ACTA UNIVERSITATIS AGRICULTURAE ET SILVICULTURAE MENDELIANAE BRUNENSIS

Volume 64 48 Number 2, 2016

http://dx.doi.org/10.11118/actaun201664020423

RELIABILITY ANALYSIS OF TEMPERATURE INFLUENCE ON STRESSES IN RIGID PAVEMENT MADE FROM RECYCLED MATERIALS

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Abstract

FLORIAN ALEŠ, ŠEVELOVÁ LENKA, ŽÁK JAROSLAV. 2016. Reliability Analysis of Temperature Influence on Stresses in Rigid Pavement Made from Recycled Materials. *Acta Universitatis Agriculturae et Silviculturae Mendelianae Brunensis,* 64(2): 423–432.

Complex statistical and sensitivity analysis of principal stresses in concrete slabs of the real type of rigid pavement made from recycled materials is performed. The pavement is dominantly loaded by the temperature field acting on the upper and lower surface of concrete slabs. The computational model of the pavement is designed as a spatial (3D) model, is based on a nonlinear variant of the finite element method that respects the structural nonlinearity, enables to model different arrangement of joints, and the entire model can be loaded by thermal load. Four concrete slabs separated by transverse and longitudinal joints and the additional structural layers including soil to the depth of about 3 m are modeled. The thickness of individual layers, physical and mechanical properties of materials, characteristics of joints, and the temperature of the upper and lower surface of slabs are supposed to be random variables. The simulation technique Updated Latin Hypercube Sampling with 20 simulations is used for the reliability analysis. As results of statistical analysis, the estimates of basic statistics of the principal stresses σ1 and σ3 in 106 points on the upper and lower surface of slabs are obtained. For sensitivity analysis the sensitivity coefficient based on the Spearman rank correlation coefficient is used. As results of sensitivity analysis, the estimates of influence of random variability of individual input variables on the random variability of principal stresses σ1 and σ3 are obtained.

Keywords: concrete, FEM, pavement, simulation, sensitivity, statistics, temperature

INTRODUCTION

Complex analysis of rigid pavements is often very difficult for design practice. Rheological properties of materials, cracking, joints, contact of concrete slabs in joints, contact of slab and subsequent material layer, temperature changes, non-homogenity of pavement base, water regime in the subgrade, environmental changes etc. influence serviceability of the structure in a decisive way. Moreover, the problem is complicated by the fact that the input data are generally random variables. Further uncertainties stem from their vagueness.

Taking into account the specific properties of the particular type of structure, the combination of the proper analytical model and modern simulation techniques seems to be an effective tool for the solution of the problem see Wojtkiewicz *et al.* (2010), Lee *et al.* (2010), Florian *et al.* (2011), Leonovich *et al.* (2013). The reliability analysis of a pavement using these methods provides the designer with reliability limits of the structural response and enables the determination of possible critical development. The results of the analysis also enable finding out which input variables require special attention due to their random variability dominantly influencing the structural behavior.

The behavior of the older type of rigid pavement formerly used in the Czech Republic is analyzed. This type of pavement is made from plain concrete, no dowels are used, and joints are made during laying of concrete. Dimensions of individual concrete slabs are 7.5×3.75 m, see Fig. 1. The structure is loaded by the self-weight of concrete slabs, by the thermal loading due to the temperature difference between the upper and the lower surface of the slab, and by the external load of 50 kN intensity at a distance of 0.25 m from the edge of the slab – see point 26 in Fig. 1. Thus the total state of stress in the slab results from all three different sources of load acting together. Contrary to the original former design, the base layer in this study is supposed to be made from a recycled material instead of a natural one. It is made from recycled concrete of fractions 0–16 mm.

The influence of uncertainties in input variables on the behavior of the pavement is respected in the analysis with help of numerical simulation techniques McKay *et al.* (1979). The simulation technique Updated Latin Hypercube Sampling with 20 simulations is used Florian (1992), Florian (2005). Total of 17 basic random input variables describing layer thicknesses, mechanical properties of materials, characteristics of joints and temperature on both surfaces of concrete slabs are taken into account in the study. They are described by the assumed cumulative distribution functions (generally three-parametric) and by the appropriate statistical parameters.

The statistical and sensitivity analysis of principal stresses σ1 and σ3 in concrete slabs is performed to show possibilities of reliability methods in the analysis of real pavement structures. The stresses are evaluated in 53 points on the upper and lower surface (106 points in total) of concrete slabs, see Fig. 1. Sign convention is chosen so that positive stresses are tensile, while the negative stresses are compressive. Principal stress σ1 represents an extreme value of tensile stress in the given point of the structure, while σ3 is an extreme value of compressive stress, that arise due to the spatial state of stress. Although the calculation and measurement of deflections on the pavement has the most important role in today's engineering practice, the calculation of stresses seems to be in fact much more important. Principal stress represents the extreme normal stress at a given point of the structure and thus it is the crucial characteristic which would be used in dimensioning process. If the principal stress exceeds the tensile strength of the material, a local tensile crack is created. If it exceeds the compressive strength of the material (only hypothetically for pavements), the material is locally crushed.

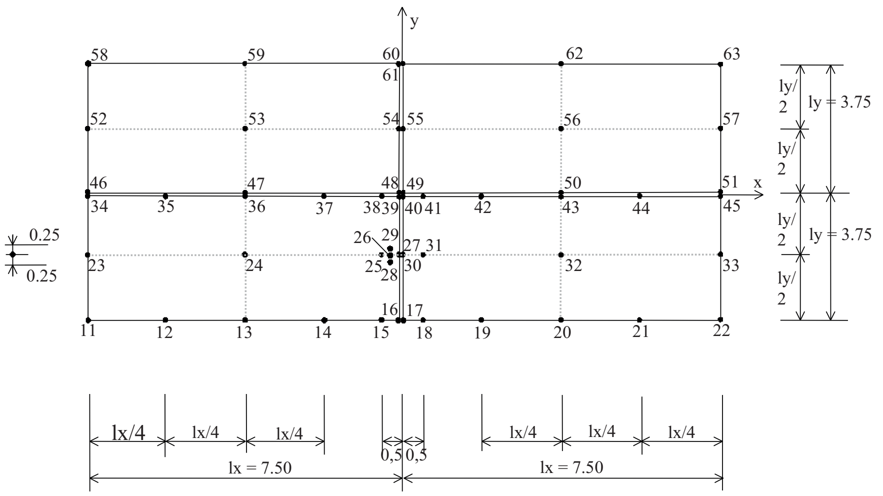
The result examples of statistical as well as sensitivity analysis of principal stresses are provided. These results are compared with the results obtained from deterministic analysis. The aim is to demonstrate the practical applicability of reliability methods in analyzing pavement behavior, their possibilities to supplement or even replace experimental methods, the nature of the information obtained, and finally, the possibility to quantify the reliability of pavements.

MATERIALS AND METHODS

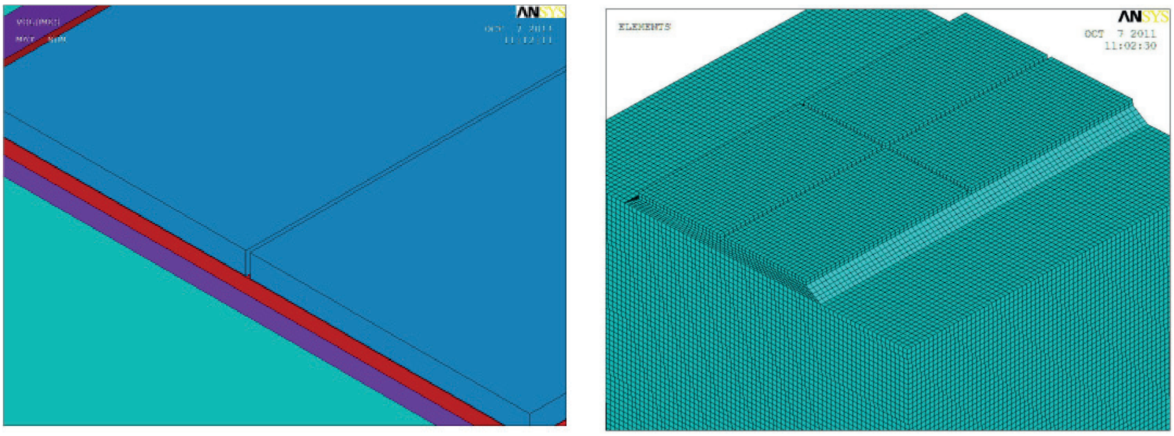
Analytical Model

The computational model is based on the nonlinear finite element method. It is developed in *ANSYS* system. Four concrete slabs, all other layers and longitudinal and transverse joints are modeled as 3D space – see Fig. 2. Joints, contact of slabs in joints, contact of slabs and subsequent layer, and the thermal loading are modeled in detail. Its main features are:

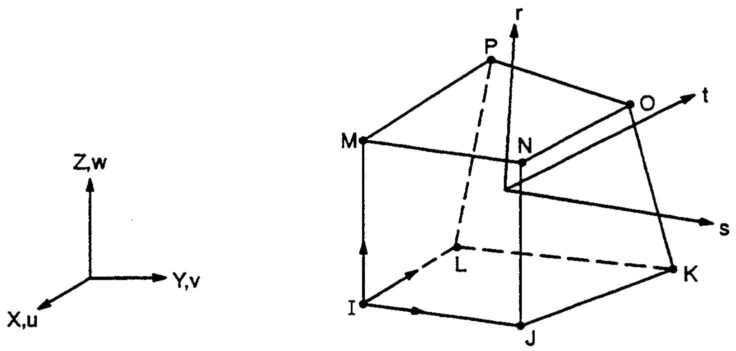
- parametric model, open and flexible,
- nonlinear model that respects the structural nonlinearity,
- designed as a spatial (3D) model that models the four adjacent concrete slabs with subsequent material layers and the surrounding soil,
- allows to model different arrangement of joints,



1: *Points for evaluation of stresses*



2: *Details of FE model*



3: *BRICK45 finite element*

- interaction of adjacent slabs is realized with help of special contact elements which prevent the transmission of tensile stress,
- contact between a slab and a subsequent layer is also realized using the contact elements, which allow realistic modeling of lifting of corners and the centre of the slab,
- each material layer can be of any thickness and of any material,
- pavement can be loaded by thermal loading,
- external load can be applied at any point of the slab,
- displacements, rotations, strains, normal or shear stresses etc. can be evaluated at any point in any layer of the pavement structure.

The geometry of the structure is modeled by finite elements BRICK45, see Fig. 3. It is the eight-mode element with three degrees of freedom (UX, UY, UZ) in each node.

Attention is paid to modeling of concrete slab contact with subsequent material layer and interaction of adjacent slabs in joints. In these areas the so-called contact problem occurs, in which tensile stresses cannot be transmitted. This is the case of structural nonlinearity and the pavement modeling therefore becomes nonlinear. Thus the numerical solution is divided into individual iteration steps and the Newton-Raphson method is used in each step. The actual contacts are modeled with a help of special contact elements. The contact element connects appropriate nodes of finite element mesh and operates in compression as the imaginary spring with given stiffness. But if it is pulled, the nodes behave independently and tensile stresses are not transmitted.

The proposed model of joints tries to simulate at least some of the complex phenomena that occur there. The adjacent slabs interact due to their mutual contact, due to the material in joints, and also due to stress transmitted through the other layers of the structure.

Reliability Analysis Techniques

The simulation technique Updated Latin Hypercube Sampling Florian (1992), Florian (2005) with 20 simulations is used for the reliability analysis. It is an improved variant of Latin Hypercube Sampling McKay *et al.* (1979). The method keeps the methodology of Latin Hypercube Sampling, but uses an improved strategy of generating input samples based on specially modified tables of random permutations of rank numbers. The modified tables consist of random permutations that are mutually statistically independent. The use of Updated Latin Hypercube Sampling generally results in a further increase in accuracy, quality and reliability of the results obtained from the reliability analysis. The detailed description of Updated Latin

Hypercube Sampling can be found in Florian (1992), Florian (2005).

To measure the relative influence of random variability of each input variable on the random variability of the output (principal stresses in 106 points in concrete slabs), the sensitivity coefficient based on the Spearman rank correlation coefficient is proposed Florian and Navrátil (1993). It is not limited to the linear relationship like the Pearson correlation coefficient. The sensitivity coefficient is defined as

$$r_s = 1 - \frac{6\sum_i d_i^2}{N(N-1)(N+1)}, \quad (1)$$

where r_s is the sensitivity coefficient among the k-th input variable and the output, d_i is the difference between the rank numbers of the k-th input variable and the rank numbers of the output, and N is the number of simulation.

The sensitivity coefficient ranges within interval −1 to +1. The higher is the coefficient (in absolute value), the higher is the sensitivity of the output on the appropriate input variable. The sign of the coefficient indicates positive or negative influence. The sensitivity coefficient lower than 0.30 (in absolute value) can be explained as practically no influence, higher than 0.30 as a low influence, higher than 0.50 as a moderate influence, higher than 0.70 as a high influence, and the sensitivity coefficient higher than 0.90 as a dominant influence.

Input Random Variables

Total of 17 variables are considered to be random input variables, see Tab. I (units in MPa, mm, °C). Their statistical parameters are carefully evaluated taking into account the data obtained from in-situ measurements, experimental tests, data from technological handbooks and scientific publications and the corresponding standards.

The influence of the current level of construction process and technological discipline is also taken into account. To derive appropriate statistical parameters of input variables (with the exception of Young modulus of recycled concrete used in base layer of the pavement), the following procedure is utilized. At first, the limits are specified (minimum, maximum and mean value) in which input variables will occur with a high probability, see Tab. I. Then, based on the assumption that values smaller than the minimum value and higher than the maximum value can occur only with a low probability, and choosing the appropriate probability distribution function (N – normal, LN – three-parametric lognormal, TN – truncated normal), the other required statistical parameters are determined – coefficient of variation (COV) and skewness. The normal and truncated normal CDF are used for symmetrically distributed variables, the three-parametric lognormal CDF for the other case. Statistical parameters of Young modulus of recycled concrete are based on the statistical evaluation of data obtained from laboratory tests Florian *et al.* (2015). For simplicity, the mutual statistical independence of input variables is considered with the following exception – the temperatures of the upper and the lower surface of the slabs are supposed to be fully statistically dependent.

The derived statistical parameters of input variables used in the presented study take into account uncertainties due to their random

I: *Statistical parameters of input variables*

No.	Layer	Input Variable	Mean	Min	Max	PDF	COV	Skewness
X1	Concrete Slab	Thickness	220	180	250	LN	0.09	–0.6
X2		Young modulus	37500	30000	45000	N	0.12	0.0
X3		Poisson's coefficient	0.20	0.19	0.21	N	0.02	0.0
X4	Base	Thickness	200	150	250	N	0.15	0.0
X5		Young modulus	150	-	-	N	0.40	0.0
X6		Poisson's coefficient	0.30	0.25	0.35	N	0.08	0.0
X7	Sub-base	Thickness	250	150	650	LN	0.24	1.0
X8		Young modulus	120	80	200	LN	0.18	0.9
X9		Poisson's coefficient	0.30	0.25	0.35	N	0.07	0.0
X10	Subgrade	Young modulus	80	30	150	LN	0.32	0.5
X11		Poisson's coefficient	0.35	0.30	0.45	LN	0.075	0.9
X12	Joints	Width transversal	20	15	25	N	0.15	0.0
X13		Width longitudinal	1.5	0	3	TN	0.75	2*
X14		Young modulus	150	50	1000	LN	0.39	1.0
X15		Coefficient of friction	0.5	0.1	0.9	N	0.34	0.0
X16	Temperature	Upper surface	11	–10	40	LN	0.95	0.5
X17		Lower surface	10	5	20	LN	0.25	0.9

* truncation parameter

nature and also the uncertainties due to our incomplete knowledge of the structure, insufficient experimental research, modeling errors, vagueness of input data etc.

RESULTS OF STATISTICAL ANALYSIS

The statistical analysis provides us with the following statistics of principal stresses in individual points on concrete slabs:

- mean (MPa),
- standard deviation (MPa),
- coefficient of variation (dimensionless),
- skewness (dimensionless),
- minimum and maximum value (MPa),
- suitable type of probability distribution function (PDF).

The mean value describes the average tendency of stresses, the standard deviation and the coefficient of variation describe their variability, skewness indicates the asymmetry of their population under and above the mean value, and finally the minimum and maximum values describe the possible interval within the stresses can occur. In addition, the deterministic analysis (DA) with input variables set to their nominal (mean) values is performed.

The suitable type of probability distribution is chosen with help of the comparative tests from a set of competing distributions Florian and Novák (1988). In our study this set includes normal (N), three-parametric lognormal (LN), truncated normal (TN), three-parametric Weibull (W) and three-parametric Pearson III (P3) probability distribution. Based on the chosen distribution, the 1, 5, 95 and 99% quantiles are determined, which are usually the most important ones in civil engineering practice. Quantiles allow making probabilistic conclusions. E.g. for 1% quantile, the 1% probability exists that the value is less than the quantile, while the 99% probability exists that the value is larger than the quantile.

Principal Stress σ1

Illustrative results of statistical analysis of principal stress σ1 (maximal tensile stress) in some important points on the lower surface of concrete slabs are shown in Tab. II. In Tab. III and Tab. IV, the 1, 5, 95 and 99% quantiles in some important points on the upper and lower surface are presented.

Nominal values of stress obtained from deterministic analysis (DA) on the lower surface have the character of tensile stress in all points, while on the upper surface there are both tensile and compressive stresses as well. Nominal values are within the interval 0.3 to 1.6 MPa on the lower surface and within the interval –0.05 to 1.1 MPa

II: *Statics of principal stress σ1 in points on lower surface (MPa)*

Point	Mean	DA	Standard Deviation	Skewness	Minimal Value	Maximal Value	PDF
11	0.7370	0.5813	0.3382	1.00	0.3421	1.5845	W
13	0.5061	0.3325	0.5541	0.48	0.0000	1.4701	TN
16	0.6369	0.5030	0.3148	0.70	0.2463	1.3530	W
17	0.6724	0.5115	0.3216	0.66	0.2704	1.3913	W
20	0.5092	0.3397	0.5567	0.47	–0.0002	1.4795	TN
23	0.9142	0.3274	0.7751	1.49	0.2189	2.9583	W
24	0.7045	0.3630	0.8385	0.76	–0.0247	2.3967	W
25	1.5225	1.2201	1.1264	0.96	0.2825	3.9933	W
27	1.9464	1.5362	1.0752	1.21	0.8909	4.5842	W
29	1.5940	1.1856	1.0623	1.25	0.4106	4.2867	W
30	1.0685	0.5631	0.8921	1.22	0.2395	3.2154	W
32	0.6980	0.3818	0.8218	0.70	–0.0267	2.3163	W
33	0.9134	0.3274	0.7735	1.49	0.2191	2.9505	W
34	1.2611	0.5362	0.9877	2.12	0.3847	4.6728	P3
36	0.8097	0.3389	0.9592	1.24	–0.0023	3.2137	W
39	1.2062	0.5400	1.0671	2.31	0.3844	4.9698	P3
40	1.2682	0.5678	1.1120	2.41	0.4303	5.2694	P3
43	0.8098	0.3478	0.9598	1.22	–0.0703	3.1888	W
45	1.2592	0.5362	0.9867	2.12	0.3844	4.6630	P3
48	1.1950	0.4924	1.0666	2.30	0.3529	4.9539	P3
49	1.2164	0.4862	1.1135	2.37	0.3546	5.1952	P3
51	1.2578	0.5433	0.9794	2.12	0.3897	4.6386	P3
58	0.4931	0.5258	0.5774	0.00	–0.7656	1.6383	N
63	0.4933	0.5257	0.5772	0.00	–0.7643	1.6381	N

III: *Quantils of principal stress σ1 in points on upper surface (MPa)*

Point	Quantil			
	1%	5%	95%	99%
13	–0.6887	–0.5134	2.4526	3.4121
20	–0.6764	–0.4849	2.1508	2.9288
24	–0.7451	–0.5654	2.9138	4.1053
32	–0.7391	–0.5495	2.6938	3.7477
36	–0.7971	–0.6184	2.8331	4.0139
37	–0.6756	–0.5263	2.3422	3.3214
43	–0.7975	–0.6100	2.6885	3.7735
47	–0.8048	–0.6156	2.7387	3.8455
50	–0.7981	–0.6089	2.6800	3.7563
53	–0.7395	–0.5508	2.6987	3.7577
56	–0.7383	–0.5498	2.6916	3.7473
59	–0.6698	–0.4870	2.1672	2.9699
62	–0.6711	–0.4881	2.1730	2.9784

IV: *Quantils of principal stress σ1 in points on lower surface (MPa)*

Point	Quantil			
	1%	5%	95%	99%
25	–0.1283	0.0681	3.6595	4.8601
26	0.3349	0.4904	4.0570	5.3658
29	0.2143	0.3382	3.6598	4.9556
30	–0.1019	0.0058	2.8003	3.8764
34	0.3356	0.3648	3.2390	4.8831
36	–0.4398	–0.3268	2.6739	3.8400
39	0.2854	0.3033	3.3503	5.2175
40	0.3481	0.3616	3.5050	5.5008
45	0.3320	0.3617	3.2348	4.8744
46	0.3436	0.3717	3.2251	4.8650
47	–0.4314	–0.3228	2.6544	3.8259
50	–0.4327	–0.3226	2.6602	3.8286
51	0.3409	0.3698	3.2190	4.8501

on the upper surface. In both cases, the nominal values are close to the minimum values obtained from statistical analysis. Mean values of stress on both surfaces in all points have the character of tensile stress and are generally always greater than the nominal values. They are within the interval 0.4 to 2.0 MPa on the lower surface and within the interval 0.2 to 1.4 MPa on the upper surface. The largest nominal as well as mean values reach their maximum at points near the external load application.

The interval in which principal stress σ1 may occur is considerable high – see the interval between the minimum and maximum values in Tab. II. Larger tensile stresses and thus higher probability of tensile cracks occurring arise on the lower surface of concrete slabs. In the case of the lower surface, the maximum values are up to 5.3 MPa, in the case of upper surface they are up to 3.3 MPa. The largest tensile stresses on the lower surface arise in points near the external load application (points 25–29), in points on all slabs near the longitudinal joint (points 33–51) and especially in the corners of slabs (points 34, 39, 40, 45, 46, 48, 49, 51). The largest tensile stresses on the upper surface arise in points near the external load application (points 26, 30) and in points on the transverse axis of symmetry of all slabs (points 13, 14, 20, 24, 36, 37, 43, 47, 50, 53, 56, 59, 62).

The stresses in all points show considerable variability and also nonzero skewness. Skewness has (with some exception) a positive sign and in some points is quite large. As a result of significant positive skewness, the often used normal probability distribution (a priori assuming zero skewness) is not suitable to describe the random variability of principal stress σ1. Weibull probability distribution seems to be the most appropriate.

The stresses in some points on the upper but especially on the lower surface reach magnitude where the creation of tensile cracks is highly probable. Taken into account the tensile strength of standard quality concrete, there is at least probability of 1% of tensile crack creation on the upper surface, see Tab. III, and at least probability of 5% of tensile crack creation on the lower surface, see Tab. IV.

Principal Stress σ3

Illustrative results of statistical analysis of principal stress σ3 (maximal compressive stress) in some important points on the upper surface of concrete slabs are shown in Tab. V. The nominal values of stress obtained from deterministic analysis (DA) on the upper surface have the character of compressive stress in all points, while on the lower surface there are also minimal tensile stresses in points near the external load application (points 28, 30). Nominal values are within the interval –0.1 to –2.2 MPa on the upper surface and within the interval 0.1 to –0.5 MPa on the lower surface. In both cases, the nominal values are close to the minimum values obtained from statistical analysis. Mean values of stresses on both surfaces have the same character as nominal values but they are generally greater. They are within the interval –0.7 to –2.8 MPa on the upper surface and within the interval 0.1 to –1.0 MPa on the lower surface.

The interval in which principal stresses σ3 may occur is considerably high – see the interval between the minimum and maximum values in Tab. V. Larger compressive stresses arise on the upper surface of concrete slabs. In this case, the maximum values are up to –7.3 MPa, in the case of lower surface they are up to –4.9 MPa. The largest compressive stresses on the upper surface arise in points on the transverse axis of symmetry of all slabs and in points close the contact of all slabs (points 13, 20, 24, 32, 38, 41, 47, 50, 53, 56, 59, 62). On the lower surface the largest compressive stresses arise in some corners of slabs (points 11, 22, 58, 60, 61, 63).

The stresses in all points show considerable variability and also nonzero skewness. Skewness in points on the upper surface has (with some

V: *Statics of principal stress σ3 in points on upper surface (MPa)*

Point	Mean	DA	Standard Deviation	Skewness	Minimal Value	Maximal Value	PDF
13	–1.1498	–0.1085	1.8849	–1.90	–7.0537	0.0006	W
14	–1.0367	–0.1303	1.7250	–1.94	–6.4393	–0.0004	W
15	–0.8540	–0.0709	1.6205	–2.19	–6.2834	0.0361	N
18	–0.8531	–0.0948	1.6029	–2.18	–6.2052	0.0377	P3
19	–1.0084	–0.1006	1.6986	–1.95	–6.3279	0.0008	W
20	–1.1538	–0.1115	1.8875	–1.90	–7.0586	0.0009	W
24	–1.2252	–0.3183	1.8835	–1.72	–6.7689	–0.0010	W
25	–1.3254	–1.0120	1.5899	–1.49	–6.1474	0.3362	W
26	–2.7713	–2.2155	0.9942	–2.14	–6.0725	–2.1881	P3
27	–1.5332	–1.2314	1.0052	–1.07	–3.9730	–0.5648	W
28	–1.3070	–1.0131	1.2482	–1.46	–5.0746	–0.0603	W
29	–1.2657	–1.0134	1.3311	–1.26	–5.0886	0.1848	W
32	–1.2302	–0.3129	1.8858	–1.71	–6.7494	–0.0010	W
36	–1.1521	–0.1188	1.8827	–1.70	–6.5366	0.1294	W
37	–1.0582	–0.1822	1.7849	–1.88	–6.5716	0.1092	W
38	–0.9452	–0.1694	1.8066	–2.33	–7.2465	0.1411	P3
39	–0.8460	–0.1127	1.5618	–2.51	–6.4681	–0.0002	N
40	–0.8791	–0.1501	1.5636	–2.46	–6.4586	–0.0372	N
41	–0.9365	–0.1826	1.7701	–2.31	–7.0709	0.1203	P3
42	–1.0284	–0.1471	1.7414	–1.86	–6.3358	0.1091	W
43	–1.1512	–0.1240	1.8731	–1.68	–6.4679	0.1265	W
47	–1.1749	–0.1196	1.9291	–1.71	–6.7266	0.1285	W
48	–0.8307	–0.0707	1.5352	–2.49	–6.3252	–0.0362	N
56	–1.2357	–0.2485	1.9122	–1.71	–6.8331	–0.0010	W

exception) the negative sign and in some points is quite large. Weibull probability distribution seems to be the most appropriate to describe random variability of principal stress σ3.

RESULTS OF SENSITIVITY ANALYSIS

Principal Stress σ1

Sensitivity analysis of principal stress σ1 (maximal tensile stress) is performed in all points on the upper and lower surface of concrete slabs. Stresses on the both surfaces are generally affected by random variability of the input variables differently. Illustrative results are presented for some important points on the lower surface only, see Tab. VI.

Some points on the lower surface are influenced only by the minimum number of input variables. The stress in points near the external load application, the centers of slabs and the longitudinal edge of slabs (points 12–14, 20, 24–29, 32, 37, 38, 53, 56, 58, 63) are influenced only by two or three variables. The largest number of input variables – eight – has an influence in the outer corners of the loaded slab as well as the adjacent slab (points 11, 16, 22).

Most points on the lower surface are influenced by the temperature field acting on the upper (X16) and lower (X17) surface of the concrete slabs. Influence of these variables can be considered as dominant. Influence of the modulus of elasticity of the joints material (X14), Poisson coefficient for sub-base layer (X9), and the width of transverse joints (X12) can be generally considered as moderate, in the case of the modulus of elasticity and the joint width in some points as high. Low influence show the modulus of elasticity of subgrade layer (X10), the thickness, the modulus of elasticity and Poisson coefficient of concrete slabs (X1, X2, X3), the thickness of base layer (X4), and the coefficient of friction in joints (X15).

Principal Stress σ3

Sensitivity analysis of principal stress σ3 (maximal compressive stress) is performed in all points on the upper and lower surface of concrete slabs. Stresses on the both surfaces are generally affected differently by random variability of the input variables. Illustrative results are presented for some important points on the lower surface only, see Tab. VII.

In the points on the upper surface the influence of temperature field acting on the upper (X16) and lower (X17) surface of the concrete slabs are dominant. Influence of other variables can be considered as minimal or negligible.

VI: *Sensitivity coefficients of principal stress σ1, lower surface*

Point	Input Variable																
	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14	X15	X16	X17
11	0.10	0.20	0.00	**0.43**	0.02	0.09	0.20	0.26	0.24	0.22	0.13	**0.41**	0.06	**0.41**	0.00	**0.46**	**0.46**
16	0.10	0.24	0.00	**0.35**	0.08	0.16	0.15	0.17	0.24	0.21	0.00	**0.51**	0.00	**0.44**	0.03	**0.42**	**0.42**
17	0.16	0.21	0.01	0.28	0.08	0.18	0.18	0.20	0.20	0.18	0.03	**0.57**	0.03	**0.44**	0.10	**0.39**	**0.39**
22	0.10	0.20	0.00	**0.43**	0.02	0.09	0.20	0.26	0.24	0.22	0.13	**0.41**	0.06	**0.41**	0.00	**0.46**	**0.46**
23	0.14	0.06	0.04	0.17	0.02	0.03	0.12	0.10	0.34	0.24	0.02	0.29	0.05	0.23	0.05	**0.69**	**0.69**
24	0.09	0.14	0.01	0.06	0.08	0.01	0.04	0.04	0.02	0.02	0.01	0.04	0.06	0.04	0.02	**0.99**	**0.99**
26	**0.31**	0.06	0.08	0.09	0.11	0.11	0.04	0.02	0.07	0.00	0.05	0.11	0.00	0.03	0.00	**0.93**	**0.93**
28	0.22	0.03	0.08	0.13	0.04	0.13	0.02	0.05	0.06	0.09	0.06	0.13	0.03	0.07	0.01	**0.93**	**0.93**
32	0.11	0.14	0.02	0.06	0.11	0.01	0.04	0.06	0.03	0.01	0.04	0.03	0.06	0.04	0.03	**0.99**	**0.99**
33	0.14	0.06	0.04	0.17	0.02	0.03	0.12	0.10	**0.34**	0.24	0.02	0.29	0.05	0.23	0.05	**0.69**	**0.69**
34	0.13	0.09	0.13	0.13	0.02	0.00	0.19	0.20	0.18	0.12	0.01	**0.51**	0.03	**0.54**	0.16	**0.42**	**0.42**
39	0.15	0.10	0.12	0.18	0.01	0.01	0.15	0.11	0.17	0.11	0.03	**0.43**	0.05	**0.55**	0.14	**0.53**	**0.53**
40	0.16	0.12	0.10	0.16	0.00	0.02	0.14	0.14	0.17	0.13	0.02	**0.43**	0.04	**0.56**	0.10	**0.52**	**0.52**
45	0.13	0.09	0.13	0.13	0.02	0.00	0.19	0.20	0.18	0.12	0.01	**0.51**	0.03	**0.54**	0.16	**0.42**	**0.42**
46	0.12	0.09	0.14	0.14	0.03	0.00	0.19	0.20	0.16	0.10	0.03	**0.51**	0.02	**0.55**	0.18	**0.42**	**0.42**
48	0.15	0.09	0.18	0.15	0.03	0.03	0.20	0.18	0.17	0.09	0.03	**0.47**	0.06	**0.53**	0.22	**0.45**	**0.45**
49	0.15	0.09	0.16	0.16	0.01	0.03	0.18	0.15	0.18	0.07	0.06	**0.44**	0.07	**0.54**	0.21	**0.48**	**0.48**
51	0.12	0.09	0.14	0.14	0.03	0.00	0.19	0.20	0.16	0.10	0.03	**0.51**	0.02	**0.55**	0.18	**0.42**	**0.42**
63	0.06	0.03	0.14	0.14	0.20	0.01	0.07	0.06	0.02	0.04	0.08	0.07	0.06	0.24	0.01	**0.93**	**0.93**

VII: *Sensitivity coefficients of principal stress σ3, lower surface*

Point	Input Variable																
	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14	X15	X16	X17
1603	0.09	0.15	0.14	0.10	0.11	0.20	0.04	0.15	0.28	**0.31**	0.06	0.22	0.11	0.06	0.07	**0.71**	**0.71**
2403	0.16	0.21	0.01	0.08	0.09	0.11	0.07	0.09	0.20	0.01	0.06	0.26	0.13	0.00	0.05	**0.91**	**0.91**
2503	0.02	0.18	0.09	0.07	0.17	0.18	0.23	**0.34**	0.07	0.23	0.01	**0.62**	0.17	0.06	0.13	**0.52**	**0.52**
2603	0.29	**0.54**	0.05	0.07	0.24	0.11	0.17	0.11	0.16	0.11	0.23	0.28	0.00	0.05	0.25	**0.40**	**0.40**
2703	0.12	0.03	0.03	0.18	0.29	0.02	0.17	0.06	0.05	0.11	0.03	0.15	0.20	0.12	0.21	**0.87**	**0.87**
3003	0.05	0.28	0.16	0.11	0.10	0.27	0.07	0.14	0.06	0.07	0.05	**0.57**	0.25	0.18	0.21	**0.58**	**0.58**
3203	0.16	0.21	0.01	0.08	0.09	0.11	0.07	0.09	0.20	0.01	0.06	0.26	0.13	0.00	0.05	**0.91**	**0.91**
3903	0.02	**0.34**	0.22	**0.40**	0.02	0.17	0.17	**0.30**	0.06	0.18	0.06	**0.37**	0.13	0.04	0.11	**0.55**	**0.55**
4003	0.10	**0.43**	0.21	**0.31**	0.07	0.13	0.13	**0.37**	0.08	0.10	0.06	**0.42**	0.12	0.07	0.10	**0.49**	**0.49**
4803	0.11	**0.30**	0.25	**0.35**	0.04	0.22	0.22	**0.31**	0.02	0.21	0.03	**0.35**	0.10	0.03	0.18	**0.56**	**0.56**

On the lower surface the influence of temperature field (X16, X17) is still dominant, but the influence of other variables is no longer negligible. The influence of the width of transverse joint (X12), as well as the mechanical properties of concrete slabs (X2, X3), the thickness of base layer (X4), and the modulus of elasticity of sub-base layer (X8) may be mentioned.

DISCUSSION

Using combination of the accurate computational models (e.g. based on non-linear FEM) and the effective simulation techniques it is possible to analyze the behavior of structures with respect to the uncertainties in input variables. Reliability analysis allows to obtain more realistic view of the behavior of structures, allows designing structure for a predetermined reliability level, enables setting of limits (extremes) in which the structure will behave, and finally allows to supplement or even replace the experimental research.

The presented reliability analysis of principal stresses in concrete slabs of rigid pavement shows that the stresses are of positive as well as negative sign, i.e. tensile and compressive stresses occur despite the fact that dynamic loading is not considered. This is caused by the thermal loading due to the temperature difference between the upper and the lower surface of concrete slabs.

The principal stresses are influenced by random variability of input variables in a different way.

That is, random variability influences differently principal stress σ1 and principal stress σ3, differently principal stresses on the upper and lower surface of concrete slabs and differently in individual points in which sensitivity coefficients are calculated. The dominant influence shows the random variability of temperature field acting on the upper and the lower surface of the concrete slabs.

In some points of the lower (but also of the upper) surface of slabs large tensile stresses occur, that reach a maximum value of about 5.3 MPa. Especially on the lower surface in the points near the external load application and in all corners of the slabs the creation of tensile cracks is highly probable. The presented study also shows that the standard deterministic analysis with the input variables set to their nominal (mean) values generally does not provide information about the average behavior of the structure.

CONCLUSION

Rheological properties of materials, cracking, joints, contact of concrete slabs in joints, contact of slab and the subsequent material layer, temperature changes, non-homogenity of pavement base, water regime in the subgrade, environmental changes etc. influence behavior of the rigid pavements in a decisive way. Moreover, the problem is complicated by the fact that the input data are generally random variables. The combination of the proper analytical model and modern simulation techniques seems to be an effective tool for the solution of the problem.

The behavior of the older type of rigid pavement formerly used in the Czech Republic is analyzed. The structure is loaded by the self-weight of concrete slabs, by the thermal loading due to the temperature difference between the upper and lower surface of the slab, and by the external load of intensity 50kN. Contrary to the original former design, the base layer is supposed to be made from a recycled material instead of a natural one in this study.

The computational model is based on the nonlinear finite element method. Four concrete slabs, all other layers and longitudinal and transverse joints are modeled as 3D space. Attention is paid to modeling of concrete slab contact with subsequent material layer and interaction of adjacent slabs in joints. In these areas the so-called contact problem occurs, in which tensile stresses cannot be transmitted. This is the case of structural nonlinearity and the pavement modeling therefore becomes nonlinear.

The influence of uncertainties in input variables on the behavior of the pavement is respected in the analysis using Updated Latin Hypercube Sampling Method with 20 simulations. Total 17 basic random input variables describing layer thicknesses, mechanical properties of materials, characteristics of joints and temperature on both surfaces of concrete slabs are taken into account in the study. Their statistical parameters are carefully evaluated taking into account the data obtained from in-situ measurements, experimental tests, data from technological handbooks and scientific publications and the corresponding standards. The influence of the current level of construction process and technological discipline is also taken into account. The normal and truncated normal CDF are used for symmetrically distributed variables, the three-parametric lognormal CDF for the other case.

The statistical and sensitivity analysis of principal stresses σ1 and σ3 in concrete slabs is performed. The stresses are evaluated in 53 points on the upper and lower surface (106 points total) of concrete slabs. Principal stress σ1 represents an extreme value of tensile stress in the given point of the structure, while σ3 is an extreme value of compressive stress, that arise due to the spatial state of stress.

The nominal values of principal stresses σ1 obtained from deterministic analysis are within the interval 0.3 to 1.6 MPa on the lower surface and within the interval –0.05 to 1.1 MPa on the upper surface. Mean values of stresses on both surfaces in all points have the character of tensile stress and are generally always greater than the nominal values. They are within the interval 0.4 to 2.0 MPa on the lower surface and within the interval 0.2 to 1.4 MPa on the upper surface. The largest nominal as well as mean values reach their maximum at points near the external load application.

The interval in which principal stresses σ1 may occur is considerable high. Larger tensile stresses and thus higher probability of tensile cracks occurring arise on the lower surface of concrete slabs. In the case of lower surface, the maximum values are up to 5.3 MPa, in the case of upper surface they are up to 3.3 MPa.

The principal stresses σ1 in some points on the upper but especially on the lower surface reach magnitude where the creation of tensile cracks is highly probable. Taken into account the tensile strength of standard quality concrete, there is at least probability of 1% of tensile crack creation on the upper surface and at least probability of 5% of tensile crack creation on the lower surface.

The principal stress σ1 in most points on the lower surface is influenced by the temperature field acting on the upper and lower surface of the concrete slabs. Influence of these variables can be considered as dominant.

The nominal values of principal stresses σ3 obtained from deterministic analysis are within the interval –0.1 to –2.2 MPa on the upper surface and within the interval 0.1 to –0.5 MPa on the lower surface. Mean values of stresses on both surfaces have the same character as nominal values but they are generally greater. They are within the interval –0.7 to –2.8 MPa on the upper surface and within the interval 0.1 to –1.0 MPa on the lower surface.

The interval in which principal stresses σ3 may occur is considerably high. Larger compressive stresses arise on the upper surface of concrete slabs. In this case, the maximum values are up to –7.3 MPa, in the case of lower surface they are up to –4.9 MPa. The largest compressive stresses on the upper surface arise in points on the transverse axis of symmetry of all slabs and in points close the contact of all slabs. On the lower surface the largest compressive stresses arise in some corners of slabs.

The principal stresses σ3 on the both surfaces are affected by random variability of the input variables differently. In the points on the upper surface the influence of temperature field acting on the upper and lower surface of the concrete slabs are dominant. Influence of other variables can be considered as minimal or negligible. On the lower surface the influence of temperature field is still dominant, but the influence of other variables is no longer negligible. The influence of the width of transverse joint, as well as the mechanical properties of concrete slabs, the thickness of the base layer, and the modulus of elasticity of the sub-base layer may be mentioned.

Acknowledgement

The research was supported by the project TA010226 "Optimization of design and realization of low capacity road pavements" of the Technology Agency of Czech Republic.

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