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A SLACKS-BASED MEASURE DEA METHODOLOGY FOR IDENTIFICATION OF RETURNS TO SCALE IN THE SLOVAK BANKING SECTOR

Martin Boďa[1]

[1] Department of Quantitative Methods and Information Systems, Faculty of Economics, Matej Bel University in Banská Bystrica, Národná 12, 974 01 Banská Bystrica, Slovak Republic

Abstract

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Taking into consideration that knowing the nature of scale of operations is important both to size optimization and to technical efficiency measurement, the paper aims to identify for individual commercial banks of the Slovak banking sector the scale of their operations as exhibited by their production activities during the period from 2000 and 2012. To this end, the DEA procedure of Tone and Sahoo utilizing the concept of the degree of scale elasticity is enhanced and combined with the framework of the SBM model. The analysis of scale is conducted for the major 11 Slovak commercial banks under the prism of the production approach to banking operations, in which the economic assumption that three solid phases can be singled out in the development of the Slovak banking sector is made. The results confirm that the largest Slovak commercial banks were in the investigated period already "too large" so as to gain potentially some benefit from expanding their depository and creditory services by increasing their inputs.

Keywords: returns to scale, degree of scale elasticity, SBM model, production approach, Slovak banking sector

INTRODUCTION

It is held in the literature that in executing their production activities commercial banks act as rational agents who pursue their economic goals with an emphasis placed upon pecuniary objectives. With this determination commercial banks endeavour to maximize their profits at least in the short run and they abide by this normative of their behaviour under several constraints that are particular of banking operations, to wit banking regulation and riskiness of banking business. As commercial banks make decisions concerning their production plans and allocation of their economic resources, they strive after efficiency as the pre-condition of profit maximization. Their efficiency comes in various forms and out of the differing notions notable to this paper is technical efficiency. Technical efficiency is interpreted as the capacity of producing without waste, which suggests that a given level of outputs is secured through the smallest level of inputs or, vice versa, the highest level of outputs is achieved with a given level of inputs. Technical efficiency is determined by the position of a concrete production relative to (the efficient subset of) the technological frontier and is quantified as a standardized distance between this actual production and its Pareto-Koopmans optimal possibility marked with absence of waste in physical terms.

Another associated property of banking production is the scalability of operations, i.e. the ability to produce prevalently at constant or variable returns to scale. Abstracting from other conditions of banking enterprise, it is desirable from a societal standpoint that banking production is accomplished at constant returns to scale inasmuch as in such a case average production is maximum and under perfect competition (when the prices of outputs and inputs are exogenous) commercial banks attain zero profits in the long run. The knowledge about the scale of operations is of import to commercial banks on several grounds.

- In the first place, if a commercial bank operates at increasing returns to scale, there is still room for increasing the size of its operations as a more dynamic increase of outputs is associated with the initial expansion of inputs. For a commercial bank operating under decreasing returns to scale it is advisable to shrink in activities or to undertake some other form of size optimization since any boost in the volume of inputs does not bring about a satisfactory increase of outputs. This sort of information is valuable from a managerial viewpoint and may be ancillary in judging whether a particular size of the commercial bank is economically convenient.
- The performance of commercial banks (as well as their branch offices) is nowadays frequently treated not only in terms of profit generation but also through technical efficiency status. This follows from a great amount of research interest that arose in connection to technical efficiency measurement in the banking sector. The methods devised to this end (and data envelopment analysis in particular) require the analyst to specify the nature of operations in terms of scale. This specification is often set heuristically, a priori to a full and thorough consideration of the issue, or both constant returns to scale and variable returns to scale are considered as options and results are compared. This is undertaken in the spirit of defence that constant returns to scale offer a view into benchmark (societally ideal) technical efficiency whilst variable returns to scale induce empirical (and true) technical efficiency, the difference between which represent another form of efficiency, the so-called scale efficiency. Still, this specification of returns to scale is pivotal for technical efficiency measurement, which constitutes the second reason for analysing the nature of scale of operations for commercial banks.

In line with these considerations, the centre of attention of this paper rests with the Slovak banking sector and its production characteristics. There were several studies with their attention devoted to technical efficiency measurement in the Slovak conditions that employed some approach of data envelopment analysis (DEA) either for the entire banking sector (e.g. Stavárek, 2006; Zimková, 2014, 2015) or on the level of branch offices (e.g. Ševčovič, Halická and Brunovský, 2001; Kočišová, 2012), but they failed to recognize the importance of this issue and opted for variable returns to scale by default. To the best knowledge of the author, the investigation into the nature of returns to scale in the Slovak banking sector has not been conducted and this issue relevant to technical efficiency measurement escaped the attention of researchers.

In filling this gap, the goal of the paper is to identify for individual commercial banks of the Slovak banking sector the scale of their operations and locate their position on the technological frontier in view of scale. In tackling the problem of a small sample size (as the Slovak banking sector is not plentiful in the number of commercial banks), the assumption of three recognizable and separable phases in the development of the Slovak banking sector during the period from 2000 until 2012 is adopted and supported with argumentation. The identification procedure of returns to scale is thus carried out for these three identified phases, under the belief that each phase is characteristic of a specific production technology constant throughout the phase, and with this the "unified DEA" approach of Tone and Sahoo (Tone and Sahoo, 2004) is utilized. These authors in the non-parametric framework of the standard BCC model of DEA derived the degree of scale elasticity, a measure of scale developed alongside neoclassical thoughts on production and used frequently for the purpose of identification of returns to scale. As such, the procedure of Tone and Sahoo is two-step. In the first step, the production activity is projected on to the efficient subset of the (empirical) production technology (if necessary); and, in the second step, its degree of scale elasticity is calculated by means of the output-oriented BCC model. Whilst in the second step the use of the output-oriented BCC model is crucial owing to its geometric and definitional linkage to returns to scale, the authors use the oriented BCC model in the first step as well. Instead of this model, the paper highlights the slacks-based measure (SBM) model of Tone (Tone, 2001) for the first step as it is non-radial and does not suffer from limitations of the BCC model.

The contributions of the paper are then twofold. In the empirical dimension, an investigation of the position of Slovak commercial banks with respect to scale of their operations is conducted from the perspective of one of the most prominent approaches to interpreting banking production: the so-called production approach. From the methodological point of view, the procedure of Sahoo and Tone for identification of scale is enhanced by introducing a less restrictive projection into its first step.

The paper is structured into five sections. This introductory section expands into the second section containing the general presentation of the approach that is adopted for identification of scale in the case of Slovak commercial banks. The third section contains methodological notes and the ensuing fourth section gives the obtained results and comments their implications. The final, fifth, section summarizes.

Scale and its Identification

It is assumed that production technology of commercial banks converts m inputs into s (desirable) outputs. All production variables are represented by non-negative numbers and any input vector denoted generally as **x** is a point in $\Re^m_+$ and, similarly, any output vector **y** is a point in $\Re^s_+$. A production activity is then an ordered pair [**x**, **y**] in the full input-output space $\Re^m_+ \times \Re^s_+$. The production technology of commercial banks is represented by the set of all feasible production activities $T = \{[\mathbf{x}, \mathbf{y}] \in \Re^m_+ \times \Re^s_+ : \mathbf{x} \text{ can produce } \mathbf{y}\}$. The assumptions that are warranted on economic grounds and placed upon *T* are as follows: possibility of inaction, closedness, free disposability of inputs and outputs, impossibility of free production, non-emptiness and boundedness of associated output possibility sets as well as convexity (see Debreu, 1959, pp. 39–42; McFadden, 1978, p. 7). The boundary of *T* defined à la Debreu and Farrell in terms of maximum attainable radial contractions of inputs and maximum attainable radial expansions of outputs is referred to as the technological frontier. It is the set of feasible production activities $[\mathbf{x}, \mathbf{y}] \in T$ that satisfy $(\forall\theta \in (0, 1])([\theta\mathbf{x}, \mathbf{y}] \notin T)$ and simultaneously $(\forall\eta \in (1, \infty))([\mathbf{x}, \eta\mathbf{y}] \notin T)$. Production activities lying on the production frontier need not still be technically efficient because there may be room for non-radial improvement of (at least) one of their inputs or (at least) one of their outputs. Production activities that come to satisfy the definition of Koopmans (1951, p. 60) stipulating that no further improvement of production is attainable are called here technically efficient or technically efficient in the sense of Pareto and Koopmans. Such production activities belong to the efficient subset of technological frontier that is composed of production activities $[\mathbf{x}, \mathbf{y}] \in T$ that satisfy $(\forall\mathbf{x}_0 : [\mathbf{x}_0, \mathbf{y}] \in T)(\mathbf{x} \le \mathbf{x}_0])$ and simultaneously, $(\forall\mathbf{y}_0 : [\mathbf{x}, \mathbf{y}_0] \in T)(\mathbf{y} \ge \mathbf{y}_0])$.

For reasons analytical, it is needful to characterize production technology in respect of the scale of production. Returns to scale describe the behaviour of production as the scale of production changes in situations when all input levels are variable and chosen by the production unit. Returns to scale are the qualitative property of production that pertains either to the production technology as such (and are related to all production activities that it comprises) or to a sole unique production activity. Only the second understanding is addressed here in the paper wherein the ambition is to establish whether a particular production activity operates at constant returns to scale, increasing returns to scale or decreasing returns to scale. The last two cases are commonly cast into the case of variable returns to scale. Before providing the definition for these three basal situations, to change the scale of operations for a given production activity $[\mathbf{x}, \mathbf{y}] \in T$ is to multiply [**x**, **y**] by a non-negative number *k*, and to increase (resp. decrease) the scale of operations is to restrict further *k* to be larger than 1 (resp. smaller than 1). It is concurred in the literature (e.g. Sahoo, Mohapatra and Trivedi, 1999, pp. 380, 383; Tone and Sahoo, 2004, p. 758) that it is only sensible to define the returns to scale status only for production activities that are positioned on the efficient subset of the technological frontier. Hence, assume that a production activity $[\mathbf{x}, \mathbf{y}] \in T$ is technically efficient in the sense of Pareto and Koopmans. This production activity is then said to operate at:

- *increasing returns to scale* if one can arbitrarily increase the scale of operations, i.e. for any $k > 1$ also $[k\mathbf{x}, k\mathbf{y}] \in T$;
- *decreasing returns to scale* if one can arbitrarily decrease the scale of operations, i.e. for any $0 < k < 1$ also $[k\mathbf{x}, k\mathbf{y}] \in T$; and, finally,
- *constant returns to scale* if for one can arbitrarily change the scale of operations, i.e. for any $k > 0$ also $[k\mathbf{x}, k\mathbf{y}] \in T$.

The trio of definitions is borrowed and adapted from Debreu (1959, p. 40) and is complemented by several analytical tools devised to identify for a particular technically efficient activity its local status with respect to returns to scale. Mention is made here of two approaches that are instrumental in understanding the meaning of these definitions. The interested reader is invited to consult Takayama (1993, pp. 157–162) for an overview of other approaches.

For a technically efficient production activity $[\mathbf{x}, \mathbf{y}] \in T$, Banker (1984, p. 36) introduces for any $\beta > 0$ the corresponding maximum attainable expansion coefficient $\alpha(\beta) = \max\{\alpha : [\beta\mathbf{x}, \alpha\mathbf{y}] \in T\}$ and sets up the coefficient

$$\delta = \lim_{\beta\to 1} \frac{\alpha(\beta) - 1}{\beta - 1}, \tag{1}$$

which is clearly the derivative of $\alpha(\beta)$ evaluated at $\beta = 1$, i.e. $\delta \equiv \alpha'(1)$. Constant returns to scale are associated with $\delta = 1$ (as proportionate increase in all inputs causes an increase in all outputs of the same proportions), whilst increasing (resp. decreasing) returns are effective when $\delta > 1$ (resp. $\delta < 1$). This coefficient measures – in radial (i.e. proportionate) terms – a maximum local expansion of inputs that is possible for a given production activity without adjusting its inputs.

Another possibility is to use the measure known as the degree of scale elasticity (hereinafter addressed as "DSE") or passus coefficient, which is also the instrument used throughout the paper to recognize the nature of scale for individual production activities. The latter designation is used in older literature, e.g. by Frisch (1965). This coefficient is originally defined for a single-output production (with possibly multiple inputs) and is generalized by Tone and Sahoo (Tone and Sahoo, 2004) to multiple-output (and possibly multiple-input) technologies, which is the reference basis here in the paper. All production variables must be required positive. Assume for a while that the production technology transforms $m \ge 1$ inputs into $s = 1$ outputs. The DSE ε is then the elasticity of the output quantity with respect to one of the inputs when all inputs vary proportionally and is defined as:

$$\varepsilon = \frac{d^{pr}y / d^{pr}x_1}{y / x_1} = \frac{d^{pr}y / d^{pr}x_2}{y / x_2} = \ldots = \frac{d^{pr}y / d^{pr}x_m}{y / x_m}, \tag{2}$$

wherein $d^{pr}y$ represents the infinitesimal increment in the output quantity associated with infinitesimal proportional input increment $d^{pr}x_i$ for the input x_i ($i \in \{1, 2, \ldots, m\}$). Note that the numerators of the expression in (2) represent the marginal product and the denominators are simply the average products of individual input quantities. Especially, when in addition to $s = 1$ also $m = 1$, this definitional formula then specializes to

$$\varepsilon = \frac{dy / dx}{y / x} = \frac{\text{marginal product}}{\text{average product}}. \tag{3}$$

However, insomuch as only proportionate changes are taken into consideration, this means that $d^{pr}x_1/x_1 = d^{pr}x_2/x_2 = \ldots = d^{pr}x_m/x_m$, and justifies the equality signs in (2). When all input quantities increase at the same proportion with the relative input proportion (i.e. the input mix) being unchanged, but the production scale increases. In this way, DSE provides an expression for the way the output quantity changes when an infinitesimal change occurs in the production scale. This coefficient thus measures elasticity with respect to scale. A production activity with $\varepsilon > 1$ (resp. $\varepsilon < 1$) thus operates in a local increasing (resp. decreasing) returns to scale environment, and constant returns to scale answers to the case when $\varepsilon = 1$.

Note that both these definitions as well as other methodological approaches treat the scale property of production activities in a radial way as they base themselves on equiproportionate (rather than differential) adjustments of input and output quantities requiring that the respective input and output mixes be kept constant. There, however, has been a debate amongst theoretical economists, though tentatively settled in favour of equiproportionateness (for a review see Sahoo, Mohapatra and Trivedi, 1999, pp. 380–382; Tone and Sahoo, 2003, pp. 167–169).

In generalizing (2) to a multiple-output and multiple-input case, the formula in (3) is made use of and the focal challenge is how to approximate the marginal product *dy/dx* and the average product *y/x*. Tone and Sahoo (Tone and Sahoo, 2004) employ the concept of a supporting hyperplane to render this approximation workable. Assume a production activity $[\tilde{\mathbf{x}}, \tilde{\mathbf{y}}] \in T$. It is desired that this production activity be technically efficient in the sense of Pareto and Koopmans. A hyperplane in the full $m + s$ dimensional input-output space passing through this reference point is expressed by the equation $\mathbf{u}'(\mathbf{y} - \tilde{\mathbf{y}}) - \mathbf{v}'(\mathbf{x} - \tilde{\mathbf{x}}) = 0$, where $\mathbf{v} \in \Re^m$ and $\mathbf{u} \in \Re^s$ are coefficient vectors. If a scalar quantity u_0 is introduced by the formula $u_0 = -\mathbf{u}'\tilde{\mathbf{y}} + \mathbf{v}'\tilde{\mathbf{x}}$, then the hyperplane can be reexpressed as:

$$\mathbf{u}'\mathbf{y} - \mathbf{v}'\mathbf{x} + u_0 = 0. \tag{4}$$

This hyperplane divides the input-output space into two halfspaces. If the hyperplane (4) contains the production possibility set T in only one of the halfspaces, then it is said to be a supporting hyperplane of T at the point $[\tilde{\mathbf{x}}, \tilde{\mathbf{y}}]$. Assume now that the vectors $\mathbf{v}$ and $\mathbf{u}$ are such that for any production activity $[\mathbf{x}, \mathbf{y}] \in T$ it is satisfied that $\mathbf{u}'\mathbf{y} - \mathbf{v}'\mathbf{x} + u_0 \le 0$, then truly (4) is the supporting hyperplane dictated by $[\tilde{\mathbf{x}}, \tilde{\mathbf{y}}]$. For some $[\mathbf{x}, \mathbf{y}] \in T$, the scalar quantity $\eta = \mathbf{u}'\mathbf{y}$ can be interpreted as the associated virtual output and the scalar quantity $\xi \equiv \mathbf{v}'\mathbf{x}$ as the associated virtual input. Consequently, from the supporting hyperplane equation $\eta - \xi + u_0 = 0$ one immediately has that:

$$\text{marginal product} = \frac{d\eta}{d\xi} = \frac{d(\xi - u_0)}{d\xi} = 1 \tag{5}$$

and

$$\text{average product} = \frac{\eta}{\xi} = \frac{\eta}{\eta + u_0}. \tag{6}$$

Therefore, the degree of scale elasticity according to single-input single-output formula (3) goes into:

$$\varepsilon = \frac{\text{marginal product}}{\text{average product}} = 1 + \frac{u_0}{\eta}. \tag{7}$$

One has, however, to put formula (7) into practice and on the basis of the observed set of production activities $S_n = \{[\mathbf{x}_i, \mathbf{y}_i]\}_{i=1}^{i=n}$ on n production units he has to establish appropriate values for $\mathbf{v}$, $\mathbf{u}$, and u_0 by a suitable estimation procedure. This estimation is done in the framework of the standard BCC model that is linked and built upon the concept of supporting hyperplane. In this, introduce the matrices of inputs and outputs respectively as $\mathbf{X} = (\mathbf{x}_1|\mathbf{x}_2| \ldots |\mathbf{x}_n)$ and $\mathbf{Y} = (\mathbf{y}_1|\mathbf{y}_2| \ldots |\mathbf{y}_n)$. Subsequently, estimate the production technology as a convex linear combination of all observed production activities contained in S_n by:

$$est.T = \{[\mathbf{x}, \mathbf{y}] \in \Re_+^m \times \Re_+^s : \mathbf{x} \ge \mathbf{X}\boldsymbol{\lambda}, 0 \le \mathbf{y} \le \mathbf{Y}\boldsymbol{\lambda}, \mathbf{1}'\boldsymbol{\lambda} = 1, \boldsymbol{\lambda} \ge 0\}, \tag{8}$$

where $\mathbf{1}$ stands for a vector of ones. If an observed production activity is not positioned on the Pareto-Koopmans efficient portion of *est.T*, it is projected on to it and made technically efficient; if, however, this activity is found technically efficient with respect to *est.T*, no projection is necessary. For individual technically efficient production activities directly or for technically efficient projections of production activities an output oriented BCC model is then run in order to estimate their supporting hyperplanes. In checking whether individual projection activities are technically efficient with respect to *est.T* and in effecting projections – unlike Tone and Sahoo – this paper suggests employing the non-radial SBM model of Tone (Tone, 2001) based on the slacks-based measure. This will be elucidated later on.

In the meantime, assume that $[\mathbf{x}_r, \mathbf{y}_r]$ is either technically efficient with respect to *est.T* or made technically efficient by an appropriate projection. The output oriented BCC model is solved for it in a multiplier form as:

$$\mathbf{v}'\mathbf{x}_r - u_0 = ! \min_{\mathbf{v}\in\Re^m, \mathbf{u}\in\Re^s, u_0\in\Re}$$

subject to

$$-\mathbf{v}'\mathbf{X} + \mathbf{u}'\mathbf{Y} + \mathbf{1}'u_0 \le \mathbf{0}', \mathbf{u}'\mathbf{y}_r = 1, \mathbf{v} \ge \mathbf{0}, \mathbf{u} \ge \mathbf{0}, u_0 \text{ free in sign.} \tag{9}$$

Here $\mathbf{0}$ stand for a zero vector (of appropriate length). Since $[\mathbf{x}_r, \mathbf{y}_r]$ assumes the position on the efficient portion of *est.T*, it follows from the properties of this linear program that there exists an optimal solution $(\mathbf{v}^*, \mathbf{u}^*, u_0^*)$ with $\mathbf{v}^{*\prime}\mathbf{x}_r - u_0^* = 1$ and that for any $[\mathbf{x}_p, \mathbf{y}_p] \in S_n$ it holds that $\mathbf{u}^{*\prime}\mathbf{y}_p - \mathbf{v}^{*\prime}\mathbf{x}_p + u_0^* \le 0$ and that (4) evaluated at $(\mathbf{v}^*, \mathbf{u}^*, u_0^*)$ is the supporting hyperplane to *est.T* at $[\mathbf{x}_r, \mathbf{y}_r]$ indeed. Taking into consideration the constraints of this linear program, for $(\mathbf{v}^*, \mathbf{u}^*, u_0^*)$ one has $est.\eta = 1$ and the degree of scale elasticity transforms in its estimation formula into:

$$est.\varepsilon = 1 + u_0^*. \tag{10}$$

However, as Tone and Sahoo (Tone and Sahoo, 2004) point out, in many occasions, there exist multiple optima of u_0^* and they suggest determining the lower bound $\underline{u}_0^*$ and the upper bound $\overline{u}_0^*$ by solving the linear program

$$u_0 = ! (\max) \min_{\mathbf{v}\in\Re^m, \mathbf{u}\in\Re^s, u_0\in\Re}$$

subject to

$$-\mathbf{v}'\mathbf{X} + \mathbf{u}'\mathbf{Y} + \mathbf{1}'u_0 \le \mathbf{0}', -\mathbf{v}'\mathbf{x}_r + \mathbf{u}'\mathbf{x}_r + u_0 = 0, \mathbf{u}'\mathbf{y}_r = 1, \\ \mathbf{v} \ge \mathbf{0}, \mathbf{u} \ge \mathbf{0}, u_0 \text{ free in sign.} \tag{11}$$

The upper (lower) scale elasticity is then estimated respectively as:

$$est.\underline{\varepsilon} = 1 + \underline{u}_0^* \text{ and } est.\overline{\varepsilon} = 1 + \overline{u}_0^*. \tag{12}$$

Although Tone and Sahoo (Tone and Sahoo, 2004) employed the oriented BCC model to check technical efficiency of production activities and used their projections to make these production activities in case of need technically efficient, here a more sophisticated SBM model proposed by Tone (Tone, 2001) is utilized to this end. For a production activity $[\mathbf{x}_r, \mathbf{y}_r] \in S_n$ this model solves the following task of fractional programming (though in actual applications Charnes-Cooper transformed to the task of linear programming):

$$\rho = \frac{1 - \frac{1}{m}\sum_{i=1}^{i=m} s_{ri}^{\mathbf{x}} / x_{ri}}{1 + \frac{1}{s}\sum_{j=1}^{j=s} s_{rj}^{\mathbf{y}} / y_{rj}} = ! \min_{\mathbf{s}^{\mathbf{x}}\in\Re^m, \mathbf{s}^{\mathbf{y}}\in\Re^s, \boldsymbol{\lambda}\in\Re^n}$$

subject to

$$\mathbf{s}^{\mathbf{x}} = \mathbf{x}_r - \mathbf{X}\boldsymbol{\lambda} \ge \mathbf{0}, \mathbf{s}^{\mathbf{y}} = \mathbf{Y}\boldsymbol{\lambda} - \mathbf{y}_r \ge \mathbf{0}, \mathbf{1}'\boldsymbol{\lambda} = 1, \boldsymbol{\lambda} \ge \mathbf{0}, \tag{13}$$

in which $s_{ri}^{\mathbf{x}}$, x_{ri} denote the *i*-th element of the vectors $\mathbf{s}^{\mathbf{x}}$, $\mathbf{x}_r$, respectively, and in which $s_{rj}^{\mathbf{y}}$, y_{rj} stand for the *j*-th element of the vectors $\mathbf{s}^{\mathbf{y}}$, $\mathbf{y}_r$, respectively. The intensity vector λ is used to create convex linear combinations of observed production activities and ρ is the slack-based measure of DEA, which lent its name to the entire model. This coefficient takes values from interval [0, 1] and measures technical efficiency. Assume that for a production unit the optimal solution $(\mathbf{s}^{\mathbf{x}*}, \mathbf{s}^{\mathbf{y}*}, \boldsymbol{\lambda}^*)$ was obtained with the corresponding value ρ^*. Then, if $\rho^* = 1$ happens to be the case, this production unit is technically efficient in the sense of Pareto and Koopmans (with respect to S_n). If this production unit fails to satisfy $\rho^* = 1$, it can be brought into technical efficiency by effecting the following projection

$$\mathbf{x}_r \to \mathbf{x}_r - \mathbf{s}^{\mathbf{x}*}, \mathbf{y}_r \to \mathbf{y}_r + \mathbf{s}^{\mathbf{y}*}. \tag{14}$$

The exposition must be concluded by two remarks that clear certain ambiguities the attentive reader must be aware of.

- First, the use of the output-oriented BCC model is not in collision with the use of the non-radial SBM model. Whereas the former is utilized here for constructing an *economically interpretable* supporting hyperplane that in a geometric way characterizes the scale property of production (see Cooper, Seiford and Tone, 2007, pp. 134–136) and for subsequent investigations of scale, the latter is employed for checking whether individual commercial banks are technically efficient in respect of the estimated production technology *est.T* and for projecting them on the efficient frontier of *est.T*.
- Second, in the literature there is apparently a biased tendency to employ radial DEA models such as the CCR model or the BCC model that stipulate the choice of either input orientation or output orientation. Projections on to the efficient frontier of *est.T* are sought in two stages, in the first stage, depending on the chosen orientation, the maximum radial (i.e. proportionate) reduction of inputs or the maximum radial expansion of outputs is determined and then, in the second stage, slacks are non-proportionate adjustments of inputs and outputs are addressed additionally. Either inputs or outputs are in this process favoured, which is justified – from a managerial point of view – that DEA gives some recommendations on how to improve inputs and outputs and there are situations when only inputs or when outputs may be managed and influenced.

This is not the case here as the only interest here is to transport (project) individual production activities into the state of technical efficiency with respect to *est.T*. Orientation on inputs or outputs makes little sense in the present situation and it is the model that does not stipulate the choice of orientation that serves best the purpose of projection. In addition to its non-orientedness, the SBM model has another advantage that it is not radial and the problem of finding an appropriate projection addresses in a single stage.

Methodological Notes

Two factors influenced gravely the methodological route taken in the paper:
1. the size of the Slovak banking sector and
2. a possibility to slice the period from 2000 to 2012 into three economically consistent phases in the development of the Slovak banking sector.

On the one hand, the small number of commercial banks and branch offices of foreign banks prohibited analysis from being carried out in a traditional way on a yearly basis. In the Slovak banking sector, during the period from 2000 through 2003 a total of operating commercial banks ranged between 20 and 23, during the period from 2004 through 2008 there were between 21 and 26 operating commercial banks and, eventually, between the years 2009 and 2012 the number of commercial banks ranged between 28 and 29. The rising number of commercial banks throughout the period of 13 years from 2000 to 2012 was caused by branch offices of foreign banks that began their undertaking in Slovakia, although their share and influence on the Slovak banking market was mostly negligible. The core of the Slovak banking sector is formed by 11 banks which make up more than 90% of operations of the entire sector. The overview of these banks (with abbreviations adopted henceforth) is displayed in Tab. I.

On the second hand, the development of the Slovak banking sector during the period from 2000 to 2012, its economic changes and political reforms that it had to go through, points clearly to three separable phases with specific environmental conditions. It is reasoned here in the paper that within each distinctive phase the production technology may be thought of as intact and invariant with respect to a time shift, this being so thanks to the inertia of the economic environment. Under this view, the period from 2000 to 2012 may be broken down into three such successive non-overlapping phases: 2000–2003, 2004–2008, 2009–2012.

- The first phase (2000 to 2003) reflects the last quivers of the restructuralization of major banks and of the privatization of selected banks in Slovakia that commenced in the 1990s. Whereas political decisions on the transfer of illiquid assets into a specialized state institution were taken in the second half of the 1990's and the transfer itself took place especially in the years 1998 and 1999, the cleaning-up of balance sheets of the Slovak major banks ended only at the beginning of the 2000s and was assisted by privatization of most influential Slovak banks and their integration into the global banking sector. This phase was also accompanied by final stages of the transformation process of the Slovak economy.
- The second phase (2004–2008) begins by the accession of Slovakia into the European Union and terminates by the entry of Slovakia into the euro area. Slovak banks during this period implemented a new system of corporate governance, moved to the communication with targeted client segments and towards electronic banking services. There were significant changes in balance sheet and off-balance sheet operations, in the structure of services and in the orientation on investment banking, mortgage banking as well as asset management. Commercial banks were intensively engaged in the preparations for the entry of Slovakia into the euro area as of 1 January 2009.
- The third phase (2009–2012) is marked by the successful adaptation of the Slovak banking sector to the euro environment and by the manifestations of the economic crisis in the Slovak economy. Though the process of the euro-conversion was smooth and trouble-free, the global economic

I: *The "core" commercial banks of the Slovak banking sector between 2000 and 2012*

Commercial bank	Abbreviation
Citibank Europe plc, foreign bank subsidiary (before 2009 Citibank (Slovakia), a. s.)	CITI
Československá obchodná banka, a. s. (in 2009 merged with Istrobanka, a. s.)	CSOBISTRO
OTP Banka Slovensko, a. s.	OTP
Poštová banka, a.s.	POSTB
Prima banka Slovensko, a. s. (before 2011: Dexia banka Slovensko, a. s.)	PRIMA
Privatbanka, a.s. (before 2005 Banka Slovakia, a. s.)	PRIVAT
Sberbank Slovensko, a. s. (before 2013 VOLKSBANK Slovensko, a. s.)	SBERBANK
Slovenská sporiteľňa, a. s.	SLSP
Tatra banka, a.s.	TB
UniCredit Bank Slovakia, a. s. (a 2007 merger of UniBanka, a. s. & HVB Bank Slovakia, a. s.)	UNI
Všeobecná úverová banka, a. s.	VUB

crisis caused small Slovak banks to face existence problems.

Combining these two decisive factors, the identification of scale undertaken in the paper was effected for the 11 commercial banks whose list is given in Tab. I and for the three phases identified. The hypothesis on the time-invariance of the production technology permitted the pooling of individual commercial banks in the identified phases into one sample and resulted in using "bank-years". The dataset on these 11 commercial banks operating in the Slovak Republic came from TREND Holding, s. r. o., Bratislava. Only the data for 4 bank-years were not complete (CITI 2003–2006, 2009–2012). Accounting for the non-available data, the first sub-period was represented by 11 × 4 – 1 = 43 bank-years, the second phase counted in total 11 × 5 – 3 = 52 bank-years, and finally, the third phase was formed of 11 × 4 – 4 = 40 available bank-years.

The empirical analysis was effected under the production approach to transformation in banking. In the choice of input and output variables, only variables stated in physical quantities or balance-sheet items stated in monetary units were accommodated. The production approach views commercial banks as producers of banking services, i.e. depository and creditory facilities, in which they use and consume labour and physical. In effect, deposits taken and loans made are classified as outputs in the paper and fixed assets and labour force are considered as inputs. Tab. II shows the detailed information on the selection of production variables.

The choice of production variables is in accord with the convention in this area. Incidentally, this selection is corresponds with the assertion of Heffernan (2005, pp. 474–476) who states that the production approaches measures bank output by treating banks as firms using capital and labour to produce different categories of deposits and loans.

RESULTS AND FINDINGS

All data preparations and computations were performed in program R (R Core Team, 2013). The R scripts for computing technical efficiencies and projections in the set-up of the SBM model and for computing the degree of scale elasticity as well as its lower and upper bounds were compiled by the author on his own.

The descriptive summary of the four production variables considered is provided in Tab. III structured for the three phases identified in the development of the Slovak banking sector. For technical reasons, the three monetary variables are declared in million € (deflated to 2000 prices).

It follows from the table that during the three phases the Slovak banking sector (represented by the 11 leading commercial banks) was becoming more compact as the variability of all the four variables gradually decreased, which is suggestive of consolidation. These trends are compatible with the claim that Slovak commercial banks were gradually leaving the period of erratic conditions typical for the beginning of the investigated period of 13 years and stabilized their shares on both the deposit and credit market (to which they adjusted the volumes of input factors employed). Through the three phases the volume of both total deposits and total loans slightly ascended whilst the number of employees virtually stagnated at the unchanged level and the volume of fixed assets was rationalized and depressed. This points to a globally higher technical efficiency of the Slovak banking sector in the production of deposits and loans during the period from 2000 until 2012 regardless of whether the majority of Slovak banks operated at constant returns to scale or at variable returns to scale.

The results are displayed compactly in Tab. IV. For each year of the investigated period, this table

II: *The list of production variables recognized under the production approach*

Inputs	Outputs
(the number of) employees *)	(the total amount of) deposits †)
(the total amount of) fixed assets †)	(the total amount of) loans †)

Notes: *) The variable is measured in full time equivalents and in an arithmetic average form for the given fiscal year. †) This variable is measured in thousand € deflated to 2000 prices (by means of the gross domestic product deflator) and as reported as of 31 December of the given fiscal year

III: *The descriptive summaries on the production variables under consideration*

Variable	Employees (the number)			Fixed assets (mil. 2000 €)			Deposits (mil. 2000 €)			Loans (mil. 2000 €)		
Phase	**# 1**	**# 2**	**# 3**	**# 1**	**# 2**	**# 3**	**# 1**	**# 2**	**# 3**	**# 1**	**# 2**	**# 3**
Observations	43	52	40	43	52	40	43	52	40	43	52	40
Minimum	116	82	123	2	1	1	71	96	230	32	14	82
Maximum	6 509	5 170	4 137	319	207	200	5 469	6 432	6 204	3 052	4 241	5 054
Mean	1 708	1 692	1 693	74	61	56	1 598	2 072	2 636	653	1 285	2 157
StDev*)	1 948	1 565	1 368	87	63	54	1 691	1 939	2 056	618	1 164	1 696
CoefVar†)	114%	92%	81%	118%	104%	97%	106%	94%	78%	95%	91%	79%

Legend: *) Standard deviation. †) Coefficient of variation organizes the estimated lower and upper bounds of the degree of scale elasticity and their arithmetic average computed for the 11 Slovak commercial banks and shows the identified status of returns to scale. The preponderance of decreasing returns to scale in Tab. IV is suggestive that the largest and medium-sized commercial banks operate at decreasing returns to scale and are unable to effectively utilize their size in respect of the chosen input mixes consisting of labour force and fixed capital and the chosen output mixes made up by total deposits and total loans. The scale of their operations does not afford them to gain additional returns by increase in inputs at any factor. By (equiproportionate) raising their inputs also their outputs are expected to (equiproportionate) rise – provided that the settings of their operational environment remain the same – still, the intensity of increase in outputs is lower than that in inputs. Namely, the list of commercial banks with the status of decreasing returns to scale comprise CSOBISTRO, OTP (in the second and the third phase), POSTB (in the third phase), PRIMA (mostly during the investigated period), VUB, UNI, TB, SLSP and SBERBANK. Mostly increasing returns to scale were exhibited in the production operations of OTP (the first phase), POSTB (the first and the second phase) and PRIVAT. The status of CITI was somewhat erratic during the investigated period in order to infer firmer conclusions. Although SBERBANK and PRIMA were declared to be operating at decreasing returns to scale, in the third phase they are very close to the benchmark status of constant returns to scale.

A total of three notable implications may be inferred from these results.

First, the positions of individual commercial banks with respect to the (estimated) Pareto-Koopmans efficient frontier were during the period from 2000 to 2012 different. Most commercial banks conserved their position in the area of decreasing returns, only some smaller commercial banks were positioned in the area of increasing returns to scale (in all the three phases or in some of them). Their transition from the sphere of increasing returns to scale into the sphere of decreasing returns to scale of the production technology, and vice versa, happened alongside their economic optimization and adjustments induced by structural changes across the three periods recognized in the development of the Slovak banking sector. Eventually, only one bank (the small CITI which later transformed its status into a branch office of a foreign bank) found itself (and jittered) in the investigated period in the vicinity of constant returns to scale and switched its status between decreasing returns to scale and increasing returns to scale.

Second, if one is to occupy oneself with measuring technical efficiency of Slovak commercial banks under the production approach with reference this selection of inputs and outputs and with respect to the investigated period from 2000 to 2012, then he should opt for variable returns to scale. More ideally, especially decreasing returns to scale should be pre-set in DEA models as these are descriptive for the vast majority of Slovak commercial banks. It is evident that decreasing returns to scale capture best the empirical technology that is used in production of banking depository and creditory services. Nonetheless, one may still use DEA models in conjunction with the assumption of

IV: *The results and the identified scale for the Slovak commercial banks under consideration*

Scale elasticity in phase # 1					Scale elasticity in phase # 2					Scale elasticity in phase # 3				
Year	**Lower**	**Upper**	**Mean**	**RTS*)**	**Year**	**Lower**	**Upper**	**Mean**	**RTS*)**	**Year**	**Lower**	**Upper**	**Mean**	**RTS*)**
							CITI							
2000	1.304	6.821	4.063	IRTS	**2004**	NA	NA	NA	NA	**2009**	NA	NA	NA	NA
2001	0.175	17.706	8.941	IRTS	**2005**	NA	NA	NA	NA	**2010**	NA	NA	NA	NA
2002	0.075	∞	0.075	DRTS	**2006**	NA	NA	NA	NA	**2011**	NA	NA	NA	NA
2003	NA	NA	NA	NA	**2007**	0.321	0.482	0.401	DRTS	**2012**	NA	NA	NA	NA
---	---	---	---	---	**2008**	0.175	3.348	1.762	IRTS	**---**	---	---	---	---
							CSOBISTRO							
2000	0.605	0.666	0.636	DRTS	**2004**	0.590	0.743	0.667	DRTS	**2009**	0.686	0.957	0.821	DRTS
2001	0.453	0.757	0.605	DRTS	**2005**	0.700	0.820	0.760	DRTS	**2010**	0.701	0.959	0.830	DRTS
2002	0.440	0.635	0.537	DRTS	**2006**	0.710	0.827	0.768	DRTS	**2011**	0.689	0.958	0.823	DRTS
2003	0.404	0.600	0.502	DRTS	**2007**	0.759	0.859	0.809	DRTS	**2012**	0.796	0.959	0.877	DRTS
---	---	---	---	---	**2008**	0.788	0.877	0.832	DRTS	**---**	---	---	---	---
							OTP							
2000	0.182	0.357	0.269	DRTS	**2004**	0.428	0.616	0.522	DRTS	**2009**	0.849	1.010	0.929	DRTS
2001	0.175	17.706	8.941	IRTS	**2005**	0.540	0.705	0.623	DRTS	**2010**	0.839	1.011	0.925	DRTS
2002	0.175	17.706	8.941	IRTS	**2006**	0.488	0.666	0.577	DRTS	**2011**	0.834	1.011	0.922	DRTS
2003	0.175	17.706	8.941	IRTS	**2007**	0.554	0.716	0.635	DRTS	**2012**	0.837	1.011	0.924	DRTS
---	---	---	---	---	**2008**	0.597	0.748	0.672	DRTS	**---**	---	---	---	---

Scale elasticity in phase # 1					Scale elasticity in phase # 2					Scale elasticity in phase # 3				
Year	**Lower**	**Upper**	**Mean**	**RTS*)**	**Year**	**Lower**	**Upper**	**Mean**	**RTS*)**	**Year**	**Lower**	**Upper**	**Mean**	**RTS*)**
POSTB														
2000	0.175	17.706	8.941	IRTS	**2004**	0.175	3.348	1.762	IRTS	**2009**	0.914	1.008	0.961	DRTS
2001	0.204	0.342	0.273	DRTS	**2005**	0.175	3.348	1.762	IRTS	**2010**	0.910	1.008	0.959	DRTS
2002	0.175	17.706	8.941	IRTS	**2006**	0.175	3.348	1.762	IRTS	**2011**	0.522	1.008	0.765	DRTS
2003	0.175	17.706	8.941	IRTS	**2007**	0.297	0.500	0.398	DRTS	**2012**	0.655	1.007	0.831	DRTS
---	---	---	---	---	**2008**	0.456	0.640	0.548	DRTS	**---**	---	---	---	---
PRIMA														
2000	0.175	17.706	8.941	IRTS	**2004**	0.362	0.560	0.461	DRTS	**2009**	0.916	1.008	0.962	DRTS
2001	0.175	17.706	8.941	IRTS	**2005**	0.490	0.667	0.578	DRTS	**2010**	0.990	1.101	1.046	IRTS
2002	0.271	0.415	0.343	DRTS	**2006**	0.561	0.722	0.641	DRTS	**2011**	0.896	1.009	0.952	DRTS
2003	0.243	0.381	0.312	DRTS	**2007**	0.633	0.774	0.703	DRTS	**2012**	0.887	1.010	0.949	DRTS
---	---	---	---	---	**2008**	0.760	0.817	0.789	DRTS	**---**	---	---	---	---
PRIVAT														
2000	34.181	∞	34.181	IRTS	**2004**	7.762	∞	7.762	IRTS	**2009**	4.452	∞	4.452	IRTS
2001	6.318	8.297	7.307	IRTS	**2005**	4.651	8.921	6.786	IRTS	**2010**	1.608	6.199	3.903	IRTS
2002	9.991	14.118	12.055	IRTS	**2006**	6.327	∞	6.327	IRTS	**2011**	0.376	∞	0.376	DRTS
2003	30.077	∞	30.077	IRTS	**2007**	5.217	8.000	6.609	IRTS	**2012**	0.454	1.676	1.065	IRTS
---	---	---	---	---	**2008**	3.344	5.915	4.629	IRTS	**---**	---	---	---	---
SBERBANK														
2000	0.278	0.304	0.291	DRTS	**2004**	0.345	0.544	0.444	DRTS	**2009**	0.879	1.010	0.945	DRTS
2001	0.302	0.328	0.315	DRTS	**2005**	0.355	0.554	0.454	DRTS	**2010**	0.879	1.010	0.945	DRTS
2002	0.258	0.399	0.329	DRTS	**2006**	0.412	0.603	0.507	DRTS	**2011**	0.884	1.011	0.947	DRTS
2003	0.245	0.383	0.314	DRTS	**2007**	0.496	0.671	0.583	DRTS	**2012**	0.888	1.010	0.949	DRTS
---	---	---	---	---	**2008**	0.530	0.698	0.614	DRTS	**---**	---	---	---	---
SLSP														
2000	0.000	0.440	0.220	DRTS	**2004**	0.890	0.939	0.914	DRTS	**2009**	0.167	0.851	0.509	DRTS
2001	0.000	0.449	0.225	DRTS	**2005**	0.888	0.938	0.913	DRTS	**2010**	0.002	0.857	0.429	DRTS
2002	0.154	0.826	0.490	DRTS	**2006**	0.903	0.946	0.925	DRTS	**2011**	0.152	0.491	0.322	DRTS
2003	0.167	0.729	0.448	DRTS	**2007**	0.166	0.933	0.549	DRTS	**2012**	0.000	0.897	0.448	DRTS
---	---	---	---	---	**2008**	0.000	0.247	0.124	DRTS	**---**	---	---	---	---
TB														
2000	0.572	0.748	0.660	DRTS	**2004**	0.838	0.908	0.873	DRTS	**2009**	0.812	0.975	0.894	DRTS
2001	0.665	0.799	0.732	DRTS	**2005**	0.851	0.916	0.883	DRTS	**2010**	0.112	0.976	0.544	DRTS
2002	0.820	0.835	0.828	DRTS	**2006**	0.872	0.928	0.900	DRTS	**2011**	0.112	0.883	0.498	DRTS
2003	0.160	0.846	0.503	DRTS	**2007**	0.891	0.939	0.915	DRTS	**2012**	0.829	0.883	0.856	DRTS
---	---	---	---	---	**2008**	0.000	0.948	0.474	DRTS	**---**	---	---	---	---
UNI														
2000	0.319	0.348	0.333	DRTS	**2004**	0.519	0.689	0.604	DRTS	**2009**	0.707	1.006	0.857	DRTS
2001	0.317	0.346	0.332	DRTS	**2005**	0.523	0.693	0.608	DRTS	**2010**	0.737	0.948	0.843	DRTS
2002	0.345	0.378	0.361	DRTS	**2006**	0.503	0.677	0.590	DRTS	**2011**	0.615	1.033	0.824	DRTS
2003	0.412	0.452	0.432	DRTS	**2007**	0.550	0.930	0.740	DRTS	**2012**	0.626	1.047	0.837	DRTS
---	---	---	---	---	**2008**	0.833	0.884	0.858	DRTS	**---**	---	---	---	---
VUB														
2000	0.000	0.805	0.402	DRTS	**2004**	0.882	0.934	0.908	DRTS	**2009**	0.810	0.975	0.893	DRTS
2001	0.392	0.564	0.478	DRTS	**2005**	0.876	0.930	0.903	DRTS	**2010**	0.126	0.881	0.504	DRTS
2002	0.359	0.533	0.446	DRTS	**2006**	0.882	0.934	0.908	DRTS	**2011**	0.153	0.888	0.520	DRTS
2003	0.314	0.498	0.406	DRTS	**2007**	0.898	0.943	0.921	DRTS	**2012**	0.000	0.895	0.448	DRTS
---	---	---	---	---	**2008**	0.164	0.936	0.550	DRTS	**---**	---	---	---	---

Legend: *) RTS – returns to scale, IRTS – increasing returns to scale, DRTS – decreasing returns to scale

V: *The average scale elasticities for the Slovak commercial banks under consideration in the three phases*

Bank	Phase			Bank	Phase			Bank	Phase		
	# 1	# 2	# 3		# 1	# 2	# 3		# 1	# 2	# 3
CITI	4.359	1.082	NA	**CSOBISTRO**	0.570	0.767	0.838	**OTP**	6.773	0.606	0.925
POSTB	6.774	1.246	0.879	**PRIMA**	4.634	0.635	0.977	**PRIVAT**	20.905	6.423	2.449
SBERBANK	0.312	0.521	0.947	**SLSP**	0.346	0.685	0.427	**TB**	0.681	0.809	0.698
UNI	0.365	0.680	0.840	**VUB**	0.433	0.838	0.591	**---**	---	---	---

constant returns to scale, but this goes only with the full understanding that technical efficiencies calculated (estimated) under such an assumption does not reflect what truly holds for the production of Slovak commercial banks, but just what might be found reasonable and desirable from an economic (theoretical) point of view.

On the other hand, individual coefficients measuring the degree of scale elasticity in Tab. IV are valuable from a managerial perspective as they give an insight about reactions of output variables to changes of input variables. The smaller is the degree of scale elasticity, the lesser is the advantage of scale of which Slovak commercial banks may avail themselves. In such a case, one is free to modify the slogan "too big to fail" into "too big to expand" as with small degree of scale elasticity outputs react slowly to changes in inputs. The average values of degrees of scale elasticity for the eleven Slovak commercial banks subjected to the analysis structured according the three phases are reported in Tab. V. Putting CITI aside for its erratic behaviour, these averages imply that across the three phases of the investigated 13-year long period the scale elasticity of CSOBISTO, SBERBANK and UNI gradually increased in the direction of constant returns to scale, whereas POSTB, PRIMA as well as PRIVAT displayed a tendency to decrease their scale elasticity, again towards constant returns to scale. PRIVAT is a specific small bank that exhibited increasing returns to scale throughout the investigated period, during which it underwent a consolidation process and was able to optimize its scale and take advantage of its high scale elasticity. The largest commercial banks – SLSP, TB, VUB – showed rather law degrees of scale elasticity in all the three phases, without any apparent trending behaviour. Their production is clearly affected by the "too big to expand" maxim formulated above.

CONCLUSION

In the analyses of technical efficiency of the Slovak banking sector utilizing a DEA methodology, the previous studies – based on economic reasoning or qualified guess – adopted the assumption of either variable returns to scale or constant returns to scale to production operations of commercial banks. In some cases, concurrently both options were entertained, which permitted a comparative investigation of efficiency issues. In addition, the information on scale of operations is crucial as well in assessing the size of commercial banks and in appraising the effects of expanding or restricting production activities. In spite of the fact that the concept of returns to scale is an unavoidable ingredient to any DEA application and that it is important to economic and business management of commercial banks, the problem of determining the true nature of scale of operations in the Slovak banking sector has been neglected and has not been addressed properly. This issue has not received full understanding and it this maltreatment is remedied in the paper. Having its application in the field of Slovak banking for the period from 2000 until 2012 and from the viewpoint of the production approach, this paper modifies the DEA procedure for identification of scale proposed by Tone and Sahoo (Tone and Sahoo, 2004) and blends it with the slacks-based measure framework of DEA, which is believed to improve projections to technical efficiency required in the first step of this procedure.
The identification of scale in the Slovak banking sector was conducted under the production approach, under which banking production is treated as physical and economic transmutation of labour force and physical capital into depository and creditory services. Having adopted this scheme, the analysis covered the majority of the Slovak banking sector and was done for the 11 major Slovak commercial banks. Although branch offices of foreign banks and special banking institutions or building societies were omitted from the analysis due to high instability in provided services or atypical orientation with respect to the core of the Slovak banking sector, the analyzed sample of 11 commercial banks amounted to more than 90% of operations of the entire sector. The crucial ingredient of the methodological procedure was the division of the whole period of 13 years from 2000 to 2012 into three consecutive and separate phases of development, during which the production technology of the banking sector may be righteously thought of as constant and void of change. The breaking up the investigated period into the three phases, 2000–2003, 2004–2008 and 2009–2012, was defended by means of economic arguments pointing to changes happening between the phases. A partial justification was received when the trends in the four production variables (the number of employees, the volume

of fixed assets, the volume of total deposits and total loans) were examined and found in favour of consolidation and improvement of relative technical efficiency alongside these three phases.

The results confirm that the largest Slovak commercial banks were in the investigated period already "too large" so as to gain potentially some benefit from expanding their depository and creditory services by increasing their inputs. Most commercial banks operated at decreasing returns to scale, yet some of them moved throughout the three phases towards constant returns to scale, which is evidential in favour of their ability to implement necessary optimization measures and regulate their scale. Another implication, now in the sphere of technical efficiency measurement, is that in using DEA for the purpose measuring technical efficiency under the production approach a preference for variable returns to scale (or more appropriately, decreasing returns to scale) should be safely made as this is most characteristic of the Slovak banking sector during the investigated period from 2000 to 2012.

In the end, a caveat must be placed upon the meaning of returns to scale. One should not interchange the concept of scale elasticity with scale economy as it is done e.g. in Heffernan (2005, pp. 483). Returns to scale and scale elasticity as such are technical concepts that underlie and characterize production technology, but economies of scale are an economically-based concept that represents how production costs react to changes in inputs. As it is argued (e.g. Sahoo, Mohapatra and Trivedi, 1999, p. 380; Tone and Sahoo, 2003, p. 166), economies of scale may arise on account of five sets of factors, out of which just one is returns to scale. Another warning comes with the fact that identification of scale unfolded here in the paper takes no notice of circumstances of production that sometime happen to block the smoothness of the transformation process such as indivisibilities or congestion. Also the treatment of scale here is on the basis of equiproportionate changes and reactions, which is in line with the neoclassical convention. Contrary to these caution-urging remarks, the concept of returns to scale is still a matter of interpretation, not an issue of its validity as such.

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Contact information

Martin Boďa: martin.boda@umb.sk