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OPTIMIZATION OF THE MUNICIPAL WASTE COLLECTION ROUTE BASED ON THE METHOD OF THE MINIMUM PAIRING

Michal Petřík[1], Stanislav Bartoň[1]

[1] Department of Technology and Automobile Transport, Faculty of AgriSciences, Mendel University in Brno, Zemědělská 1, 613 00 Brno, Czech Republic

Abstract

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In the present article is shown the use of Maple program for processing of data describing the position of municipal waste sources and topology of collecting area. The data are further processed through the use of graph theory algorithms, which enable creation of collection round proposal.
In this case study is described method of waste pick-up solution in a certain village of approx. 1,600 inhabitants and built-up area of approx. 30 hectares. Village has approx. 11.5 kilometers of ride able routes, with approx. 1 kilometer without waste source. The first part shows topology of the village in light of location of waste sources and capacity of the routes. In the second part are topological data converted into data that can be processed by use of the Graph Theory and the correspondent graph is shown. Optimizing collection route in a certain graph means to find the Euler circle. However, this circle can be constructed only on condition that all the vertices of the graph are of an even degree. Practically this means that is necessary to introduce auxiliary edges – paths that will be passed twice. These paths will connect vertices with odd values. The optimal solution then requires that the total length of the inserted edges was minimal possible, which corresponds to the minimum pairing method. As it is a problem of exponential complexity, it is necessary to make some simplifications. These simplifications are depicted graphically and the results are displayed in the conclusion.
The resulting graph with embedded auxiliary edges can be used as a basic decision making material for creation of real collection round that respects local limitations such as one way streets or streets where is the waste collection is not possible from both sides at the same time.

Keywords: Graph theory, Eulerian circle, vertex, degree, weight matrix, computational complexity, minimal perfect matching, maple

INTRODUCTION

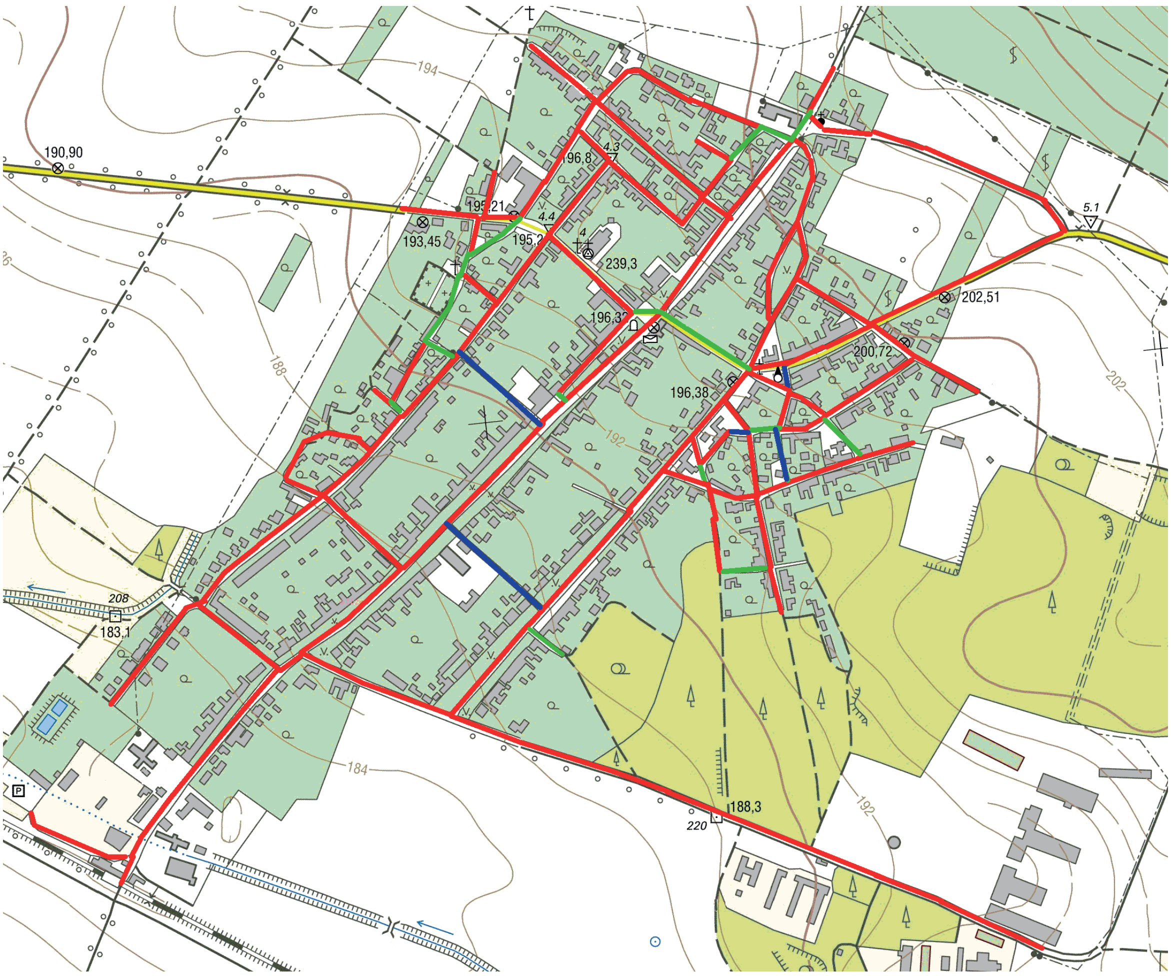
For the description of the collection vehicle will be used terminology of the Graph Theory, see (Diestel, 2010). Collecting car must pass all routes, where are the waste sources located. These routes are meeting each other at intersections. In terms of the Graph Theory, it is possible to consider these paths as edges of the graph and intersections as its vertices. Number of paths starting from one vertex – intersection – is called vertex degree. If we create a graph which matches to the position of intersections, and these are connected by certain paths, then the movement of the collection vehicle is equivalent to the problem of finding the Euler circle (Darling, 2004). If all the vertices of the graph are of an even degree, then the problem has a simple solution. If the graph has two vertices of an odd degree, then Euler circle cannot be created. It is possible to construct the Euler path, which starts in one of them and ends in the second (Cook, 2012). Problem might appear, if the corresponding graph has more than two vertices of an odd degree, which is the case of majority of the collection routes

MATERIALS AND METHODS

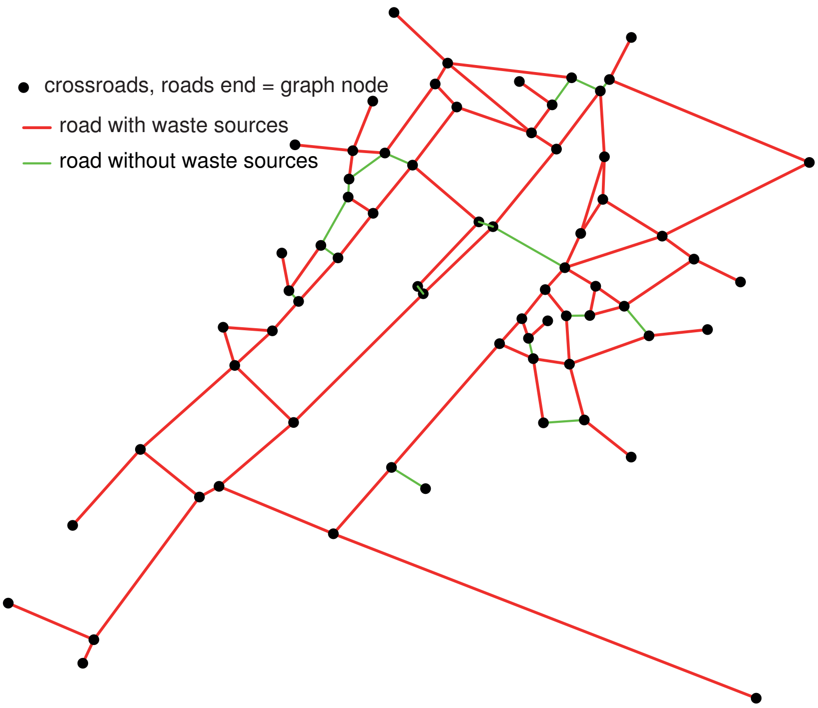
Creation of the Chart

The positions of locations of waste sources are entered in the GPS coordinates in Excel format. By use of program Maple it is possible to convert these coordinates to local rectangular coordinates, so that the origin of the coordinate system lies in the middle of the solved locations and the y-axis of the rectangular system is oriented north direction. The topology of the village can be described in a similar way by entering the coordinates of key points, namely intersections and blind paths endpoints, and creating adjacency matrix, thus, matrix that indicates the distances between the individual points. If two key points, eg. points *A* and *B*, are connected by a path, then the corresponding element of adjacency matrix has a value of positive value, which corresponds their real distance. In the case, there is no two-way communication, or in the case there is a two-way communication between two points, or there is the other way leading from point A to point B, then the matrix elements corresponding to the distance *[A, B]* and *[B, A]* will be of different values. In the case, there is no path between points, then the corresponding element adjacency matrix has a value of 0. Through the use of Maple program is also possible to make the identification of waste sites sources with the topology of the village – adjacency matrix, therefore, to distinguish roads of the village to those where is a waste source and those where is no waste source.

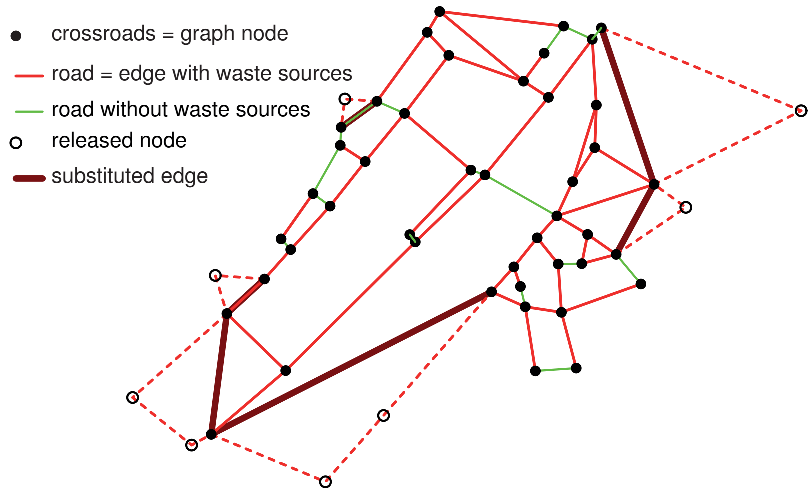
Thus, it is possible to construct graph easily. It would illustrate situation of the waste collection in the certain location. According to the graph theory, it is possible to define the edges of the graph, which correspond to the paths that are needed to be passed during the waste collection. It is also possible to assign to each graph vertex its degree – the number of edges that meet therein (Demel, 2002). Aside of these routes there are another ones in the reality – those without any waste. These routes are not included in the processed dataset. The real situation of the collection area is shown in Fig. 1. Red line shows the paths with waste, green line shows passable paths without waste sources, and blue line goes for impassable roads, which are not included in our calculations. As a cartographic input was used geo-portal ČÚZK (ČÚZK, 2016). Pick-up situation after conversion into graphic form is shown in Fig. 2. It is necessary to simplify this graph at the beginning.



1: *Pick-up situation*



2: *Graphical representation collecting situation*



3: *Simplified graph*

Simplifying Chart

The first step, all blind paths are removed from the graph. These routes must be passed, but do not offer any choice of collection route continuation. This means, that when passing the junction, which is the starting point of the blind route, it would be necessary return back to this junction.

Blind paths have no impact on the shape of the Euler circle. After overpassing the blind paths, new blind paths might appear. Therefore, this step is repeated until the resulting graph has no blind paths (Matoušek *et al.*, 2002). Paths without waste remain in the simplified graph as they can be used to form the Euler circle. Simplified graph, containing only the loops, is shown in Fig. 3. It is evident here that the number of vertices and graph edges decrease by 16. It is possible to draw-off vertices through which only two edges with waste sources are passing. We can replace them by one edge, which length will be sum of those two. This step can be repeated, until the resulting graph has vertices of minimal value of 3. Vertices which can be taken off are marked with the empty circles, the edges intended to be replaced are marked with the dashed lines, and the alternate edges are marked with the bold lines. Due to the graph vertices draw-off we can see a duplication of the graph edges in two cases, see Fig. 3. Therefore, the corresponding graph vertices will have a value of 1 higher than it is apparent from the graph at the first sight.

The next step is to decide on the appropriateness of using roads without waste. If the path without waste connects two vertices which are due to it of even degree, the path will be used. If the path without waste connects two vertices of odd degree, then by removing of this path corresponding vertices regain even degree. Paths with no waste, which then remain in the graph will be used for finding of the Euler line and therefore it is no longer necessary to distinguish between roads with and without waste in the following graph. Also nodes which are passing only two edges are deleted. If a solitary node is created in the graph, it would be draw-off as in the previous step. However, it is necessary to remember that the rank of the node is in terms of Euler line equals 2, because the edge leading into this node must be passed through twice. Therefore, we have to increase the value of the node +2 from which the edge leading to this node comes out. The resulting situation is shown in Fig. 4.

Minimum Pairing

It is evident from Fig. 4 that maximally simplified graph consists of 18 nodes of the odd degree. Therefore it is necessary to insert 9 auxiliary edges into this graph that connect these vertices. All vertices are of even degree and finding of the Euler circle for this graph will be already of simple matter.

Problem might be which nodes to connect with these auxiliary edges. If we have N number of points, then one can create $N/2$ pairs that can be combined in ways

$$\frac{N!2^{-\frac{N}{2}}}{\left(\frac{N}{2}\right)!},$$

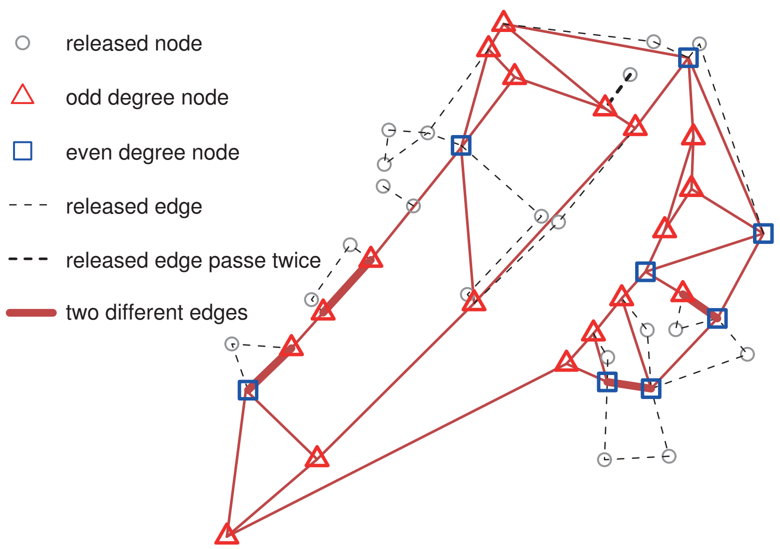
which for $N = 18$ gives 34,459,425 options. Therefore, such a problem cannot be solved by use of brute force, i.e. to write out all possible combinations and from them to find the one which leads to the total minimal length of inserted edges. Demand for the total minimal length of the inserted edges results from the requirement of creation of the Euler circle – all edges have to be passed through. Their total length is constant regardless of the order of passage. Therefore it is necessary to insert auxiliary edges into the graph, so that all nodes have even degree. In the real situation it means that some of the routes, auxiliary edges, will be necessary to pass twice. Demand for the optimization then means that the length of the twice passed routes must as short as possible. Finding connections of points with odd degree with the shortest possible total length is called the minimum pairing (Berge, 2001).

Simplifying Assumptions

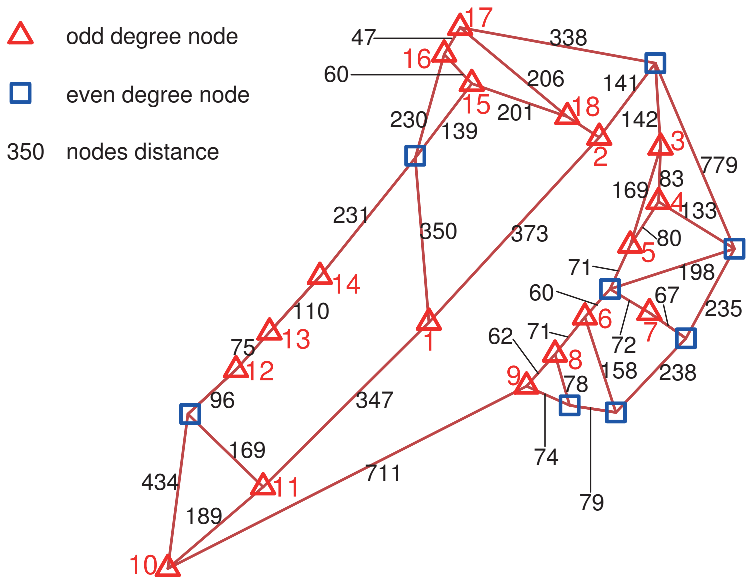
Regarding the very high number of possible pairs of the odd nodes connections it is necessary to consider the assumptions which reduce the number of their combinations. Inserted edges of the graph must follow the existing edges of the graph – there is no possibility to create new paths, therefore the assumptions are as follow:

1. Inserted edges can connect adjacent nodes.
2. If the inserted edges do not connect adjacent nodes, there could be max 1 node in between end nodes of the inserted edge.
3. Value of the inner node of the inserted edge must be even.
4. If it is possible to connect two odd nodes with more inserted edges, the shortest one is always chosen.
5. Inserted edges cannot be intersected.

We number odd degree graph nodes in the final graph. Based on the above mentioned assumptions, it is possible to create an adjacency matrix of odd nodes, see Tab. I and Fig. 5. If the path, fulfilling the



4: *Simplified graph of differentiated vertices of even and odd degrees*



5: *Final chart for minimum pairing*

I: *Adjacency matrix of the Fig. 5 graph's odd nodes*

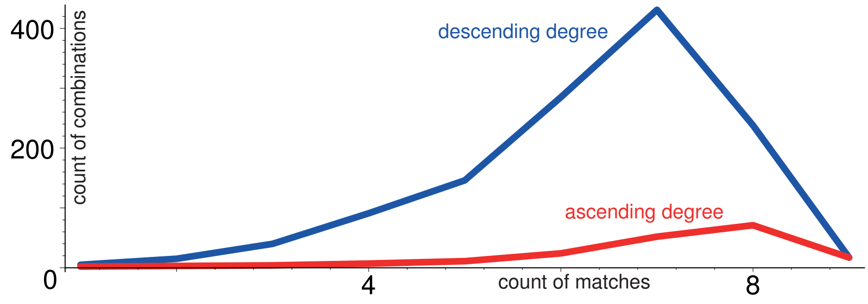
#	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1		373	∞	∞	∞	∞	∞	∞	∞	∞	347	∞	∞	581	489	580	∞	∞
2	373		283	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	479	58
3	∞	283		83	169	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	480	∞
4	∞	∞	83		80	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
5	∞	∞	169	80		131	143	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
6	∞	∞	∞	∞	131		132	71	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
7	∞	∞	∞	∞	143	132		∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
8	∞	∞	∞	∞	∞	71	∞		62	∞	∞	∞	∞	∞	∞	∞	∞	∞
9	∞	∞	∞	∞	∞	∞	∞	62		711	∞	∞	∞	∞	∞	∞	∞	∞
10	∞	∞	∞	∞	∞	∞	∞	∞	711		189	530	∞	∞	∞	∞	∞	∞
11	347	∞	∞	∞	∞	∞	∞	∞	∞	189		265	∞	∞	∞	∞	∞	∞
12	∞	∞	∞	∞	∞	∞	∞	∞	∞	530	265		75	∞	∞	∞	∞	∞
13	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	75		110	∞	∞	∞	∞
14	581	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	110		370	461	∞	∞
15	489	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	370		60	∞	201
16	580	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	461	60		47	∞
17	∞	479	480	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	47		206
18	∞	58	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	201	∞	206	
n	5	4	4	2	4	3	2	2	2	3	3	3	2	3	4	4	4	3

assumptions, between nodes does not exist, then the corresponding point of the adjacency matrix is ∞. If there are oriented edges in the graph – these edges would correspond to one-way roads in reality and then points of the adjacency matrix, corresponding to the distance of nodes *i*, *j*, and nodes *j*, *i*, would have been different. The last row of the Tab. I, which is marked red, corresponds to the number of possible inserted edges, which may come from a given node.

Algorithm

The task of the algorithm is to select from Tab. I from each column and row only one cell. The sum of the numbers from selected cells must be the lowest possible. The algorithm for finding of the minimum pairing works according to the following schedule:

1. Odd nodes are sorted in ascending order according to the number of possible inserted edges that can pass through them. Thus the node order in this case study is: *[4, 7, 8, 9, 13, 6, 10, 11, 12, 18, 2, 3, 5, 14, 15, 16, 17, 1]*.



6: *Number of possible combinations depending on the number of possible pairs*

2. It is evident from the Tab. I, that it is possible to connect the first node of the sequence –*4* with nodes *3* or *5*. Two possible connection will arise: *{4, 3}, {4, 5}*. In the first option, it is required to exclude node *3* from the possibility of creating additional joints. Node *5* is required to be excluded from the second option.
3. The second node in order – *7* offers the possibility of connection *{7, 5}* and *{7, 6}*. Therefore, possible connections are *{{4, 3}, {7, 5}}, {{4, 3}, {7, 6}}, {{4, 5}, {7, 5}}, {{4, 5}, {7, 6}}*. It is necessary to exclude combination *{{4, 5}, {7, 5}}* because each node can be used only once.
4. Similar approach is taken with the third node in order – 8. Number of combinations rises again, but some of them are excluded – *{{4, 3}, {7, 6} {8, 6}{}* and *{4, 5}, {7, 6}, {8, 6}}* – those ones where the node 6 is used twice. As more nodes are added gradually, number of possible connections rises. Also lots of them are eliminated as some nodes have been used twice.
5. This method continues until all nodes are used.

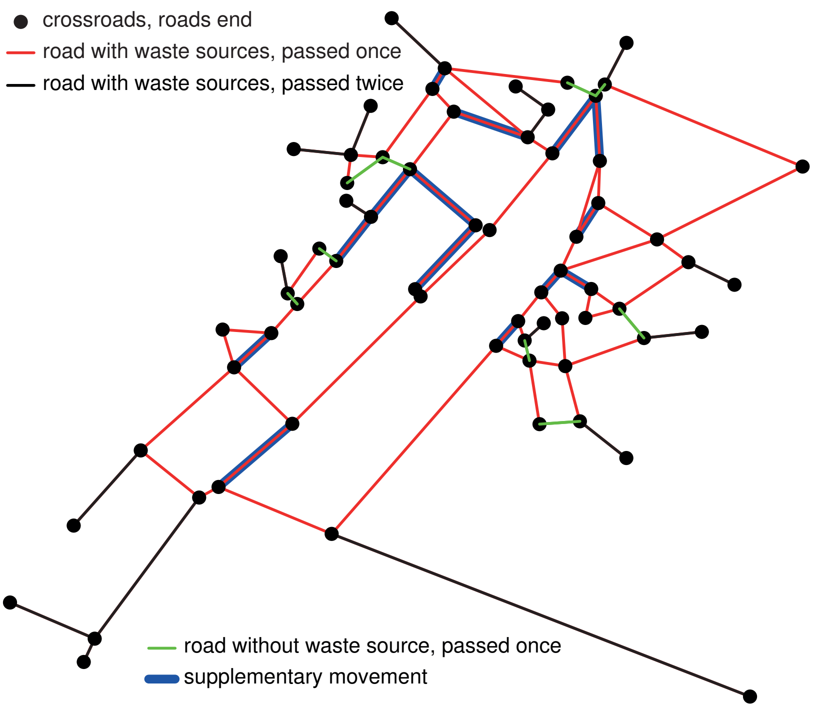
The number of possible combinations of the used nodes number is shown in the Fig. 6. This graph also demonstrates the importance of ascending order of nodes because with the descending order number of possible combinations created within the process of solving multiplies.

With the ascending order is the possible maximal number of combinations 75 for 8 pairs while for the descending order it is 409 combinations for 7 pairs.

RESULTS

There are 14 possible combinations as a result for all 9 possible pairs. Finding the minimum is a simple thing in the moment – as a result we receive pairs of embedded edges, which are connected by nodes *{1, 14}, {2, 3}, {4, 5}, {6, 7}, {8, 9}, {10, 11}, {12, 13} {15, 18}, {16, 17}*. The total length of the embedded edges is 1650 m.

Conversion of the minimum pairing result to chart of the collecting situation is illustrated



7: *Final cartage situation*

in Fig. 7. Generation of the Euler circle – the shortest collection route trajectory – is then of simple matter based on the Fleury Algorithm application (Harold *et al.*, 2007).

DISCUSSION

Theory of graphs deals with solution of this group of problems. In this case study, we are trying to find Euler circle in the graph, which generally contains more than two nodes of odd degree. Currently there are several algorithms developed that are applied in computer programs, e.g. (Lindo, 2016). Lingo program is used for solving similar problems, eg. travelling salesman problem (Lindodar, 2016), or planning flight routes (Lindofleet, 2016). As the closest solved problem appear to be Vehicle routing problem (Lindorout, 2016), or transportation problem (Lindowidgets, 2016), even though they do not implement minimum pairing methodology. The basic disadvantage of this program is very difficult import of data from Excel. As another possible program, suitable for solving this task, seems to be ArcGis Deskotp 10.01 According to the producer's documentation, design of Advanced version features extensive library of programs for optimizing trucks routes, (https://www.arcdata.cz/produkty/arcgis/desktopovy-gis/licencni-urovne). However, its Basic version costs 1500 USD, which is more than 1.5 times the price of the Maple program, (Arcgis, 2016). Therefore, the use of these programs seems to be less preferred.

It is also possible to use calculations by use of Internet connection to distant servers, e.g. (Optimap, 2015). Common feature of these applications is that they cannot simplify the problem. This means, that the problem of the original collecting situation, which contains a large number of nodes, is solved by these programs very slowly. For e.g. 55 nodes last these calculations up to tens of hours, or programs directly collapse. Only a few minutes of the computer time were needed for solution of the presented problem in the Maple program (Maple, 2011). The reason for the high speed problem solving in Maple program is the ability to work with data in a set-theoretical structure. This fact reduces the computational complexity significantly.

The principal advantage of using the Maple program is that it allows complex processing of the problem. It allows calculations and coordinates transformations, calculations of the lengths of the waste collection routes and processing of already transformed data. According to the theory of graphs and using its command library GRAPHTHEORY, so Maple enables gradual simplification of the problem, thus reduction of the number of nodes. Since the computational complexity of the problem is growing exponentially with the number of nodes, the possibility of simplification is of crucial character. A very important contribution of the MAPLE is the possibility of creating a vivid graphical output.

Converting solutions from a closed to an open issue, i.e. conversion of the Euler circle to the Euler move, is trivial. We only need to find two nearest nodes of odd degree to the arrival and departure route of the collection vehicle. Thereby, the number of nodes of odd degree is reduced by two, and we solve the same task as in the case of finding the closed path, but with lower computational demands.

The submitted algorithm is designed for solving problem of the minimum pairing, thus to find the minimum total length of the inserted auxiliary edges. Its output can be therefore input for finding the optimal real collecting routes, thus routes respecting eventual one-way paths. Algorithm solving design of this route will use outputs from the submitted algorithm and will be content of some of the future articles.

The algorithm which has been used is general and therefore it allows to solve even the most demanding tasks. Extent of the solved issue is the limited by the user's hardware and software equipment. Presented issue was solved by use of program Maple11 on the PC with operational system Windows7 64 bit, 2 GB of RAM. This configuration allows to find minimum pairing of 30–40 knots, each of odd degree, which represents municipality of approximately three times bigger size than is presented in our case. Since, in the present case is concerned quantity of waste which corresponds to the capacity of the collection vehicle, it can be stated that the used means and methodology correspond to demandingness of the solved problem. There is a need to divide municipality into smaller sections in case of larger municipality. These sections correspond with the waste production to the capacity of waste collection vehicle. These sublayers can be then solved by the submitted process.

CONCLUSION

Maple program appears to be highly effective computational tool for solving optimization problems. It is very easy to use it due to the internal data structure and command sets respecting the issue of the graph theory. It makes possible to realize a comprehensive analysis of the entire problem – from data input through its transformation, following simplifying, to the final solution. Moreover, it enables user throughout an easy control of calculation procedure both in numeric form and in particular graphically. Each step of solution is then possible to interpret in the graph form, as it is demonstrated in the present article.

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Contact information

Michal Petřík: pe3k.michal@gmail.com
Stanislav Bartoň: barton@mendelu.cz