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DIFFERENT VERSIONS OF THE SAVINGS METHOD FOR THE TIME LIMITED VEHICLE ROUTING PROBLEM

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Abstract

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The time limited vehicle routing problem (TLVRP) stems from the vehicle routing problem. The main difference is that the routes are paths (not cycles), i.e. vehicles do not return to the central city (or at least we do not observe their way back). Costs are given for the straight routes between each pair of the cities and represent the time necessary for going through. Each path must not exceed a given time limit. The sum of times for all routes is to be minimized.
This problem is NP-hard. There are many various possibilities how to design the heuristics (approximation methods) to solve it. One of the ways of how to obtain an approximation method for the TLVRP is to modify the famous savings method by Clark and Wright (1964) for this purpose. In this paper we suggest several different versions of this method, test them in some instances, and evaluate and mutually compare the results of individual versions.

time limited vehicle routing problem, vehicle routing problem, traveling salesman problem, savings method

The problem of the delivery optimization of specific material can in reality be encountered very often. The delivery is usually realized by a circular or round trip which, in comparison with the realization of each route from the supplier to the consumer, saves expenses for individual gateways from the same supplier and/or trips to one consumer. There exist many tasks of this kind and, in general, they are referred to as vehicle routing problems (VRP). Practically all of them belong to the NP-hard problems (Lenstra and Rinnooy Kan, 1981; Laporte, 1992), for which there is no efficient algorithm finding their theoretical optimum. Thus the only way to obtain efficiently or in a reasonably short time some solution is to use some of heuristics (approximation methods), which give only a "good" or "close to optimal" solution, not exactly optimum. Because of the variety of the VRPs, they are, in general, studied relatively often, recently by e.g. Borgulya (2008); Thangiah, Fergany, and Awan (2007); and Belfiore and Favero (2007).

The time limited vehicle routing problem (TLVRP) is a special case of the open vehicle routing problem (OVRP). The main feature of the OVRP is that the routes are paths (not cycles), i.e. vehicles do not return to the central city (or at least we do not observe their way back). The first proposal of an approach to solving the OVRP was by Bodin, Golden, Assad and Ball (1983), even for a case study with time windows. Brandão (2004) and Fu, Eglese and Li (2005) carried out the tabu search heuristic to solve the OVRP with constraints on the vehicle capacity and maximum route length. Tarantilis, Ioannou, Kiranoudis and Prastacos (2004) proposed annealing-based methods. Pisinger and Ropke (2007) presented a large neigbourhood search heuristic. Poli, Kennedy and Blackwell (2007), Li, Golden and Wasil (2007) and MirHassani and Abolghasemi (2011) proposed particle swarm approaches. Other most recent methods are e.g. by Zachariadis and Kiranoudis (2009), Repousis, Tarantilis, Bräysy and Ioannou (2010), Salari, Toth and Tramontani (2010) and Yu, Ding and Zhu (2011).

All the problems mentioned above had vehicle capacity constraints, in contrast to the TLVRP, as follows from the definition below.

The VLVRP is defined as follows: One central city and other n (ordinary) cities are given and for each pair of the cities a cost is given, representing the time necessary for travelling along the straight route between them. The cost matrix is supposed to be symmetric. The goal is to find a set of paths so that each of them has one of its endpoints in the central city, its length does not exceed a given time limit L and each city lies exactly on one of the paths except for the central city.

Let us introduce the following notation. The central city will be indexed by 0 and the other cities by numbers from 1 to n. The cost matrix will be denoted by $\mathbf{C}$ (and so single costs c_{ij}, i, $j = 0, \ldots, n$). The decision variables x_{ij} are bivalent and indicate whether the straight route from the city i to city j is in the solution, the decision variables u_i present the time the vehicle spends travelling along the route from city 0 to city i. M is a sufficiently large constant (in comparison to L and c_{ij}). The mathematical model of the TLVRP according to Pelikán (2006), where it is first defined, is:

$$z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \to \min$$

$$\sum_{j=0}^{n} x_{ij} = 1,\ i =, 1, 2, \ldots, n$$

$$\sum_{i=0}^{n} x_{ij} = 1,\ j =, 1, 2, \ldots, n$$

$$u_i + c_{ij} - M(1 - x_{ij}) \leq u_j,\ i = 0, 1, 2, \ldots, n, j = 1, 2, \ldots, n, i \neq j$$

$$u_i \leq L,\ i = 0, 1, 2, \ldots, n.$$

The objective function z represents the total length (time) of all the routes. The first two groups of equations assure that just one edge goes from or to each city. Then the constraints which define the sense of the variables u_i as mentioned above follow and the last group of constraints assures that no route exceeds the time limit L.

This problem has many practical instances, e.g. the transportation of newspapers from a publishing house to shops, grocery products (dumplings etc.) from a manufacturer to restaurants, daily reports from affiliated branches to headquarters (this example is mentioned in Pelikán, 2006) etc. Each vehicle is required to visit all the cities on its route until a given time, but we do not mind how it gets from the end back to the start of its route to realize it next time.

Nevertheless, the TLVRP has been studied for quite a short time, and relatively little. It stems from the VRP, and thus from the traveling salesman problem (TSP). The task of TSP is to construct one cyclic route containing all the cities. Thus the heuristics (approximation methods) for the TLVRP can be derived from the methods for the VRP and the TSP in a similar manner as is usual in the case of other related NP-hard problems (e.g. Blazsik, Imreh, and Kovacs, 2008).

Generally, we can differentiate two basic types of heuristics: the first one constructs a solution (from the beginning) while the second one improves an initial solution (which is either randomly given or obtained using another approximation method). For deriving methods for the TLVRP (or some of other related problems) from methods for the TSP, the former type is more suitable because the latter one, improving the solution, utilizes special properties of TSP solutions which are hardly modifiable for the solutions of different tasks. Perhaps the most famous method improving the solution is by Lin and Kernighan (1973).

One of well-known approximation methods for the TSP constructing a solution is the savings method by Clarke and Wright (1964). The aim of this paper is to present some of its modifications for the TLVRP and test it on several examples. Its algorithm is described in the following chapter.

Other examples of heuristics for the TSP constructing a solution are the nearest neighbour method, tree approaches, nearest merger method and insertion methods. All of them were investigated by Rosenkrantz, Stearns, and Lewis (1977). The modification of the first one for the TLVRP is used in this paper to compare the results and evaluate single versions of the savings method for the TLVRP proposed here. The most accurate method for the TSP is the Christofides method (1976) based on the combination of the tree approach and the construction of minimum matching which always finds a solution at most 1.5-times longer than the optimum. Let us mention also the patching method by Karp (1979) and the loss method by Webb (1971) and Van der Cruyssen and Rijckaert (1978).

MATERIALS AND METHODS

Savings method for the TSP

The savings method (SM) is based on comparing lengths of a straight route between any two cities and a route via another selected city, using the following algorithm:

1. Choose arbitrarily one city. Denote it 0.
2. For all pairs of other cities (i, j) compute the savings $s_{ij} = c_{i0} + c_{0j} - c_{ij}$.
3. Process the edges (straight routes between pairs of cities) according to the descending order of the savings s_{ij} using the following rules: When by adding an edge we obtain a set of vertex disjoint paths, we add it to the solution.
4. Repeat the procedure until the Hamiltonian path containing all the cities except the central one has been created.

5. Finally, add the city 0 to close the cyclic route.

It is recommended to use all the cities as the city 0 and select the best result.

Savings method for the TLVRP

To solve the TLVRP the edges from/to the central city are important, because they are more frequently used than other edges. Therefore, modifying this method for the TLVRP, the choice of the central city as the city 0 from the TSP algorithm will be natural and appropriate.

Pelikán (2006) presents the following modification for the TLVRP which we will denote as a **sequential savings method** (SSM) for the purpose of this paper:

1. For all pairs of non-central cities (i, j) compute the savings $s_{ij} = c_{i0} + c_{0j} - c_{ij}$.
2. Start the construction of a new path (route for one vehicle) by choosing the edge with the maximum savings s_{ij}.
3. Repeat the following procedure: add the edge with the maximum possible savings so that it is adjacent to some of the endpoints of the path and joins a city still not lying on any other path, while the length of the path, after joining city 0 to its closer end, does not exceed the time limit L.
4. Finally, finish the construction by joining the city 0 to the closer end of the path.
5. Repeat steps from 2 to 4 (constructing single paths) until all the cities lie on some of the paths.

Nearest neighbour method (NNM)

NNM is perhaps the simplest possible way to construct cyclic routes. The following version for the TLVRP is also taken from Pelikán (2006).

1. To start the construction of a new path, join the closest city from the ones which have not yet been put on any route to the central one.
2. Repeat the following procedure: Join the closest city to the last joined one to the path, provided that the total length of the path does not exceed the time limit L. The algorithm terminates as soon as all the cities lie on some of the paths. Otherwise, go to step 1 to construct another path.

Such simple methods as the NNM often do not give good results. As regards the NNM version for the TSP, Rosenkrantz, Stearns, and Lewis (1977) have shown that if we require any accuracy ratio for the solution, there exists an instance for which the NNM cannot achieve it. We present the NNM results especially for the demonstration of how poor quality of results may be obtained by the use of approximation methods, in comparison with the SM results.

Habr frequencies approach (HFA)

Habr (1964) is often regarded as a founder of the Czech systems school. He introduced frequencies based on the comparison of an edge with all the other edges (not only with those containing one selected city as in the case of the savings). Thus, using them we should obtain very good results valuable our assessment of the SM versions.

The frequencies can be expressed using the following formula:

$$F_{ij} = \sum_{k=1}^{n}\sum_{l=1}^{n}(c_{ij} + c_{kl} - c_{il} - c_{kj})\,.$$

There exists another form, called modified frequency, more suitable for computations: $F`_{ij} = c_{ij} - r_i - s_j$, where $r_i$ and $s_j$ are the arithmetic means of the costs of $i$-th row and $j$-th column of **C**, respectively. $F`_{ij}$ can be derived from F_{ij} by linear transformation.

Habr frequencies consider all edges with the same importance. But in the case of the TLVRP the edges incident to the central city are more important (more frequently used) than the others, as mentioned above when discussing the SM. Now we will show how big this difference is: Let us suppose that the solution will consist of p path (p vehicles will be used). Let us consider a randomly chosen solution with p cycles (with a uniform probability distribution, without respect to the costs). The probability of the choice of an edge which is non-incident to the central city is $\frac{2(n-p)}{n(n-1)}$ while for the edges incident to the central city this probability is equal to p/n. Thus the edges incident to the central city are $\frac{p(n-1)}{2(n-p)}$-times more important than the others (they occur in the solution with $\frac{p(n-1)}{2(n-p)}$-times bigger probability). Therefore, for computing the frequencies for the TLVRP we propose the following formula

$$F_{ij} = \sum_{k=1}^{n}\sum_{l=1}^{n}(c_{ij} + c_{kl} - c_{il} - c_{kj}) + \\ + \frac{p(n-1)}{2(n-p)}\sum_{m=1}^{n}(2c_{ij} + c_{m0} - c_{i0} - c_{mj} + c_{0m} - c_{im} - c_{0i})$$

or a formula for the modified frequencies $F`_{ij}$ derived by an analogous linear transformation, as in the general case above, which will not be mentioned here.

We used a parallel version of the HFA (analogous to the non-limited version of the SM described in the following chapter):

1. For all pairs of the non-central cities (i, j) compute the frequencies using the formula for the TLVRP.
2. Process the edges according to the descending order of the frequencies using the following rule: When by adding an edge we obtain a set of vertex disjoint paths and the length of each path, after joining the city 0 to the its closer end, does not exceed the time limit L, then we add it to the solution. Repeat this procedure until each city lies on some of the paths and by joining two arbitrary paths the allowed time limit is exceeded.

3. In the end add the city 0 to the closer end of all the paths.

This version was also used in e.g. Kučera, Houška, and Beránková (2008).

Neighbour search heuristic (NSH)

The last method which is hereby proposed and used by the author has been inspired by Pisinger and Ropke (2007).

1. For each city find its closest city. Add edges formed by these pairs of cities subsequently according to their ascending cost. If it is not possible to add an edge to the solution because it forms a tree or a path longer than the time limit L with the edges added so far, then try to add the edge to the second closest city. If even this is not possible, then add an edge to the third closest one etc.
2. Execute the same procedure as in step 1 with the paths so far obtained. Repeat this step until there are no two paths joinable in this manner.

RESULTS AND DISCUSSION

Because sequentially proceeding methods give generally worse results than the parallel proceeding ones, one way to improve the SSM is to implement a parallel process into its algorithm so that it can construct the routes not one by one but all simultaneously. In fact, it differs from the HFA in computing the savings instead of frequencies only:

1. For all pairs of non-central cities (i, j) compute the savings $s_{ij} = c_{i0} + c_{0j} - c_{ij}$.
2. Process the edges according to the descending order of the savings s_{ij} using the following rule: When by adding an edge we obtain a set of vertex disjoint paths and the length of each path, after joining the city 0 to the its closer end, does not exceed the time limit L, then we add it to the solution. Repeat this procedure until each city lies on some of the paths and by joining two arbitrary paths the allowed time limit is exceeded.
3. In the end add the city 0 to the closer end of all the paths.

The disadvantage of the parallel proceeding methods is that they usually require more computing time. Namely, if trying to add an edge to a path, the time limit is exceeded and therefore this edge is not added, this method may try to add other edges with a low chance of being added to this route without any limitation. Thus, we will denote this method as a **non-limited savings method** (NSM).

In order to reduce these unproductive trials and thus make the method faster, we can modify it so that it always marks such a path (route) as finished, when the time limit has once been exceeded, and does not add any more edges to these marked paths. We will denote this modification as a **limited savings method** (LSM).

1. For all pairs of non-central cities (i, j) compute the savings $s_{ij} = c_{i0} + c_{0j} - c_{ij}$.
2. Process the edges according to the descending order of the savings s_{ij}.
3. <u>If</u> the edge is not adjacent to any route marked as finished
   <u>then</u> when by adding an edge we obtain a set of vertex disjoint paths and the length of each path, after joining the city 0 to the its closer end, does not exceed the time limit L, then we add it to the solution
   <u>else</u> mark the adjacent path(s) as finished.
   Repeat the procedure until each city lies on some of the paths and by joining arbitrary two paths the allowed time limit is exceeded.
4. In the end add the city 0 to the closer end of all the paths.

Time complexity of the used methods

Let us concentrate on the complexity of single methods from the theoretical point of view.

As stated above, the **nearest neighbour method** is the simplest one of the used methods. During its procedure, each of the n cities is visited once when it is added to a route and there are at most n other trials to add a city, which are unsuccessful because of exceeding the time limit L. Adding a city to a route, the operations concerning deleting this city from the following consideration takes time $O(n)$. Thus, the whole procedure requires time $O(n^2)$.

The **sequential savings method** at the beginning computes savings of all $O(n^2)$ edges, each in a constant time and thus altogether in time $O(n^2)$. Then it sorts out these edges according to their savings in time $O(n^2 \log n)$ and within this sorting it also sorts out the incident edges for each city. During steps from 2 to 5, each of the n cities is added once to a route and within this operation, the most complex action is deleting this city from the following consideration in time $O(n)$ (all the other operations, i.e. choosing this city according to the best savings and re-computing the length of the route take a constant time); and there are at most n other trials to add a city, which are unsuccessful because of exceeding the time limit L. Steps 2 to 5 take time $O(n)$ and thus the whole procedure takes time $O(n^2 \log n)$.

The **non-limited savings method** and the **Habr frequencies approach** are analogous and their computation complexity, as will now be shown, is the same. The savings as well as the (modified) Habr frequencies can be computed in time $O(n^2)$. The remaining part of the procedures of both methods differs only in the NSM using the savings and the HFA using the frequencies, otherwise they are completely identical. Again, the sorting takes time $O(n^2 \log n)$. Consequently, there is a possibility that we try to add almost all n^2 edges. Processing each of them, similarly to the previous methods, we may spend time n at most. Therefore, the whole procedure requires time $O(n^3)$ in both these methods.

In case of the **limited savings method**, the initialization activities (computing savings and sorting the edges) take time $O(n^2 \log n)$. Then, an edge is added to a route at most n times and operations concerning it again take time $O(n)$. A trial to add an edge, when the time limit L is exceeded and the route is marked as finished, happens at most n times and operations concerning it take a constant time. Thus, the overall computation time of the LSM is $O(n^2 \log n)$.

Executing the **neighbour search heuristic**, one iteration of step 1 and $O(\log n)$ iterations of step 2 are carried out. Within each iteration, the maximum number of edges which are added is n and the technical operations concerning each of these additions take time at most $O(n)$. That is why the computation complexity is $O(n^2 \log n)$.

Test computations and their results

For testing, we took two types of randomly generated cases. In both types, all cities were located in a circle with a 100-time-unit diameter (the time necessary for travelling along a given route is supposed to be directly proportional to the distance).

In the first type, the central city was in the center of this circle. Then, the area between a circle given above and another circle with the diameter of 20 time units with the same center was considered. In this area 20 cities were originally randomly generated with the uniform distribution. Then the closest pairs of the cities were joined into "regions" so that four of them at the most might form one "region" and the final number of non-central cities ("regions") was 12.

The second type differed from the first one in the location of the central city which was the most remote one from the middle (the closest one to the circle boundary) while the city in the middle was an ordinary one.

The time limit L was set in both types to 250.

Ten instances of the first type were computed using all three versions of the SM, the NNM, the HFA and the NSH. The results are summarized in Tab. I. It contains the values of the objective function (the sum of the time necessary for travelling along all the routes) in a percentage form standardized according to the HFA (i.e. 100 % present the objective function values obtained by the HFA). The HFA was selected as the base for the results comparison because it proved to be an excellent method with very good results in Kučera, Houška, and Beránková (2008). Interestingly, the objective function values obtained by the HFA ranged from 462.8 (Case 2) to 608.3 (Case 7) time units.

As we had expected, all the versions of the SM provided identical or worse results than the HFA, while, unexpectedly, in two cases the NNM gave a slightly better solution than the HFA. The NSH provided results of very different quality in single cases. Among the SM versions, the NSM proved to be the best, only slightly less than 4 % worse on average than the HFA. Both remaining versions were about 10 % worse on average than the HFA and they were even worse than the NNM, too.

However, it is interesting to note the difference between the results quality by these methods in each case. Thus, some properties which would have an influence on the quality of these solutions were searched for. We discovered that in the case of the LSM, there is an important dependence of the results on the ratio between the farthest and the nearest city from the central one among the cities lying on the convex hull of the set of all cities. High values of this ratio properly indicate that the central city does not

I: *Test case results – the first type*

	NNM	SSM	LSM	NSM	HFA	NSH	E
Case 1	112.9%	106.2%	114.6%	100.0%	100.0%	99.9%	1.68
Case 2	122.4%	117.9%	100.0%	100.0%	100.0%	92.4%	1.71
Case 3	105.5%	102.3%	116.9%	102.3%	100.0%	120.2%	1.76
Case 4	97.4%	111.7%	119.0%	111.7%	100.0%	102.2%	2.10
Case 5	99.8%	116.1%	107.3%	105.9%	100.0%	112.8%	1.26
Case 6	100.0%	106.9%	106.9%	109.8%	100.0%	110.7%	1.30
Case 7	107.3%	100.0%	100.0%	100.0%	100.0%	106.5%	1.32
Case 8	107.2%	108.8%	100.8%	100.0%	100.0%	104.2%	1.52
Case 9	104.0%	112.3%	113.5%	103.7%	100.0%	109.1%	1.24
Case 10	107.1%	118.2%	114.6%	102.5%	100.0%	99.8%	1.66

II: *Test case results – the second type*

	NNM	SSM	LSM	NSM	HFA	NSH	E
Case 1	97.6%	112.3%	127.4%	106.5%	100.0%	97.6%	2.49
Case 2	115.1%	103.7%	128.6%	100.1%	100.0%	105.1%	3.61
Case 3	115.5%	91.4%	126.3%	91.4%	100.0%	124.6%	3.97
Case 4	109.5%	106.1%	130.7%	106.1%	100.0%	108.1%	2.89

lie near the middle of the actually serviced region. This property will be called eccentricity and it is added to Tab. I. It is clear that the LSM achieves good results for instance with a low eccentricity and vice versa (perhaps with the exception of cases 2 and 9). This dependency was also discovered by the use of regression analysis.

Another way to confirm this dependency was to solve the second type test cases. They show much higher eccentricity and the LSM would have much more problems than in the first type of cases. Four such instances were computed and their results in Tab. II show that this hypothesis is true.

CONCLUSION

If we want to apply the savings approach to the TLVRP, the NSM seems to be the right version for doing it. It provides the same or a slightly worse solution that the HFA, but the procedure of computing savings is faster and less complex than that of the computing Habr frequencies.

Nevertheless, the LSM also shows to be useful because of its good results for the instances of low eccentricity. From this point of view, this is a complementary method to the tree approach investigated by Kučera, Houška, and Beránková (2008) which is suitable for tasks of high eccentricity. In addition, in most cases, it differs in solution from all the other methods studied here.

In practice, however, companies seldom pay enough attention to dealing with VRPs, especially if transportation is not their principal work load and if a transportation task of a medium size is concerned only.

Kučera and Jarkovská (2010) present a case study of NOPEK Bakery in Vysoké Mýto. They demonstrate the effectiveness of the use of suitable approximation methods during the planning of the bakery products delivery to its customers. By the optimization of one of the so-called "fast deliveries" they succeeded in the reduction of a number of vehicles needed for the delivery – about 18% – which turned out necessary. Similar savings of all "fast deliveries" in the company may lead to a considerable tenure price reduction (by 17 million CZK) and profit increase (by 0.6 mil CZK), and thus to a significant profitability growth between 2 and 2.5%. Kučera and Jarkovská (2010) also managed to ensure a balanced use of the vehicles. This made it possible for the bakery to deliver the goods to its customers in deadlines more convenient for them.

SUMMARY

The problem of the delivery optimization of specific material can in reality be encountered very often. The delivery is usually realized by a circular or round trip which, in comparison with the implementation of each route from the supplier to the consumer, saves expenses for individual gateways from the same supplier and/or trips to one consumer. There exist many tasks of this kind and in general they are referred to as vehicle routing problems (VRP). Practically all of them belong to the NP-hard problems, for which there is no efficient algorithm finding their theoretical optimum. Thus, the only way to obtain some solution efficiently or in a reasonably short time is to use some of the heuristics (approximation methods), which give only "good" or "close to optimal" solution, not exactly optimum.

In practice, however, companies seldom pay enough attention to dealing with VRPs, especially if transportation is not their principal work load and if a transportation task of a medium size only is concerned. Let us recall a case study where, using suitable heuristics, a company reduced the number of vehicles needed for the delivery by 18% thereby increasing the profitability by 2 to 2.5%.

In this paper a special case of the VRP was studied: so called time limited vehicle routing problem (TLVRP). One central and a certain number of other cities (points, places, nodes …) were determined and a time limit within which it was necessary to visit all the cities using vehicles starting form the central city (and it was of no importance how the vehicles got back to the central city). The aim was to test some heuristics applicable even for "manual" computing when a route was chosen in a company, primarily several modifications of famous savings method by Clark and Wright, in some test cases. This study also discussed how the quality of solutions obtained using single heuristics depends on the properties of the tasks.

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