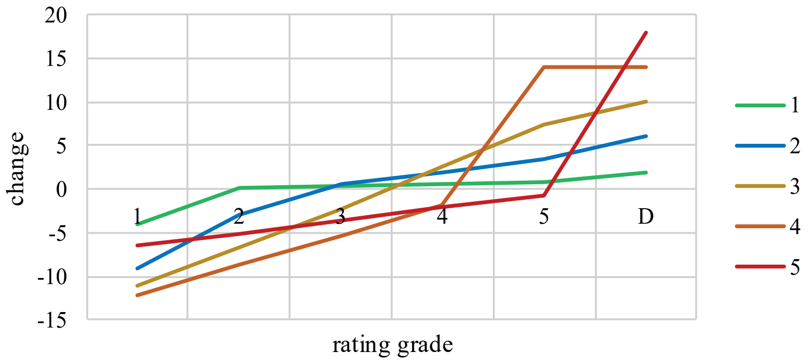
A similar description to Alternative II could be written. The difference is that the changes from the current state within the individual rows continuously increase, not decrease. The changes are also depicted in Fig. 7.

DISCUSSION

Incorporating macroeconomic (or, in general, external) information into PD estimation is a relatively modern approach and we can mention three relatively popular methodologies that can be found in the literature. The first one is based on the Merton-type models. Jakubík (2007) uses the approach of Merton for credit risk modelling and for stress testing of banks in the Czech Republic. This approach was also elaborated in Klepáč (2015) and Klepáč, Hampel (2015), where copulas were used to include information from financial markets for corporate PDs estimation, and in Pesaran *et al.* (2006), where the effects of macroeconomic shocks are investigated (among others). In Simons, Rolwes (2009) and Bruce, González-Aguado (2010) relations among macroeconomic indicators and default rates are explored with some applications in credit risk management. The second popular approach is based on the survival analysis framework. For example Duffie *et al.* (2007) use this approach to

VII: *Summary of changes in the transition matrix (Alternative IV)*

	1	2	3	4	5	D
1	–4	0.125	0.375	0.625	0.875	2
2	–9	–3	0.667	2	3.333	6
3	–11.11	–6.667	–2.222	2.5	7.5	10
4	–12.25	–8.75	–5.25	–1.75	14	14
5	–6.48	–5.04	–3.6	–2.16	–0.72	18



7: *Changes in the transition matrix per individual rating grade (Alternative IV)*

model corporate PDs and Bellotti, Crook (2009) to model default probabilities of credit card accounts. The third approach we can mention is based on Markov models and transition matrices. Fei *et al.* (2012) and Gavalas, Syriopoulos (2014) utilize this framework to analyse the relationship between transition probabilities (including default probabilities) and business cycles.

Given the main objective of this paper, it is not directly comparable with the mentioned studies. However, our paper contributes to the literature (especially to the lastly mentioned area) by proposing a straightforward and flexible two-step framework based on Markov models for multi-period PD estimation incorporating macroeconomic forecast. It is also possible to state that most of the mentioned studies deal with theoretical findings; verification is often based on simulations or case studies, like in this paper.

From a technical point of view, when using the presented methodology in practice, some difficulties may arise – especially with the availability of the official economic forecasts. The official forecasts of central banks, including the Czech National Bank (used in this paper), are usually made for "only" a few years ahead. On the other hand, reliability of longer forecasts would be low and their use would be questionable. For example, this issue needs to be tackled in the case of mortgage loans with a maturity highly exceeding the horizon of the available economic forecast. In any case, the solution would have to be based on a certain degree of simplification (for example by abstracting from the economic adjustment in larger time horizons or using long-run averages of economic variables).

As a basis for incorporating the economic forecast, the EAC was estimated by linear regression using the time series of a share of non-performing loans (NPL) in differences in and the growth of gross domestic product (GDP). In this case, yearly time series were used. To achieve higher precision in incorporating economic forecasts into PD estimation, time series with higher frequency (typically quarterly) may be used. In addition, more sophisticated time series models (e.g. vector autoregressions) including more economic variables may be utilized for EAC estimation.

One more important issue is that in most cases, the remaining maturity of a financial asset (typically a loan) is not exactly in years without any remainder (for example 3 years and 6 months). Therefore, the transition matrix needs to be estimated in a matching time horizon. This issue is addressed in the Annex, where a straightforward method of calculating the transition matrix for any time horizon is presented.

CONCLUSION

In this paper, a straightforward, flexible and intuitive computational framework for multi-period PD estimation incorporating macroeconomic forecast was proposed. The concept is based on Markov models, an estimated economic adjustment coefficient and official economic forecasts of the Czech National Bank. The economic forecast is taken into account in a separate step to better distinguish between idiosyncratic and systemic risk. This approach is also attractive from the interpretational point of view. The proposed framework can be used especially within IFRS 9 requirements (calculation of lifetime expected credit losses).

Moreover, due to the fact that most of the computations include working with matrices, the proposed computational framework is relatively easy to implement from a practical point of view in widespread software environments such as R or Matlab.

Looking at the presented alternatives, one may bring up an issue with row and column monotony towards the diagonal, which is a desired property of transition matrices (Bluhm *et al.*, 2003). Despite the fact that four alternatives were used to demonstrate the flexibility of the proposed mechanism, Alternative III can be considered as the most practical one, where – at the same time – the monotony issue should not be significant. However, to ensure monotony (especially when the initial transition matrix is calculated from historical data), an additional "smoothing" algorithm may be applied, which will be the subject of further research.

At the end it can be pointed out that this paper omits the assumption that PDs usually tend to decrease over time. For example, when a client repays his or her loan and continuously meets the contractual obligations, the closer the maturity, the lower the probability that this client defaults. Incorporation of this assumption into the presented framework (using real data) will also be the subject of future research.

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Annex: Transition matrix for any time horizon

A starting point is a calculation of logarithm of the given transition matrix and after some adjustments also its exponential. For some general matrix **X**, these calculations can be written using Taylor's expansion in a following manner:

$$\ln(\mathbf{X}) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(\mathbf{X}-\mathbf{I})^n}{n} = \mathbf{X}-\mathbf{I}-\frac{1}{2}(\mathbf{X}-\mathbf{I})^2+\frac{1}{3}(\mathbf{X}-\mathbf{I})^3- \cdots,$$

$$\exp(\mathbf{X}) = \sum_{n=0}^{\infty} \frac{\mathbf{X}^n}{n!} = \mathbf{I}+\mathbf{X}+\frac{1}{2}\mathbf{X}^2+\frac{1}{3!}\mathbf{X}^3+\cdots ,$$

where **I** is an identity matrix (ones on the diagonal, otherwise zeros). As Engelman, Ermakov (2011) shows – based on the theory of Markov chains – the transition matrix for any time horizon can be calculated if $\ln(\mathbf{P}_{t+1})$ is the so-called generator matrix (**G**). A matrix can be considered a generator matrix if the following three conditions are met:

- all diagonal entries are non-positive, i.e. $g_{i,i} \leq 0$ for $i = 1, \ldots, r$;
- all non-diagonal entries are non-negative, i.e. $g_{i,j} \geq 0$ for $i,j = 1, \ldots, r$, where $i \neq j$;
- row sums are equal zero, i.e. $\sum_{j=1}^{r} g_{i,j} = 0$ for $i = 1, \ldots, r$.

Based on **G**, the transition matrix for any time horizon $t + \theta$ can be calculated as

$$\mathbf{P}_{t+\theta} = \exp(\theta \cdot \mathbf{G}).$$

If, however, $\ln(\mathbf{P}_{t+1}) \neq \mathbf{G}$, which is the usual case, a technique called regularization can be used. The idea of regularization is to replace $\ln(\mathbf{P}_{t+1})$ with a similar matrix that meets the conditions of **G.** Kreinin, Sidelnikova (2001) proposed a simple and intuitive regularization algorithm that consists of three steps:

1. calculation of $\mathbf{G} = \ln(\mathbf{P}_{t+1})$;
2. replacement of all negative non-diagonal elements with zero;
3. adjustment of all other non-zero elements according to

$$g_{i,j} \leftarrow g_{i,j} - \left|g_{i,j}\right| \cdot \frac{\sum_{i=1}^{r} g_{i,j}}{\sum_{i=1}^{r} \left|g_{i,j}\right|} .$$

After this procedure, **G** meets the above-mentioned conditions and the transition matrix for any time horizon can be estimated.

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