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PREDICTION OF OIL DEPLETION IN GHANA BASED ON HUBBERT'S MODEL

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Abstract

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This study seeks to fulfil four specific tasks: to predict the year of peak oil production in Ghana, to estimate the quantity that will be produced at this peak point, to investigate the year of total oil depletion and last but not the least is to assess the total recoverable oil resources at the point of depletion. The study applied Hubbert's model of oil production and depletion. There was however modification to the approach in that Nonlinear Least Squares were used to estimate the logistic growth parameter instead of linear regression techniques. The modification was due to the fact that the historical data of oil production in Ghana did not follow a linear trend. The study acknowledges the influence of technology, economic, and political factors in the shorter time scales but hypothesizes that the amount of recoverable oil resources dominates all other factors in the long run. Hence the focus was placed on physical or geological constraints. The study results show that peak production in Ghana will occur in 2022. A maximum of about 100 million barrels will be produced per year though currently the annual production is about 40 million barrels. The predicted logistic curve also shows that total oil will be depleted by 2060. Ghana has total recoverable oil resources of about 1.8 billion. The data for the study was obtained from the U.S Energy Information Administration covers the period of production from 1992 to 2014.

Keywords: Hubbert's curve, Hubbert's linearization, oil production, oil depletion, peak production, Ghana

INTRODUCTION

Ghana discovered oil in large commercial quantities in 2007 and started large scale production in 2010. It is however worth mentioning that oil production in Ghana dates back to the late 1970s in the Saltpond field. Production from this field was however low and barely exceeded 7000 barrels per day and it further reduced to lower levels as the years went by. Major and sustained exploration activities started with the formation of the Ghana National Petroleum Corporation (GNPC) in 1985 and has continued until today.

In 2007 Ghana discovered the largest quantities of crude oil in the country's history at the Jubilee Field which was estimated to hold a potential of 1.8 billion barrels (Natural Resource Governance Institute, 2015). The main crude oil grade, at Jubilee, is light and sweet. It has an American Petroleum Institute (API) Gravity of 36.4 degrees and Sulphur content of 0.26% by weight, with no unusual characteristics. After producing an average of only 8,880 barrels per day (bpd) in 2010, Ghana's oil production increased to 84,737 barrels per day by late 2011. Total oil production was anticipated to rise to 120,000 barrels per day in 2012 but this was not achieved. According to GNPC (2015), total annual oil lifting was 24,450,155 barrels in 2011, 26,430,934 barrels in 2012 and 36,048,290 barrels in 2013. As at the end of 2014, daily oil production was about 110,000 barrels.

The economic impact of an oil boom cannot be overemphasized as it has a huge potential to turn the economic fortunes of nations. Just like every nonrenewable resource, oil production is time bound and will be depleted after years of production. The rate of depletion depends on several factors which include geological, technological and economic.

This study seeks to predict Ghana's oil production peak and depletion using the techniques employed by Hubbert (1956) with modification to some of the aspects criticized by opponents. Hubbert's model has remained a source of emphasis on dwindling oil reserves. It is interpreted by several other researchers in the area as a technical constraint on oil production. It focuses more on the limits placed by geological factors. This model assumes a bell shape for oil production. In other words, production of oil rises like every other resource, reaches a peak and declines. The technical constraints which Hubbert claimed were the reason for the bell shape were supported by other researchers such as Cleveland and Kaufmann (1991), Kaufmann (1991), Kaufmann (1995), Moroney and Berg (1999). The idea of geological constraints was also highlighted by (Pesaran and Samiei, 1995). These views acknowledge the impact of economic factors in the short time scale but emphasized that in the long term physical and geologic constraints limit the production of oil. In other words, they implicitly assume that exogenous technical constraints limit production below a level that would be the optimum production if there were no technological restrictions.

Hubbert's logistic model is the most popular approach. Numerous variants and extensions sometimes involving economic variables adopt this technical view of oil production. Brandt (2007) surveys this wide class of models by comparing 139 oil producing regions. He finds that the symmetric Hubbert model provides on the average a better fit than alternative mathematical functions (Fisher, 2008). In support of Hubbert's model, Moroney and Berg (1999) using US lower 48 states data, assume that production capacity is a function of the size of the proven reserves. Their model results were close to the predictions in Hubbert's model.

There have been research works that made arguments for the role of economic factors in determining developments in oil production. Using US lower 48 states data, Kaufmann (1991) and Pesaran and Samiei (1995) find that the gap between the actual production and the logistic curve could be explained by economic and political variables. The addition of economic variables to their model enriched their study but failed to explain why production followed a logistic trend. According to their study, production is not directly determined by the technical characteristic of the resource as portrayed in Hubbert's model but by profitability which depends mainly on three economic factors: the oil price, the extraction costs and the interest rate.

Even though these issues were raised by mainly opponents of Hubbert's analysis, the predictions made in his model were accurate. He employed both logistic production function and linearization of US oil data to calculate the total recoverable oil resources, peak year and peak production.

Hubbert's analysis does not say that technology, economic, and political factors do not affect production within shorter time scales, but it hypothesizes that the amount of recoverable oil resources dominates all other factors in the long run (see Deffeyes, 2005; Deffeyes, 2002). This assumption is expressed in the differential form of the logistic equation, which forms the basis of the mathematical model used in the analysis.

This study thus employs the approach used by Hubbert with modification to the assumption of linearity of the dataset. A plot of historical oil production data in Ghana exhibited a nonlinear relationship thus warranting the need to modify the assumption. The study also allowed for growth in the ultimate recoverable resources rather than a fixed value. The main objective of the study is fourfold: to predict the year of peak oil production in Ghana, to estimate the quantity that will be produced at this peak point, to investigate the year of total oil depletion and last but not the least is to assess the total recoverable oil resources at the point of depletion.

The study acknowledges the impact of economic factors such as the international price of crude oil, demand, investment into production capacity, and level of technology available on oil production globally and in Ghana. This model however like Hubbert emphasized the physical and geological constraints to oil production.

MATERIALS AND METHODS

The data for the study was obtained from the United States Energy Information Administration and the Ministry of Finance in Ghana. It covers the period from 1992 to 2014. The annual oil production (P_t) was obtained by multiplying the daily oil production data by 365 days. The cumulative oil production (Q_t) was measured by summing up all annual production up to 1992 (the starting year) and then adding each year's production to the sum of the previous years.

The study fits a logistic growth function to oil production as done in Hubbert's model (Hubbert, 1956). Cumulative oil production is given in equation 1. It is symmetric about the peak point and reaches its maximum when half the oil is produced

$$Q_t = \frac{Q_\infty}{1+e^{w\cdot(\tau-t)}}, \tag{1}$$

where Q_∞ is the ultimate recoverable oil resources or the total oil that can be recovered when the resource is exhausted. t denotes the current year (2014), ω is logistic growth rate. The derivative (dQ/dt) of cumulative oil production gives current production P_t which is used to capture the development in oil production at a given year. Hubbert's curve for predicting current oil production is given in equation (2).

$$P_t = Q_\infty \omega \frac{1}{(e^{-(\omega/2)(\tau-t)} + e^{(\omega/2)(\tau-t)})^2}, \quad (2)$$

As can be observed, equation 2 is nonlinear in nature. The equation was therefore solved using Nonlinear Least Squares estimation though Hubbert used linearization as described by Deffeyes (2005). The scatter plot of P_t/Q_t and Q_t from historical oil production data in Ghana did not give a straight linear regression line such as observed in Hubbert's model. Linearization techniques would therefore lead to imprecise estimates hence the use of Nonlinear Least Squares by this study.

Nonlinear Least Squares regression is characterized by the fact that the prediction equation depends nonlinearly on one or more of the unknown parameters. It usually arises when there are physical reasons for believing that the relationship between the response and the predictors follow a particular functional form. The unknown parameter ω in the nonlinear regression model was estimated from the data by minimizing the sum of squares of the prediction errors. The estimates were computed by iteration using optimization methods (see Ratkowsky, 1983; Bates and Watts, 1998; and Seber and Wild, 1989).

Using Nonlinear Least Squares to estimate equation 2 where, P_t is the dependent variable, then the model had a functional form which is not linear with respect to the unknown parameter (ω) as in equation 3.

$$P_t = f(\vec{Q}_\infty; \vec{\omega} + \varepsilon_t), \quad (3)$$

where f is a given function, Q_∞ is the explanatory variable, ω is the unknown parameter, ε_t denotes the error of specification. t from equation 2 is the current year, 2014.

Given that

$$\alpha = \frac{(Q_\infty - Q_0)}{Q_0}, \quad (4)$$

where Q_0 is the initial cumulative oil production, then the time to peak production measured in years (γ) was calculated as

$$\gamma = \ln(\alpha). \quad (5)$$

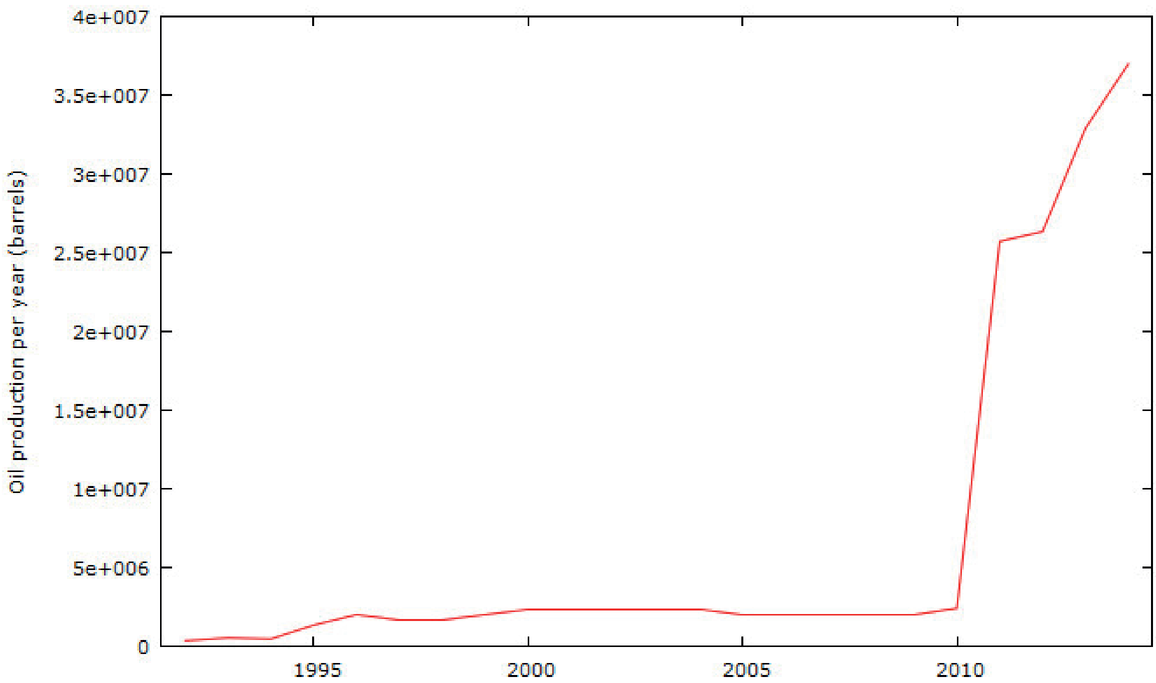
The exact year of peak production (τ) was then derived as the sum of the current year (t) and the time to peak production (γ). See Poyrazoglu, (2011).

$$\tau = t + \gamma, \quad (6)$$

where t = 2014. All estimations were done in Gretl software.

RESULTS

The results are discussed with regards to the different views of factors affecting oil production and depletion. The focus is more on that of Hubbert due to the fact that the study employed a similar approach. There is however comparison with research results obtained by opponents of this approach.



1: *Historical data of oil production per year in Ghana*
Source: United States Energy Information Administration

Time Series Plot of Historical Oil Production Data in Ghana

Fig. 1 represents a time series plot of historical data on oil production in Ghana per year in millions of barrels. It covers the period from 1992 to 2014. The period 1992–2010 was marked by low oil production. The second period which is from 2011–2014 shows a sharp rise in the level of production. Total oil production as given by the Ghana National Petroleum Corporation was 24,450,155 barrels in 2011, 26,430,934 barrels in 2012 and 36,048,290 barrels in 2013. Daily production in 2014 was about 110,000 barrels leading to about 40 million barrels of total annual production.

Scatter Plot of Oil Production Data in Ghana

Fig. 2 shows a scatter plot of P_t/Q_t against Q_t where P_t represents the annual oil production and Q_t the cumulative oil production at a given year. It shows that in the case of Ghana, linearization will not be a good fit for the dataset. This thus informed the decision to employ Nonlinear Least Squares to estimate the logistic growth parameter as in equation 2. The modification of linearization was necessary since it was one of the bases for the criticism of Hubbert's analysis.

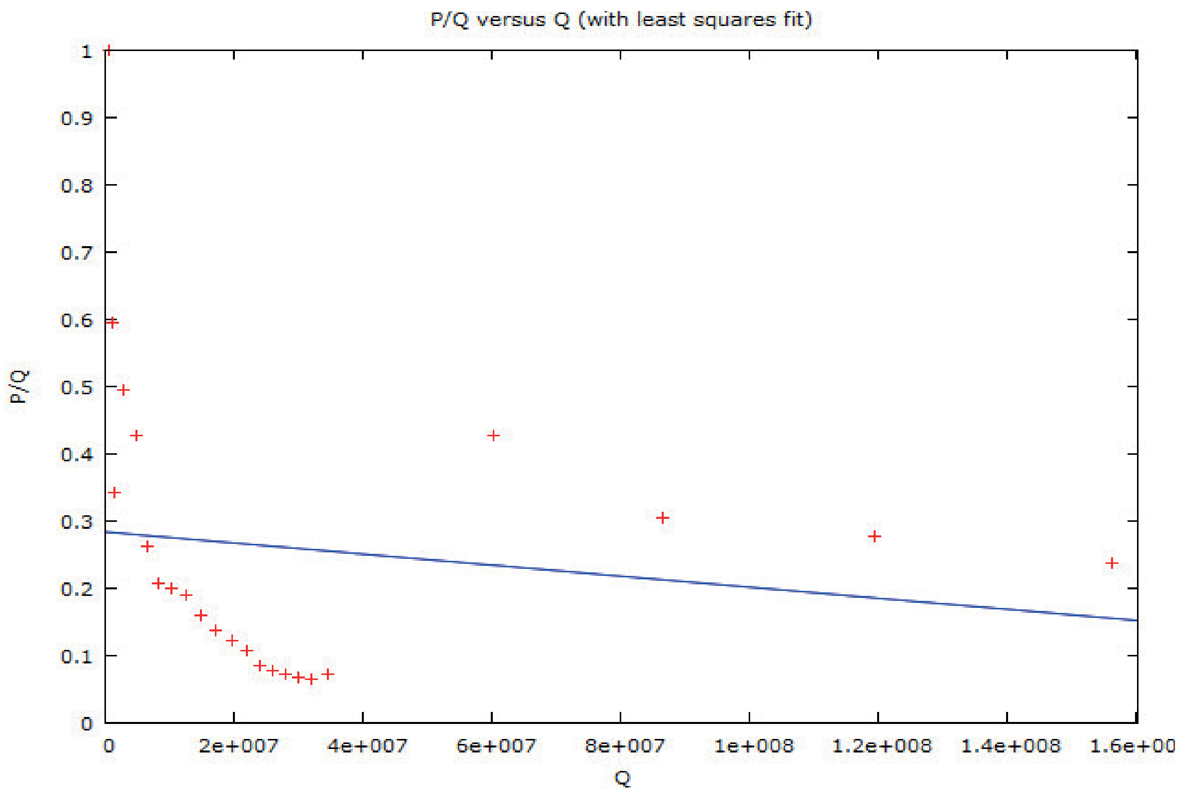
Nonlinear Least Squares Estimation of the Logistic Growth Parameter

Tab. I shows the results of the Nonlinear Least Squares estimation of the logistic growth paramter (ω) in equation 2. The estimated parameter aided in the prediction of the future developments of oil production in Ghana as seen in Fig. 3.

Prediction of Oil Production Developments in Ghana

The time to peak production (γ) from equation 5 is 8 years implying that the year of peak production (τ) which is given by the sum of the current year, t (2014) and the time to peak production, γ(8) will be 2022.

Realizing the nonlinear trend in the Ghanaian oil data plot in Fig. 2, the study fit equation 2 to the historical time series data of oil production and simply extended the line as in the approach by Hubbert to predict developments in future oil production. This prediction was realized with

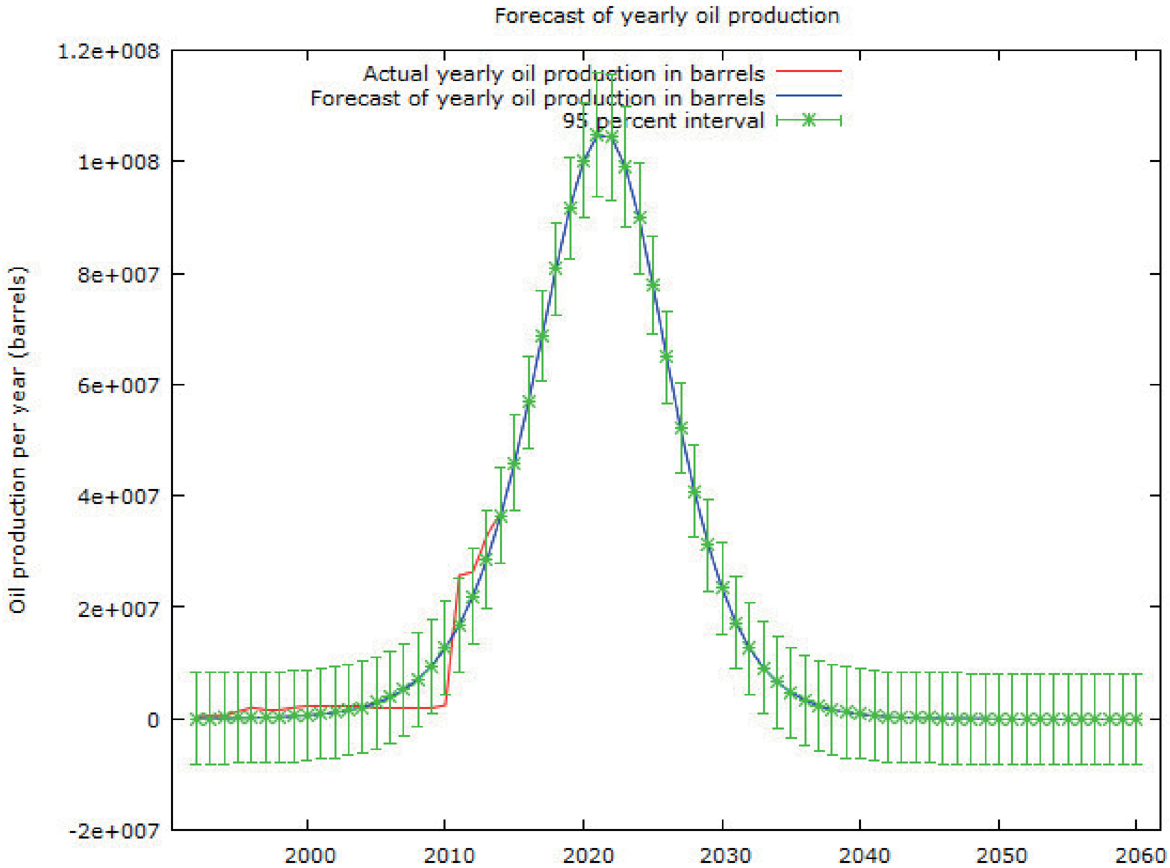


2: *Scatter plot of P/Q vrs Q*
Source: Author generated from historical oil production data in Ghana

I: *Nonlinear Least Squares model*

Parameter	Estimate	Std. Error	t-ratio	P-value	
ω	0.309	0.012	24.850	< 0.001	***

The R-squared for the NLS mode was 0.88. Std. Error represents standard errors, t represents student t distribution and P represents probability.



3: *Forecast of oil production per year*

high degree of accuracy based on the fact that the predicted production fit actual historic production data (see Fig. 3). It shows that oil production will peak in 2022 and the total annual production will be about 100 million barrels in that year. Production levels will be at low levels (around 10,000 barrels per day) after 2045 and it will be completely depleted by 2060.

Looking at Fig. 3, the study agrees with Lynch (2003; 2004) that the production profiles of oil fields around the world in history do not most often have an exact symmetric shape. Fig. 1 clearly shows that the production profile of Ghana might not exhibit a complete case of symmetry. It will be characterized by asymmetric sharp rises and long tails at some points. This thus means that there will be a little deviation from the prediction made by this study though the deviations will not be significant at the 5% significance level.

DISCUSSION

Moroney and Berg (1999) employed Hubbert's approach and obtained similar results even though they added economic factors. Kaufmann (1991) and Pesaran and Samiei (1995) added the price and technological effect to their model and found out that the results produced were still similar to the predictions made by Hubbert. They therefore concluded that those factors are important but other factors, (geological factors) dominate developments in oil production.

Attempting to apply Hubbert's model to global oil production development could be challenging since developments in each oil producing country vary. Rehrl and Friedrich (2006) from their study observed that Hubbert's approach predicts accurately at the individual country level such as in the case of Ghana or the United States of America but performs poorly when applied at the aggregate or global production level.

Hubbert's analysis came under severe criticism for omitting economic factors in the development of oil production. It emphasizes the influence of physical scarcity or the remaining faction of ultimate recoverable resources in dominating long term trends over economic factors such as the international price of crude oil, demand, investment into production capacity, and level of technology available (Fisher, 2008). His predictions were however accurate and the justification for his model was on the basis of his predicted results fitting the historical oil production data of the United States of America. The predicted results from this study also fit the historical oil production data at about a 95% confidence level.

In response to the arguments about the over simplification of Hubbert's model, Deffeyes (2005) emphasized that the best model is always the simplest one that can describe observations. The fact that Hubbert's model fit the historical data of US is enough evidence for the validity of the model. It will also be pointed out that economic changes (demand, investment, and price changes), as well as technological developments, occurred throughout the time periods represented by data. Such developments are therefore necessary but do not matter very much in comparison to geologic scarcity (Deffeyes, 2005).

The second criticism of this approach is its assertion of and reliance on a fixed value for ultimate recoverable resources for all points in time. Ultimate recoverable resources according to optimists should be a changing number and not fixed as in the approach by Hubbert. This criticism was supported with documented growth of proven reserves figures over history (IEA, 2004; IEA, 2005; Williams, 2003; USGS, 2000; Lynch, 2004). Given that the potential for more oil reserve discoveries in Ghana is still high, this study took the stance by the optimists by allowing ultimate recoverable resources to grow due to expectation of improvement in technology as proposed by Lynch and Adelman (1997).

CONCLUSION

The main objective of this study is fourfold: to predict the year of peak oil production in Ghana, to estimate the quantity that will be produced at this peak point, to investigate the year of total oil depletion and last but not the least is to assess the total recoverable oil resources at the point of depletion. The study applied Hubbert's model of oil production and depletion. There was however modification to the approach in that Nonlinear Least Squares were used to estimate the logistic growth parameter instead of linear regression techniques. The modification was due to the fact that the historical data of oil production in Ghana did not follow a linear trend. Hubbert employed linearization techniques which attracted several criticism from other researches who emphasized their argument on the fact that the entire US oil production data did not follow a linear trend.

The study results show that peak production in Ghana will occur in 2022. A maximum of about 100 million barrels will be produced per year though currently the annual production is about 40 million barrels. The predicted logistic curve also shows that total oil will be depleted by 2060. The country has total recoverable oil resources of about 1.8 billion.

The study admits the impact changes in the level of technology, investment and other demand factors can have on the predicted results. However based on the fact that these factors occurred all these years but the predicted results still fit the historical production data, deviations as a result of these factors will not significantly affect the results. It must also mentioned that though global oil prices are declining currently, oil production level is rather increasing and now stands at about 110,000 barrels per day from about 102,000 barrels per day in 2013.

The study unlike in the analysis of Hubbert, allowed for growth in the ultimate recoverable resources to account for changes in technology and new discoveries in future.

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