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ON THE IDEALITY OF FILTERING TECHNIQUES IN THE BUSINESS CYCLE ANALYSIS UNDER CONDITIONS OF EUROPEAN ECONOMY

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Abstract

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The paper deals with the applications and comparison of various filtering techniques, which is an integral part of the business cycles identification and their further analysis. In particular, the paper examines four high-pass and band-pass filters and compares them to an ideal filter. The gain function is estimated and shown by the periodogram to assess the filters' efficiency in the extraction of undesired periods in the time series. The monthly data of industrial production indices of chosen EU countries are used to analyse the filtering techniques. The results show the Butterworth filter as a high pass and the Christiano-Fitzgerald filter as a band-pass to be closer to an ideal filter than the Hodrick-Prescott and Baxter-King filters.

Keywords: Baxter-King filter, business cycle, Butterworth filter, Hodrick-Prescott filter, Christiano-Fitzgerald filter, gain function

INTRODUCTION

Business cycle analysis is currently widely applied in the literature on economics of monetary integration. In addition to that business cycle characteristics are analysed and compared in official convergence reports and other studies published by national governments, central banks and institutions within the European Union. The methodological and theoretical framework for using the business cycle similarity, convergence and other characteristics is provided by the Optimum Currency Areas (OCA) theory firstly introduced by Robert Mundell in his pioneering article (Mundell, 1961). The empirical meaning of the business cycle measures was explained and developed in the "New" optimum Currency Area Theory for instance in papers by Tavlas (1993) and Mongelli (2002). The dissection of cyclical components within macroeconomic time series, approximating the aggregate economic activity of a nation, is an integral part of the business cycle analysis methodology. Various detrending techniques are used in the literature to dissect the cyclical component and identify the cycles in the original time series. Detrending is in line with the technical definition of the growth business cycles which are considered as fluctuations of the cyclical component of the macroeconomic time-series around its trend (Lucas, 1977). Contrary to growth cycles, the literature defines classical cycles or regular fluctuations of national economic activity measure in absolute levels (Burns and Mitchell, 1946). The vast majority of business cycle literature uses the growth cycle definition, which requires the application of some of the many existing detrending techniques. In line with the OCA theory, the correlation of growth cycles is considered an important criterion of economic similarity of integrating countries. Various measures of growth business cycle correlation were applied in recent literature dealing with the European monetary integration process. Fidrmuc and Korhonen (2006) provide a meta-analysis of studies examining the business cycle correlation between the euro area and Central and Eastern European countries. Considering the growth business cycle literature applying various detrending techniques, recent papers by Fidrmuc, Korhonen and Poměnková (2014); Bátorová, Fidrmuc and Korhonen (2013) as well as Fidrmuc and Korhonen (2010) can be mentioned.

The detrending techniques, which are widely called filters, usually differ in the way of estimating the cyclical and other components. The filtered time series also have different characteristics depending on the filters applied. Apart from the time domain, the filters operating in the frequency domain have become more popular in recent literature. The pros and cons of applying various techniques are described by Canova (1998, 1999), King and Rebelo (1993), Schenk, Hopé (2001) or recently by Estrela (2007), Watson (2007), Buss (2011) and Grochová, Rozmahel (2012). Apart from dissecting the time series, the filters are also used to estimate the potential product (Garrat *et al.*, 2014) as well as the equilibrium exchange rate (Issever Grochová and Plecitá, 2013).

Regarding the existing variety of detrending techniques and filters, the paper aims at evaluating the appropriateness of selected filters using the macroeconomic data of selected EU countries. In particular, four selected high-pass and band pass filters are applied and compared to the ideal filter defined to assess their ideality under conditions of European data. The main goal of the paper is to evaluate the appropriateness of selected detrending filters by estimating their gain functions and comparing them to an ideal filter. Regarding the comparison of the analyzed filters to an ideal filter, the paper asks: How close is the real gain of a filter when compared to an ideal gain? How well do the filters perform for real EU economies? Answering those questions and providing some evidence of functionality and efficiency of the filters, in the sense of leaving desired and blocking unwanted frequencies from the spectra of the time series, the paper intends to contribute to recent literature discussing the appropriateness of various filtering techniques and their types in the business cycle analysis.

The paper is structured as follows. After this introduction of the purpose and motivation of the research paper, the second section explains the methodology and data. The main results are described in the third section. The fourth section concludes.

METHODOLOGY AND DATA

The cyclical component of the natural logarithm of the industrial production index is estimated applying four different high-pass and band-pass filters. The filtered time series are analyzed and compared to an ideal filter to assess the ideality and appropriateness of all examined filters. In particular, the evaluation of high-pass filtering techniques (one-parameter Hodrick-Prescott filter and two-parameter Butterworth filter) and band-pass filtering techniques (Baxter-King and Christiano-Fitzgerald filters) using macroeconomic data of selected European countries for the time span of 01/1993–12/2013 is done applying the periodograms and gain functions. Regarding the limited availability of data the following EU countries are chosen for the analysis: Austria, Belgium, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Luxembourg, Netherlands, Poland, Portugal, Slovakia, Spain, Sweden and United Kingdom.

The time-series filters decompose a time-series into trend and cyclical components:

$$y_t = \tau_t + c_t, \tag{1}$$

where τ_t is the trend component and c_t is the cyclical component. The cyclical component, c_t, is considered as an approximation of the growth business cycle from the OCA theory perspective and can be written as:

$$c_t = B(L)y_t. \tag{2}$$

The ideal filter, $B(L)$, is in time domain defined as:

$$B(L) = \sum_{j=-n}^{n} B_j y_{t-j}\,,\ L^j y_t \equiv y_{t-j}, \tag{3}$$

where B_j are the impulse–response coefficients of an ideal filter computed dependently on the filter used.

In spectral representation, the time-series can be expressed as:

$$y_t = \mu + \int_{-\pi}^{\pi} e^{i\omega t} f_y(\omega) d\omega\,, \tag{4}$$

$f_y(\omega)$ being the spectral density function of y_t, and i being imaginary number. The spectrum of the cyclical component, $f_c(\omega)$, is given as:

$$f_c(\omega) = |B(\omega)|^2 f_y(\omega), \tag{5}$$

where $|B(\omega)|$ is the filter's gain function and $B(\omega) = |B(\omega)|e^{i\theta(\omega)}$, $\theta(\omega)$ being the filter's phase function. The impact of the filter on frequency domain characteristics of the series is given by the frequency response function that can be written as:

$$B(\omega) = B(e^{-i\omega}) = \sum_{j=-n}^{n} B_j e^{-i\omega j}. \tag{6}$$

The frequency response function can also be expressed as combination of a sine and cosine functions:

$$B(\omega) = C(\omega) + iS(\omega), \tag{7}$$

where

$$C(\omega) = \sum_{j=-m_1}^{m_2} w_j \cos(\omega j) \text{ and } S(\omega) = \sum_{j=-m_1}^{m_2} w_j \sin(\omega j)\,.$$

The gain function of a filter can be then also written as:

$$B(\omega) = \sqrt{(C(\omega)^2 + S(\omega)^2)}\,. \tag{8}$$

The frequency domain of band-pass filters can be then specified for monthly data as:

$$B(\omega) = B(e^{-i\omega}) = \begin{cases} 1 \text{ for } \omega \in \left[\frac{2\pi}{96}, \frac{2\pi}{18}\right] \vee \left[\frac{-2\pi}{18}, \frac{-2\pi}{96}\right] \\ 0, \text{otherwise} \end{cases}, \tag{9}$$

and of high-pass filters as:

$$B(\omega) = B(e^{-i\omega}) = \begin{cases} 1 \text{ for } \omega \in \left[\frac{2\pi}{96}, \pi\right] \vee \left[-\pi, \frac{-2\pi}{96}\right] \\ 0, \text{otherwise} \end{cases}. \tag{10}$$

Filtering techniques then approximate c_t by $\hat{c}_t$. A more detailed review of various filtering techniques used in this contribution and the methodology of their appropriateness assessment are provided in the following subsections.

Hodrick-Prescott Filter

The Hodrick-Prescott filtering technique (HP filter) filters data by removing the trend. The smoothness of the trend depends on a parameter λ, with the trend becoming smoother as $\lambda \to \infty$. Hodrick and Prescott (1997) recommended setting λ to 14,400 for monthly data. King and Rebelo (1993) demonstrate that removing a trend with the HP filter corresponds to a high-pass filter as it filters low-frequencies and passes through high-frequency components in c_t. The trend component is computed solving:

$$\min_{\tau_t}\left(\sum_{t=1}^{T}(y_t - \tau_t)^2 + \lambda\sum_{t=2}^{T-1}\left[(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})\right]^2\right). \tag{11}$$

The HP filter in a high-pass transformation can be written, in the time domain, as:

$$B^{HPhigh}(L) = \frac{\lambda(1-L)^2(1-L^{-1})^2}{1+\lambda(1-L)^2(1-L^{-1})^2}. \tag{12}$$

Differentiating eq. (7) with respect to τ_t, the cyclical component can be then expressed as two-sided moving average of y_t:

$$\hat{c}_t = B^{HPhigh}(L)y_t, \tag{13}$$

where λ is a smoothing parameter (set to 14,400 for monthly data) and L is the lag operator.

Butterworth Filter

The Butterworth (BW) filter is based on backward and forward smoothing operation of the time series filtered (Gomez, 2002). The high-pass BW filter, in the time domain, can be expressed as:

$$B_n^{BW}(L) = \frac{\lambda(1-L)^n(1-L^{-1})^n}{(1+L)^n(1+L^{-1})^n + \lambda(1-L)^n(1-L^{-1})^n}, \tag{14}$$

where L is lag operator, $n \in N$,

$$\lambda = \left\{\tan\left(\frac{\omega_c}{2}\right)\right\}^{-2n},$$

and ω_c is the cutoff point (set to $2\pi/96$ for monthly data). The approximation of high-frequency cyclical component, $\hat{c}_t$, is calculated as:

$$\hat{c}_t = B_n^{BW}(L)y_t. \tag{15}$$

Baxter-King Filter

Baxter and King (1999) divide the time-series into 3 components: trend, cycle, and irregular fluctuations. As we are interested in monthly data, business cycles for this data are, according to Burns and Mitchell (1946), defined as periodical components ($f_c(\omega)$) with frequencies lying between 1.5 and 8 years per cycle, i.e. 18 and 96 months per cycle, with K = 36. Frequencies longer than 96 months per cycle, corresponding to a trend, and shorter frequencies than 18 months per cycle, considered irregular fluctuations, are then suppressed.

To isolate the business cycle component from the remaining components, a two-sided finite-order (K) moving average is performed:

$$\hat{c}_t = \sum_{h=-K}^{K} B_h^{BK} y_{t-h}\,, K+1 \le t \le T-K. \tag{16}$$

The longer the moving average, the better the approximation but also the higher the amount of observations to be dropped. Baxter and King find that three years of observations (i.e. K = 36 months) should be dropped to make the filter perform well.

Weights obtained from the minimization problem in the frequency domain are:

$$\min_{\hat{B}_j^{BK}, j=-K,\dots,K} = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|B(e^{-i\omega}) - \hat{B}^{BK;K,K}(e^{-i\omega})\right|^2 d\omega. \tag{17}$$

Christiano-Fitzgerald Filter

The Christiano-Fitzgerald random walk (CF) filter is constructed on the same principles as the BK filter, formulating the detrending and smoothing problem in the frequency domain (2003). In contrast to the BK filter, no observations have to be dropped by the CF filter. The cyclical component is approximated as follows:

$$\hat{c}_t = \sum_{h=-K}^{K} \hat{B}_j^{CFp,f} y_{t-j}\,, \tag{18}$$

where $f = T - t$, $p = t - 1$, $\hat{B}_j^{p,f}$ being a solution of the minimization problem in the frequency domain.

$$\min_{\hat{B}_j^{CFp,f},\, j=-f,\dots,p} = \int_{-\pi}^{\pi} \left| B(e^{-i\omega}) - \hat{B}^{CFp,f}(e^{-i\omega}) \right|^2 f_y(\omega) d\omega\,. \qquad (19)$$

Filters' Appropriateness Assessment

An economic time-series consists of a wide range of cycles such as trend, business cycles, and noise. In the frequency domain, each time-series can be defined as an infinite set of periods and amplitudes. Then the trend corresponds to components with an infinite period, very short periods represent noise and the remaining frequencies are business cycles (Azevedo, 2002).

The filters presented above should eliminate either high (HP filter and BW filter) or high and low (BK filter and CF filter) frequencies. How these filters are efficient in the extraction of undesired periods is given by the gain function and shown by periodogram.

A well-performing filter is able to extract the periodical component, c_t, of a time series of desired frequencies. An ideal filter passes or blocks the stochastic cycles at given frequencies by having a gain of 1 or 0, respectively. For monthly data, high-pass filters pass through the stochastic cycles from a specified frequency $\omega_0 = 2\pi/96$ and block lower frequencies. Band-pass filters allow stochastic cycles in the specified range of frequencies [$\omega_0 = 2\pi/96$, $\omega_1 = 2\pi/18$], corresponding to a cycle length of 1.5–8 years expressed in months, to pass through and block all the other frequencies. The real filter multiplies amplitude of y_t by the gain $|B(e^{-i\omega})|$ which determines how the amplitude of a time series is increased or reduced at frequency ω. It is required that $f_c(\omega) = 0$ in the frequency range $\omega < 2\pi/96$ or $\omega < 2\pi/96 \wedge \omega > 2\pi/18$, respectively, and $f_c(\omega) = f_y(\omega)$ for desired frequencies such as $\omega \geq 2\pi/96$ and $2\pi/96 \leq \omega \leq 2\pi/18$ for monthly data, respectively, dependent on the filter used. To sum up, gain functions report the effect of a filter on the amplitude of a cycle.

The evidence of how well the cyclical component c_t was estimated, is presented as a periodogram (Estrella, 2007). A periodogram is an estimator of the spectral density function of y_t, $f_y(\omega)$. A periodogram identifies dominant frequencies in an observed time series. A relatively large value of a periodogram indicates relatively more importance for the respective frequency in explaining the periodicity in the observed time series. That is, if applied to the filtered series, only desired frequencies should appear to be dominant. If the filter completely drops the unwanted frequencies, i.e. it behaves ideally, the log periodogram is a flat line at the minimum value of –6 for unwanted frequencies. The log periodogram values greater than –6 at unwanted frequencies indicate a limited ability of the filter to block undesired frequencies outside the specified band [$\omega_0 = 2\pi/96$, $\omega_1 = 2\pi/18$]. For our purpose the log periodogram displays natural frequencies, that is standard frequencies divided by 2π.

EMPIRICAL RESULTS

In this section we present time-series filtered with the HP, the BW, the BK and the CF filters and possible ways of assessing the appropiateness of these filtering techniques – gain function and periodograms. First, in Fig. 1 and Tab. I cyclical components of logs of industrial production in the EU countries using various filtering techniques are shown. The most volatile cyclical components are obtained by the HP filter, followed by the BW filter, which produce frequent turning points in the detrended time series. Both filters leave components of higher frequencies in the time series whereas the BK and the CF filters remove them. The cyclical components produced by the band-pass filters are almost identical as they remove both low and high undesired frequencies. However, the BK filter drops three-year observations from the tails of filtered series. This also causes the BK filterer series to have the highest standard deviation. In contrast, the lowest standard deviation is observed in the BW cycles. Furthermore, the greatest changes in log of industrial production with remarkable deviations can be seen in Estonia, Hungary, Luxembourg, and Slovakia, while the smallest changes with less deviations are in Belgium, France, Netherlands, and the United Kingdom. In general, similar patterns are well visible during economic crises for all economies analyzed.

Gain Function

As mentioned above, the amplitudes of cycles are affected by the gain of the filter. The ideal and real gains of the high-pass and band-pass filters are reported in Fig. 2 and their numerical characteristics can bee also seen in Tab. II.

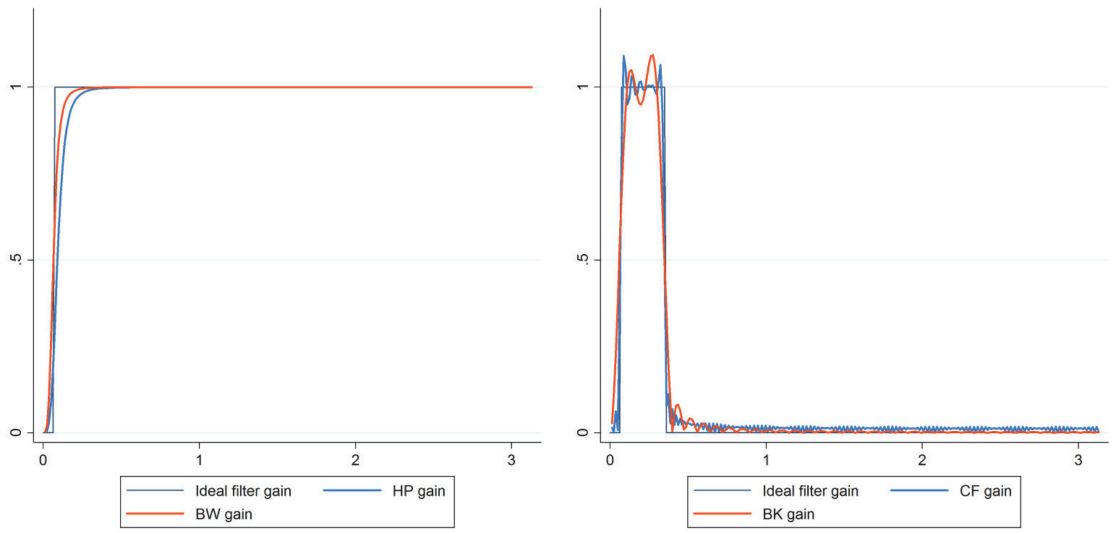
The gains of the filters used are compared to the ideal filter gains. The high-pass filters are designed to remove low frequencies. Both filters attribute a gain greater than 0 to the cutoff frequency so the overall variance of the series is exaggerated, while immediately after the cutoff frequency the components are multiplied by a gain less than 1. This is more remarkable in case of the HP filter, for which the gain equals one at higher frequencies when compared to the BW filter. Also the average gain for the desired frequencies is greater when the BW filter is used. As a result the BW filter should perform better as it passes desired frequencies with a more proper gain (of 0.997). The band-pass filters more or less precisely copy the ideal band-pass filter removing both very short and very long cycles. The CF filter is more efficient in suppressing the undesired frequencies, giving a lower gain to these amplitudes. This is also proved by the average gain of 0.015 which is less than the average of the BK filter. Moreover, even if the CF filter gives more

1: *Business cycles in the chosen EU countries*
Source: Authors' calculations

I: *Descriptive statistics of cyclical components*

Variable	Obs	Mean	Std. Dev.	Min	Max
ip_bw	4980	0.000	0.022	–0.143	0.124
ip_hp	4980	0.000	0.034	–0.216	0.149
ip_bk	3540	0.002	0.041	–0.259	0.143
ip_cf	4980	0.000	0.037	–0.253	0.144

Source: Authors' calculations

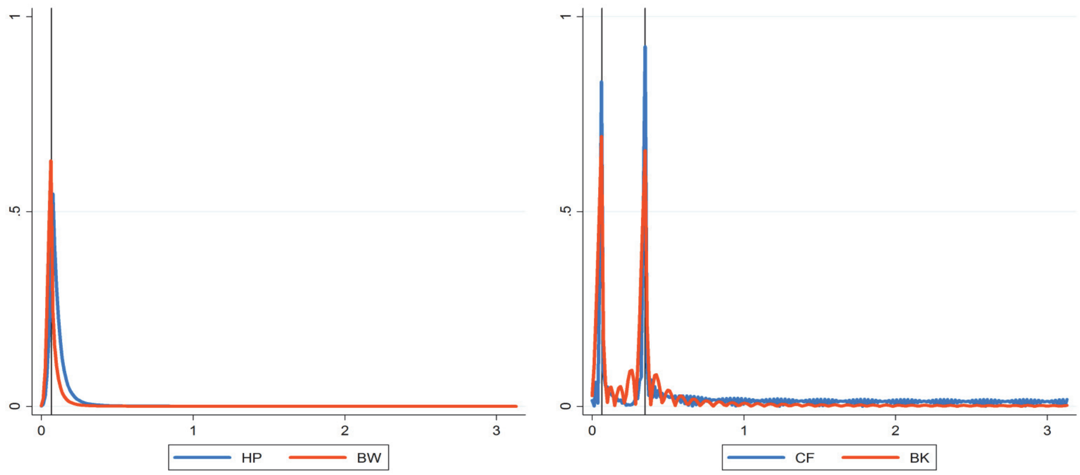


2: *Sample real and ideal gain functions B(ω) (gain on the vertical and frequency on the horizontal axis)*
Source: Authors' estimations

II: *Average values of the real gain functions*

	Mean	Std. Dev.	Min	Max	Mean	Std. Dev.	Min	Max
Variable	**undesired frequencies**				**desired frequencies**			
BW	0.242	0.233	0.001	0.631	0.997	0.020	0.760	1.000
HP	0.101	0.112	0.000	0.311	0.991	0.051	0.455	1.000
BK	0.019	0.065	0.000	0.693	0.878	0.272	0.000	1.092
CF	0.015	0.058	0.000	0.833	0.943	0.212	0.077	1.090

Source: Authors' calculations



3: *Absolute difference between real and ideal gain (gain on the vertical and frequency on the horizontal axis)*
Source: Authors' estimations

III: *Average absolute deviation from ideal filter*

Variable	Obs	Mean	Std. Dev.	Min	Max
BW	4980	0.009	0.054	0.000	0.631
HP	4980	0.011	0.055	0.000	0.545
BK	5040	0.028	0.106	0.000	1.000
CF	4980	0.025	0.084	0.000	0.923

Source: Authors' calculations

than ideal importance to the amplitudes at the cutoff frequencies, the gain of desired frequencies corresponds better to those of the ideal filter, which can be seen in the average gain of 0.943. Thus the CF filter is demonstrated to be more efficient than the BK filter, as it passes through more components with fluctuations of 18 to 96 months. When comparing high-pass and band-pass filters, it is obvious that high-pass filters do not suppress the noise and thus show higher volatility in the filtered series.

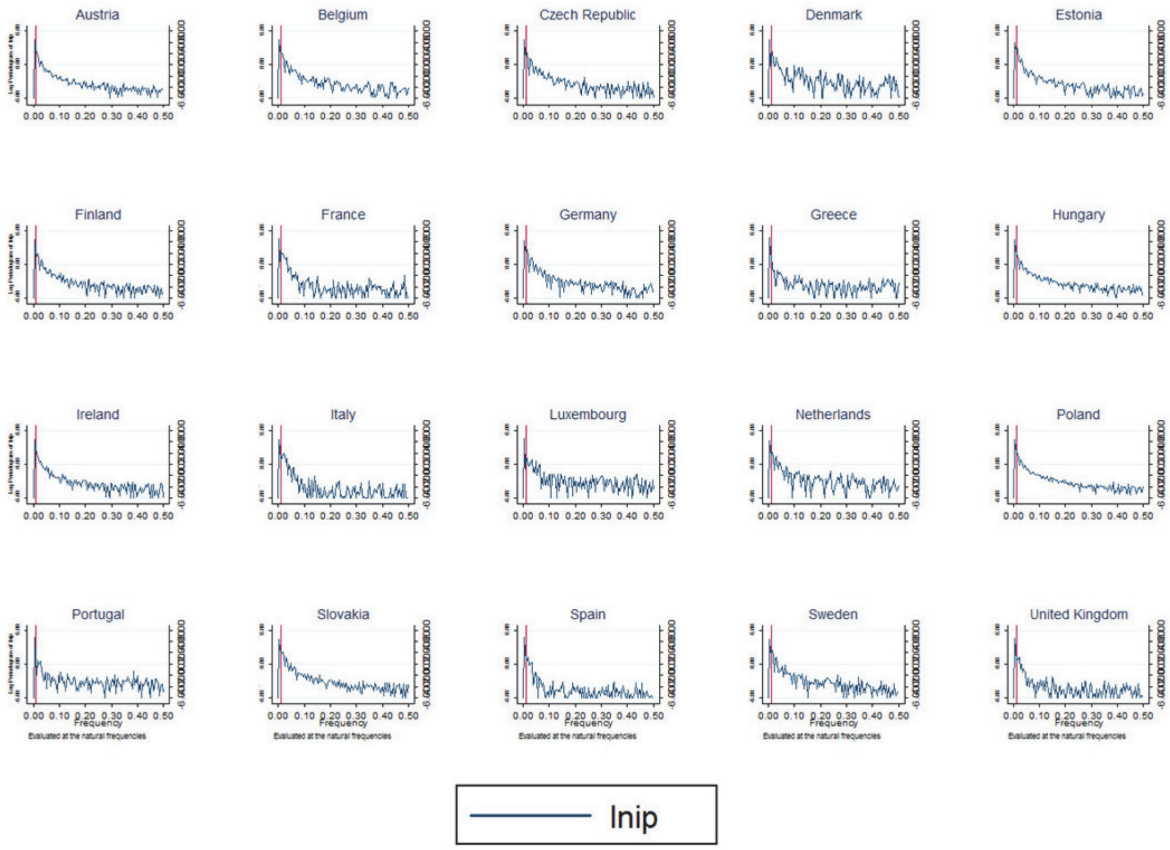
The differences between the chosen and ideal filters is summed up in the following Fig. 3 and Tab. III that show how the gains deviate from the ideal gain.

As can be seen, the BW filter gain is further from the ideal gain in the cutoff frequency but the deviation is present for less frequency, which favours the BW filter when compared to the HP filter. The same pattern can be observed for the CF filter gain, whose distance from the ideal filter is greater than the BK, but the deviation is less frequent.

Periodograms

After this general comparison of the filtering techniques, we can follow with the second part of the filters' approriateness assessment, i. e. periodograms or, in other words, sample spectral density functions. Figs. 4 and 5 present periodograms of logs of industrial production in the chosen countries. The aim is to show, how the filters are able to eliminate undesired and pass through desired frequencies. We report periodograms of filtered and non-filtered series so that we can compare them, verifying, first, how well the particular filter suppresses undesired and passes through desired frequencies, and, second, how the original amplitudes are modified by the filters. In particular, the HP and BW filters should eliminate all frequencies less than $2\pi/96$, i.e. natural frequencies less than 1/96, and for greater frequencies greater then the cutoff frequency of the original series should be identical to the filtered one.

As can be seen in Fig. 5, the HP filter suppresses well the undesired low frequencies in Finland,

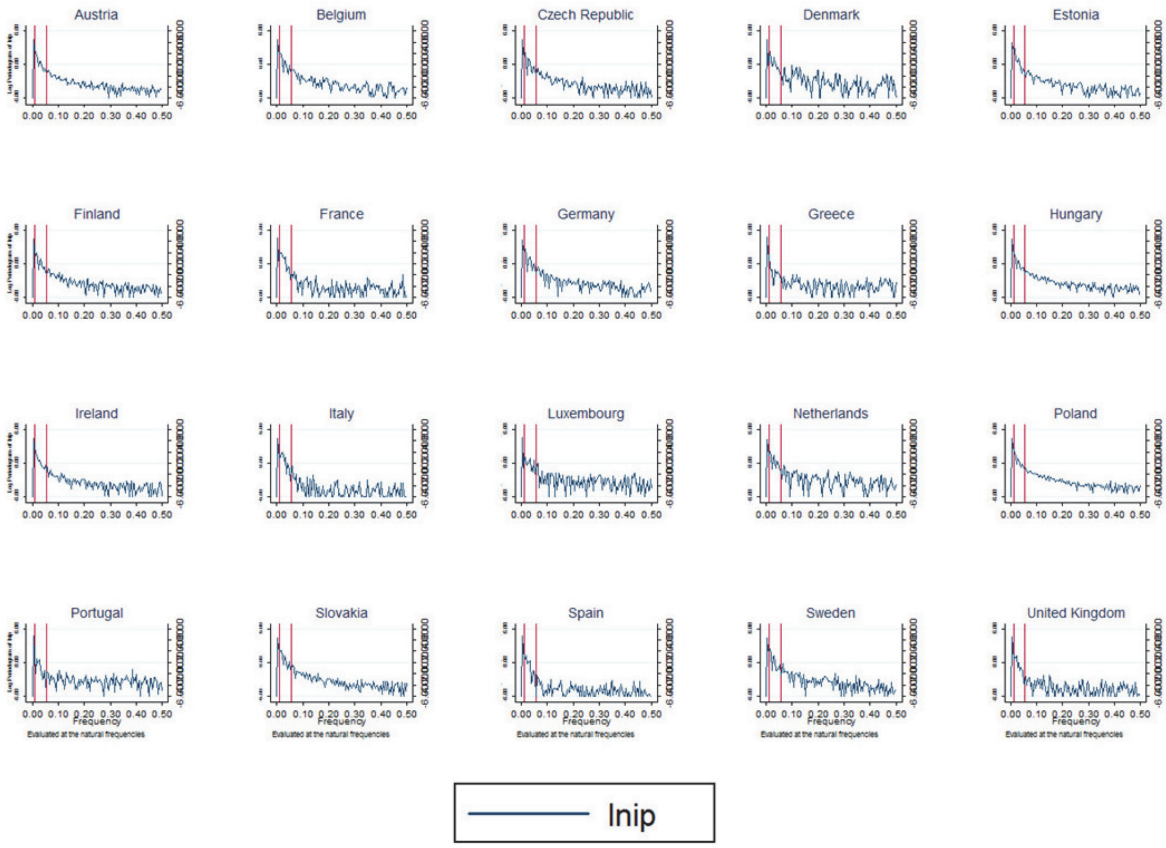


4: *Sample spectral density functions of nonfiltered series*
Source: Authors' calculations

Germany, Italy, Netherlands, Slovakia and Sweden. The BW filters performs better as it filters out the long cycles also in Austria, Belgium, Denmark, Estonia, France, Greece, Hungary, Ireland, Luxembourg, Poland, Portugal and the United Kingdom. The only imperfect performance can be seen in the Czech Republic and Spain. In contrast, the HP filter fails to drop low frequencies in Estonia.

As for the band-pass filters shown in Fig. 7, only the range of frequencies between 2π/96 and 2π/18 which corresponds to the interval of natural frequencies (1/96, 1/18), should be passed. In general, undesired low frequencies are eliminated similarly by both filters. The CF filter is more efficient in blocking the undesired high frequencies in 11 out of 20 cases. In two cases the CF and BK filters perform equally well. Only in 7 series better results were provided by the BK filter.

5: *Sample spectral density functions of series filtered by the HP and BW filters*
Source: Authors' calculations



6: *Sample spectral density functions of non-filtered series*
Source: Authors' calculations

7: *Sample spectral density functions of series filtered by the CF and BK filter*
Source: Authors' calculations

CONCLUSION

The optimum currency area theory widely uses various business cycle characteristics to assess the appropriateness of countries to join the common monetary union. Especially, the latter development of the theory known as the "New" OCA theory focuses on empirical application of various OCA criteria (Mongelli, 2002). Mostly, the similarity of business cycles and their convergence is used to evaluate the countries. In reality, the business cycles characteristics of the Euro area member countries as well as candidate countries, particularly the Central and Eastern European countries, are frequently assessed in the regular convergence reports as well as in the research papers (Fidrmuc and Korhonen, 2006). However, the theory is not unified in an appropriate filtering technique to be applied to dissect the cyclical component of the analysed macroeconomic time series when identifying the business cycles. In this paper, four different detrending techniques are evaluated and compared to an ideal filter. Particularly, the gain function is estimated and shown by the periodogram to assess the filters' efficiency in the extraction of undesired periods in the time series. The results show the advantages and disadvantages of the analysed high-pass and band-pass filters. In general, the high pass filters leave the cyclical component as well as the high "noise" frequencies, whereas the band pass filters remove the high frequencies from the spectra. Focusing on the high pass filters the Butherworth filter acts closer to an ideal filter than the Hodrick-Prescott filter. The time series filtered by the Butterworth filter show larger but fewer differences from the ideal filter than the Hodrick-Prestcott filter. One might consider this finding as a surprise, since the Hodrick-Prescott filter is among the most popular filters used in business cycle analysis in the literature. Considering the band-pass filters, the Christiano-Fitzgelrald filter shows fewer differences from an ideal filter than the Baxter-King filter. In addition to that, 6 years of observations have to be cut off when using the Baxter-King filter to avoid the end-sample problem. In general, the Butterworth and Christiano-Fitzgerald filter are more appropriate to leave the desired and block undesired frequencies from the spectra. These filters also do not change the amplitudes of the series too much, compared to the other analysed filters. Comparing the Butterworth and the Christiano-Fitzgerald filter, the latter remains as the most appropriate filter from the sample. In 15 of 20 cases the Christiano-Fitzgerald filter behaves closely to an ideal filter and eliminates the majority of unwanted frequencies. In summary, our comparative analysis shows the necessity to interpret the results of business cycle analysis with respect to different characteristics and impacts of applied filters. Therefore the paper intends to warn against absolute and mechanistic interpretation of business cycle analysis, when different filtering techniques are used. Instead, the paper recommends interpreting the results in the context of employed methodology and its possible limitations.

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