METHODOLOGY OF THE CAM MECHANISM DESIGN OF ROTARY RAKES

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Abstract

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This paper describes the method of derivation of mathematical description of a cam orbit in rotary rakes (hay aggregators). At first, the authors describe basic construction elements of the rotary hay rake mechanism and their mutual links and relationships. Thereafter, they define the origin and the orientation of the system of coordinates, in which all calculations are carried out. In the next step they define basic requirements concerning the assurance of an optimum functioning of cam mechanisms as well as their transformation into mathematical equations. These requirements represent a base for the mathematical formulation of an optimum transition curve and it is emphasized that an optimum formulation of parameters of this curve is very important. In the course of calculation, they use also a normalized transition curve, which is used for the optimizations of the total number of parameters of the transition curve. Thereafter, they take into account mechanical and operational parameters of the hay aggregators and convert the optimum transition curve to that part of the space curve, which agrees at best with these parameters. Finally, the whole cam orbit is constructed using individual segments and presented as a sequentially defined space curve. Its individual parts concur sequentially to the level of the second derivation and are described as explicit mathematical functions of mechanical and operational parameters of the hay aggregator. The definition of the **system of coordinates**, the execution phase of calculations and the final shape of the cam orbit are illustrated in graphs.

system of coordinates, cam orbit, transition curve, sequentially defined functions, space curve

In the technological line of forage harvesting, the rake should assure an efficient and considerate raking of spread-harvested forage. The working width of one rake segment should be as great as possible so that the manipulation with the dry material would be minimized and the shattering of dry leaves reduced.

Because of their high performance, rotary rakes are the most frequently used of all hay aggregators (Šmíd, 2007).

They have the following structural elements: cam guide, cam itself, cam arm, carrying attachment, and working fingers (Fig. 1).

In the course of one working cycle, the carrying arms rotate together with cam arms around a fix cam guide and in this way they transfer the motion to gathering arms fitted with working fingers. These working fingers transpose the harvested material (i.e. hay) and push it aside (ca 2/3 of the cam pathway).

In the remaining part of the cam pathway (ca 1/3), the gathering arms turn due to the action of cam arms by approximately 90° and the flexible working fingers leave the harvested material and move back to a horizontal position. The harvested material is thrown to a grabbing cloth and forms a swath of hay (Šmíd & Bartoň, 2008 b).

The shape of cam guide is very important because it assures not only a correct working of the rotary rake but also its high operational reliability.

MATERIALS AND METHODS

Importance of cam guides

The cam guide represents a stationary part of all types and models of rotary rakes. It is mounted symmetrically around the vertical axis of rotation of bearing arms and assures their rotation around their

longitudinal axis and, thus, also lifting and tilting of working fingers into the swath operation. This means that the shape of cam guide determines not only the beginning and the end of swath operation but also the velocity of lifting and tilting of working fingers. The angular speed and acceleration of bearing arms and working fingers (as well as the torsion forces affecting these bearing arms along their longitudinal axis) change in dependence on the shape of the cam guide. This means that the cam guide influences not only operational parameters of rotary rakes but also their longevity and resistance against mechanical damage, above all due to a long-term load of cams and bearing arms (Šmíd & Bartoň, 2010).
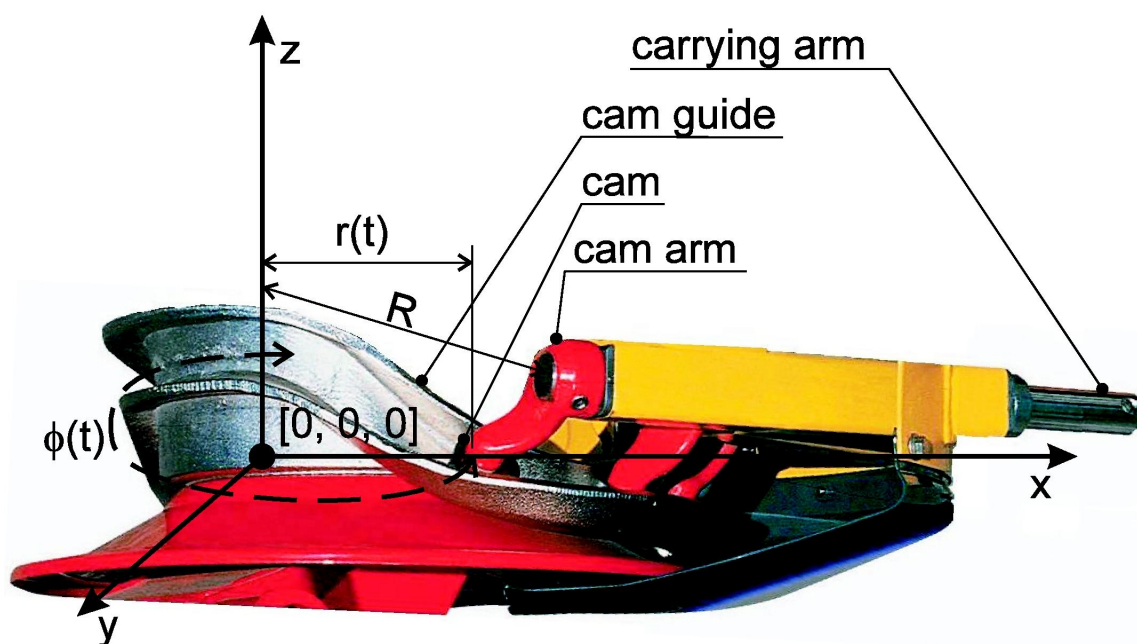
Definition of the system of coordinates

Šmíd & Bartoň (2009 a) mentioned that, because of rotation of cam arm and bearing arm around their vertical axis, the best solution of hay aggregators design is to use the cylindrical system of coordinates and to place the axis of rotation at the origin of coordinates to the height equal to the horizontal position of the cam arm. The axis of the system of coordinates can be identified with the axis of rotation. The cam position, i.e. the shape of the cam orbit, can be examined from the viewpoint of the relationship existing between the cam height $z(t)$ on the one hand and its distance from the axis of rotation $r(t)$ and the angle of sight $\phi(t)$ on the other. The calculation into the rectangular system of coordinates is then performed using the relationship $[x = r(t)\cos(\phi(t)), y = r(t)\sin(\phi(t)), z(t)]$. The x-axis will be directed to the point occupied by the cam at the moment of horizontal position of the bearing arm, i.e. in time $t = 0$. The angle of sight ϕ will

be then derived (measured) from the direction of the x-axis.

Shape requirements

The end of the cam arm, which rotates the bearing arm, moves round the axis of rotation on a circular orbit with the radius R. The axis of rotation and the end point of cam arm define the vertical plane, which rotates around the axis of rotation. However, in this plane the cam makes only a pendulous movement that can be characterized by the angle of cam arm displacement or by the $z(t)$ coordinate of the cam. The course of this coordinate plays a decisive role in the determination of the course of the $z(t)$ coordinate.

When starting from the aforementioned definition of the origin of the system of coordinates, it is quite obvious that both extreme positions (i.e. the upper dead point and the lower dead point) are identical as far as the absolute value of the coordinate z is concerned. For this value we can use the symbol h and the moment when the cam must leave the lower dead point will be denoted as t1. The time interval of its transition to the upper dead point can be denoted as dt. The moment when the cam must leave the upper dead point will be denoted as t2. This means that within the time interval t1+dt <= t <= t2 the cam is in the upper dead point. Similarly, the time interval of its transition from the upper to the lower dead point can be denoted as dt2. In case that we choose the direction of motion in such a way that the cam will reach at first the upper dead point, individual moments may be arranged into the series 0 <= t1 < t1 + dt < t2 < t2 + dt2 < T, where T is the period of one rotation. Evidently, it is necessary that z(0) = z(T) (Šmíd & Bartoň, 2009 b).

1: *Basic components and definition of the system of coordinates*

To prevent the occurrence of immediate impulses of force affecting the cam it is necessary to assure that the guiding of cam will have the form of a smooth curve. Although such a curve can be defined stepwise, it must be continuous (including the continuity of its first and the second derivations). In this case the cam orbit will disintegrate into four, mutually independent segments. The first of them is the ascending transition segment, the second one represents the holding time in the upper dead point, the third one is the ascending transition segment and the fourth one represents the holding time in the lower dead point. The mutual linkage of these four segments must respect the condition of continuity (also including the continuity of its first and the second derivations).

Mathematical derivation of the shape of the transition segment

From the mathematical point of view it is possible to define conditions of the course of the $z(t)$ coordinate of a general transition curve as

$$p1 := z(t1) = -h$$

$$p2 := z(t1 + dt) = h$$

$$p3 := \left(\frac{d}{dt} z(t)\right)_{t = t1} = 0$$

$$p4 := \left(\frac{d}{dt} z(t)\right)_{t = t1 + dt} = 0$$

$$p5 := \left(\frac{d^2}{dt^2} z(t)\right)_{t = t1} = 0$$

$$p6 := \left(\frac{d^2}{dt^2} z(t)\right)_{t = t1 + dt} = 0.$$

Condition $p1$ – The coordinate z is at the moment t1 in the lower dead point.

Condition $p2$ – The coordinate z is at the moment t1 + dt in the upper dead point.

Condition $p3$ – The continuity of the 1st derivation. The cam velocity must not change non-continuously (i.e. by jumps).

Condition $p4$ – This condition is the same as that concerning continuity of the ascending transition segment and the holding time in the upper dead point.

Conditions $p5$ and $p6$ are similar to those concerning continuity of the second derivation (the cam acceleration of must not change in a discontinuous manner).

It is a problem to find such a suitable function, which would meet the aforementioned conditions.

It seems that a polynomial could be suitable to solve this problem because of the existence of 6 conditions of at least 5th degree. For this reason let's try to use polynomial of the 6th degree:

$$z := t \rightarrow c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4 + c_5 t^5 + c_6 t^6.$$

Basing on conditions of continuity, it is possible to define the unknown coefficients $c_0 \ldots c_6$.

$$p1 := c_0 + c_1 t1 + c_2 t1^2 + c_3 t1^3 + c_4 t1^4 + c_5 t1^5 + c_6 t1^6 = -h.$$

$$p2 := c_0 + c_1 (t1 + dt) + c_2 (t1 + dt)^2 + c_3 (t1 + dt)^3 + c_4 (t1 + dt)^4 + c_5 (t1 + dt)^5 + c_6 (t1 + dt)^6 = h$$

$$p3 := c_1 + 2 c_2 t1 + 3 c_3 t1^2 + 4 c_4 t1^3 + 5 c_5 t1^4 + 6 c_6 t1^5 = 0$$

$$p4 := c_1 + 2 c_2 (t1 + dt) + 3 c_3 (t1 + dt)^2 + 4 c_4 (t1 + dt)^3 + 5 c_5 (t1 + dt)^4 + 6 c_6 (t1 + dt)^5 = 0$$

$$p5 := 2 c_2 + 6 c_3 t1 + 12 c_4 t1^2 + 20 c_5 t1^3 + 30 c_6 t1^4 = 0$$

$$p6 := 2 c_2 + 6 c_3 (t1 + dt) + 12 c_4 (t1 + dt)^2 + 20 c_5 (t1 + dt)^3 + 30 c_6 (t1 + dt)^4 = 0.$$

When considering conditions p1 and p2, it is obvious that we can simplify the first two equations by dividing all six aforementioned conditions by h and introducing new coefficients $k_0 \ldots k_6$ pursuant the relationship

$$su1 := \frac{c_0}{h} = k_0, \frac{c_1}{h} = k_1, \frac{c_2}{h} = k_2, \frac{c_3}{h} = k_3, \frac{c_4}{h} = k_4, \frac{c_5}{h} = k_5, \frac{c_6}{h} = k_6.$$

The other equations will remain de facto unchanged. This significant simplification does not mean that the onset of time measurement would be shifted to the moment of the beginning of the transitory segment of cam movement (i.e. substitution)

$$su2 := t1 = 0,$$

which will result in a really radical reduction of the solution difficulty. A shift to the original beginning of time measurement can be done by means of the substitution t = t – t1.

This means that after performed substitutions the above conditions will change to the term:

$$p1 := k_0 = -1$$

$$p2 := k_0 + k_1 dt + k_2 dt^2 + k_3 dt^3 + k_4 dt^4 + k_5 dt^5 + k_6 dt^6 = 1$$

$$p3 := k1 = 0$$

$$p4 := k_1 + 2k_2\,dt + 3k_3\,dt^2 + 4k_4\,dt^3 + 5k_5\,dt^4 + 6k_6\,dt^5 = 0$$

$$p5 := 2k_2 = 0\,.$$

$$p6 := 2k_2 + 6k_3\,dt + 12k_4\,dt^2 + 20k_5\,dt^3 + 30k_6\,dt^4 = 0.$$

As one can see, this simplification is very important. Coefficients k_0, k_1 and k_2 can be directly substituted by 1 or 0. Regarding coefficients $k_3 \ldots k_6$, the remaining three equations p2, p4 and p6, can be solved as follows:

$$k_0 := -1 \qquad\qquad k_1 := 0$$

$$sol := \left\{ k_3 = -\frac{k_6\,dt^6 - 20}{dt^3},\ k_4 = \frac{3\left(k_6\,dt^6 - 10\right)}{dt^4},\ k_5 = -\frac{3\left(-4 + k_6\,dt^6\right)}{dt^5},\ k_6 = k_6 \right\}.$$

This means that, after the substitution, the transition curve will be

$$z := -1 - \frac{\left(k_6\,dt^6 - 20\right)t^3}{dt^3} + \frac{3\left(k_6\,dt^6 - 10\right)t^4}{dt^4} - \frac{3\left(-4 + k_6\,dt^6\right)}{dt^5} + k_6\,t^6\,.$$

It is also obvious that it was not necessary to use the coefficient k_6. However, it is possible to find out if it could not be used for an improvement of parameters of the transition curve.

Regarding the fact that the transition curve is valid only for times z of the interval $0 <= t <= dt$, it is possible to carry out a substitution t=q dt (with the condition $0 <= q <= 1$) and develop a normalized transition curve zs.

$$zs := -1 - \left(k_6\,dt^6 - 20\right)q^3 + 3\left(k_6 dt^6 - 10\right)q^4 -$$
$$-3\left(-4 + k_6\,dt^6\right)q^5 + k_6\,q^6 dt^6\,.$$

Now it is possible to compare courses of zs(k6,q) for various values of dt, e.g. dt = 0.4 and dt = 0.5.

As one can see in Fig. 2, the optimum course of normalized transition function could be for the value $k_6 = 0$ (outlined as a bold black line). In Fig. 2, the red and blue curves correspond with values dt = 0.4 and dt = 0.5, respectively.

Determination of the optimum value of k_6 coefficient

It is necessary to find out such value of k_6 coefficient, which assures that the value of the maximum increase of normalized transition function would be as low as possible. The maximum slope of the transition curve will be in the inflex point of normalized curve, i.e. in the place where

$$zsqq := \frac{\partial^2}{\partial q^2} zs = 0$$

or

$$zsqq := -6\left(k_6\,dt^6 - 20\right)q + 36\left(k_6\,dt^6 - 10\right)q^2 -$$
$$-60\left(-4 + k_6\,dt^6\right)q^3 + 30k_6\,q^4\,dt^6 = 0.$$

The above equation enables to find out that q value, which means that the equation is fulfilled

$$solq := q = \left[0, 1, \frac{1}{10}\frac{-40 + 5k_6\,dt^6 + \sqrt{1600 + 5k_6{}^2\,dt^{12}}}{k_6\,dt^6}, -\frac{1}{10}\frac{40 - 5k_6\,dt^6 + \sqrt{1600 + 5k_6{}^2\,dt^{12}}}{k_6\,dt^6} \right].$$

The first and the second value of *q* result from the setting of the form of normalised transition function and meet conditions *p5* and *p6*. For that reason it is necessary to verify only the third and the fourth variant.

The value q is inserted into the expression of the slope of normalized transition function, which is determined by

$$zsq := \frac{\partial}{\partial q} zs = -3\left(k_6\,dt^6 - 20\right)q^2 + 12\left(k_6\,dt^6 - 10\right)q^3 -$$
$$-15\left(-4 + k_6\,dt^6\right)q^4 + 6k_6\,q^5 dt^6\,.$$
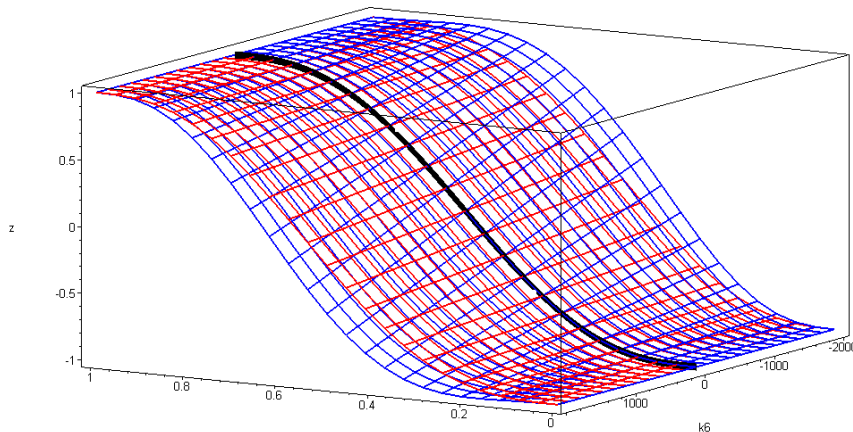
2: *A normalized transition function for values dt = 0.4 (in red) and dt = 0.5 (in blue) as dependent on k6 and q*

After the introduction of the third variant of solution (the fourth variant is quite identical)

$$q3 := q = \frac{1}{10} \frac{-20 + 5 k_6 dt^6 + \sqrt{400 + 5 k_6^2 dt^{12}}}{k_6 dt^6}$$

the slope will be changed to the expression

$$zsqq3 := -\frac{96}{k_6^2 dt^{12}} + \frac{3}{125}\sqrt{400 + 5 k_6^2 dt^{12}} + \frac{156}{25}\frac{\sqrt{400 + 5 k_6^2 dt^{12}}}{k_6^2 dt^{12}} - \frac{1152}{5}\frac{\sqrt{400 + 5 k_6^2 dt^{12}}}{k_6^4 dt^{24}} + \frac{4608}{k_6^4 dt^{24}} + \frac{12}{5}.$$

Regarding the coefficient k_6, the maximum slope of normalized transition function will occur in the situation when the derivation of the curve slope (according to k_6) is equal to zero

$$\frac{\partial}{\partial k_6}\left(\begin{array}{c}-\frac{96}{k_6^2 dt^{12}} + \frac{3}{125}\sqrt{400 + 5 k_6^2 dt^{12}} + \frac{156}{25}\frac{\sqrt{400 + 5 k_6^2 dt^{12}}}{k_6^2 dt^{12}} - \\ \frac{1152}{5}\frac{\sqrt{400 + 5 k_6^2 dt^{12}}}{k_6^4 dt^{24}} + \frac{4608}{k_6^4 dt^{24}} + \frac{12}{5}\end{array}\right) = 0$$

and thus

$$dzs3 := \frac{3}{25}\left(\begin{array}{c}1600 k_6^2 dt^{12}\sqrt{400 + 5 k_6^2 dt^{12}} + k_6^6 dt^{36} - 12800 k_6^2 dt^{12} - \\ 260 k_6^4 dt^{24} + 3072000 - 153600\sqrt{400 + 5 k_6^2 dt^{12}}\end{array}\right) \Big/ \Big/ \left(k_6^5 dt^{24}\sqrt{400 + 5 k_6^2 dt^{12}}\right) = 0.$$

If the numerator of the fraction on the left side of the above equation is equal to zero then the last expression mentioned above is also equal to zero

$$n := 14400 k_6^2 dt^{12}\sqrt{400 + 5 k_6^2 dt^{12}} + 9 k_6^6 dt^{36} - 115200 k_6^2 dt^{12} - 2340 k_6^4 dt^{24} + 27648000 - 1382400\sqrt{400 + 5 k_6^2 dt^{12}} = 0.$$

This equation may be converted to the expression

$$n := 81 k_6^8 dt^{48}\left(k_6 dt^6 + 10\right)\left(k_6 dt^6 - 10\right)\left(k_6^2 dt^{12} - 420\right) = 0.$$

which enables an easy calculation of the coefficient k_6

$$solk := k_6 = \left[0, -\frac{10}{dt^6}, \frac{10}{dt^6}, -\frac{2\sqrt{105}}{dt^6}, \frac{2\sqrt{105}}{dt^6}\right].$$

The first solution, i.e. $k_6 = 0$, can be used only in the case that there is a limit

$$\lim_{k_6 \to 0}\frac{3}{25}\frac{\left(\begin{array}{c}1600 k_6^2 dt^{12}\sqrt{400 + 5 k_6^2 dt^{12}} + k_6^6 dt^{36} - 12800 k_6^2 dt^{12} - \\ 260 k_6^4 dt^{24} + 3072000 - 153600\sqrt{400 + 5 k_6^2 dt^{12}}\end{array}\right)}{k_6^5 dt^{24}\sqrt{400 + 5 k_6^2 dt^{12}}} = 0,$$

because, after the insertion of $k_6 = 0$, the numerator on the left side of the equation takes the zero value. As one can see, the value $k_6 = 0$ can be used.

Maximum values of the slope of normalized transition function y can be obtained after the insertion of calculated values of coefficient k_6 into the expression zsqq3. In this case it is necessary to use for the calculation the limit because the numerators of fragments take after the insertion also the zero value. Thereafter, the remaining two values of coefficient k_6 can be introduced into the expression zsqq3 to calculate its value.

$$zsqq3\_1 := \frac{15}{4} = 3,750000000$$

$$zsqq3\_2 := \frac{1188}{625} + \frac{198\sqrt{900}}{3125} = 3,801600000$$

$$zsqq3\_3 := \frac{1188}{625} + \frac{198\sqrt{900}}{3125} = 3,801600000$$

$$zsqq3\_4 := \frac{2692}{1225} + \frac{46\sqrt{2500}}{1225} = 4,075102040$$

$$zsqq3\_3 := \frac{2692}{1225} + \frac{46\sqrt{2500}}{1225} = 4,075102040.$$

Now it is necessary to determine the value of q for calculated coefficients k_6. For the zero value, it is again necessary to use limit relations:

$$q3\_1 := q = \frac{1}{2}$$

$$q3\_2 := q = \frac{3}{5}$$

$$q3\_3 := q = \frac{2}{5}$$

$$q3\_4 := q = \frac{\sqrt{105}\left(3 + \sqrt{105}\right)}{210}$$
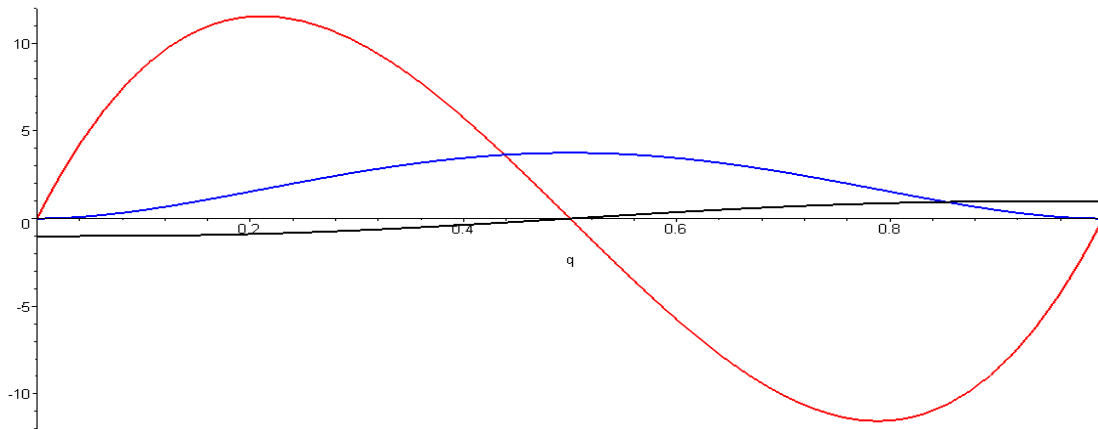
$$q = 0,6463850114$$

$$q3\_5 := q = \frac{\sqrt{105}\left(-3 + \sqrt{105}\right)}{210}$$

$$q = 0,3536149894.$$

As one can see, the value $k_6 = 0$ is fully satisfactory. In this case the gradient of the transition curve is the lowest and occurs in the middle of the transition interval.

This means that the final form of the transition function is

$$z := -1 + \frac{20 t^3}{dt^3} - \frac{30 t^4}{dt^4} + \frac{12 t^5}{dt^5}$$

3: *A normalized transition function (in black) and its first (blue) and the second (red) derivations*

and/or that the normalized transition function is

$$zs := -1 + 20q^3 - 30q^4 + 12q^5.$$

Now it is possible to draw the normalized transition function (including its first (blue) and the second (red) derivation)

$$zsq := 60q^2 - 120q^3 + 60q^4$$

$$zsqq := 120q - 360q^2 + 240q^3.$$

A general form of transition curve

The general form of transition curve can be obtained after the substitution $t = t + t1$ and multiplication of the whole curve by the value h, i.e.

$$Z := \left(-1 + \frac{20(-t1+t)^3}{dt^3} - \frac{30(-t1+t)^4}{dt^4} + \frac{12(-t1+t)^5}{dt^5} \right) h.$$

The shape of the descending transition curve

The descending gradient of transition curve can be derived in a practically identical manner; it is only necessary to exchange values +1 and -1 in the first and the second conditions. The corresponding transition curve is expressed as

$$z2 := 1 - \frac{20t^3}{dt^3} + \frac{30t^4}{dt^4} - \frac{12t^5}{dt^5}$$

and, after the substitution $t = t + t2$, $dt = dt2$, will take a general form

$$Z2 := \left(1 - \frac{20(-t2+t)^3}{dt2^3} + \frac{30(-t2+t)^4}{dt2^4} - \frac{12(-t2+t)^5}{dt2^5} \right) h.$$

Cam orbit coordinates

The course of the cam orbit coordinate can be derived on the base of transition functions Z and $Z2$; this course is expressed as a sequentially defined function

$$Vz := h \begin{pmatrix} -1 & t \leq t1 \\ -1 + \dfrac{20(-t1+t)^3}{dt^3} - \dfrac{30(-t1+t)^4}{dt^4} + \dfrac{12(-t1+t)^5}{dt^5} & t > t1 + dt \\ 1 & t < t2 \\ 1 - \dfrac{20(-t2+t)^3}{dt2^3} + \dfrac{30(-t2+t)^4}{dt2^4} - \dfrac{12(-t2+t)^5}{dt2^5} & t < t2 + dt2 \\ -1 & t \leq T \\ -1 & otherwise \end{pmatrix}.$$

The joint of cam arm (Fig. 1) rotates on a horizontal orbit with the radius R, at the height z = 0. Joint coordinates can be expressed as

$$KRV := \left[R \cos\left(\frac{2\pi t}{T} \right), R \sin\left(\frac{2\pi t}{T} \right), 0 \right],$$

where ω is the angular velocity of rotation,

$$\omega := \frac{2\pi}{T},$$

where T = period of rotation. If the length of the cam arm is written as r, then this length must be constant, regardless to the cam position. The maximum angle that can be formed between the cam arm and the horizontal plane can be defined as α. This means that the cam arm will tilt from the horizontal position by ± α. Then it must be valid that

$$h := r \sin(\alpha).$$

The distance between the cam and the rotation axis will change in dependence on the angle of cam tilt. If this distance will be denoted as ρ then it is valid that

$$r^2 = (R - \rho)^2 + vz^2,$$

where vz is the coordinate z of the cam orbit; quite naturally, vz=Vz (the substitution for Vz would be too long). The last equation enables to calculate the distance of the cam ρ from the rotation axis.

Within the selected system of coordinates, the cam coordinates (i.e. of its orbit) will be

$$Vxyz := \left[\left(R - \sqrt{r^2 - vz^2} \right) \cos \left(\frac{2\pi t}{T} \right), \left(R - \sqrt{r^2 - vz^2} \right) \sin \left(\frac{2\pi t}{T} \right), vz \right],$$

where

$$vz = r \sin(\alpha) \begin{pmatrix} -1 & t \leq t1 \\ -1 + \dfrac{20(-t1+t)^3}{dt^3} - \dfrac{30(-t1+t)^4}{dt^4} + \dfrac{12(-t1+t)^5}{dt^5} & t > t1 + dt \\ 1 & t < t2 \\ 1 - \dfrac{20(-t2+t)^3}{dt2^3} + \dfrac{30(-t2+t)^4}{dt2^4} - \dfrac{12(-t2+t)^5}{dt2^5} & t < t2 + dt2 \\ -1 & t \leq T \\ -1 & otherwise \end{pmatrix}.$$

Finally, it is possible to draw the cam orbit, trajectory of the cam arm joint, and 200 positions of the cam arm during one revolution, e.g. for values

$$su := \left[T = 1, \quad t1 = 0, \quad dt = 0.2, \quad t2 = 0.55, \quad dt2 = 0.4, \quad R = 0.3, \quad r = 0.15, \quad \alpha = \frac{\pi}{3} \right].$$

RESULTS AND DISCUSSION

The method described above is a direct analytical calculation of the shape of cam orbit in dependence on required design values and requirements. It results in the final shape of the cam orbit, which is described by means of common mathematical functions. The coordinates of cam orbit can be defined without the need to apply special design programmes. Although the calculation can be carried out in a conventional manner (i.e. on paper), the application of any simple computer can the whole process significantly shortens and accelerates. According to Šmíd & Bartoň (2008 a), the simplest method of processing seems to be the application of any spreadsheet processor (e.g. Excel or Calcul), which are markedly cheaper than various specialized programmes, (e.g. Automa 2011, Solid Works 2011 or Sims 2011). Moreover, the work with these simple spreadsheet programmes is also much easier.

Regarding the fact that the final shape of cam orbit is not given numerically (in contradistinction to common outputs of programmes generating the cam mechanisms, viz. AV Engineering 2011, Generators (Generátory) 2011, and/or Křivky (Curves) 2011) but as a function of basic design dimensions of the mechanism), it is possible to carry out also subsequent analyses and mathematical modeling of the behavior of other constructional elements or operational forces, of corresponding moments of forces and/or outputs required for the motion of whole subsequent structural units. This is not possible when using cam orbits defined in the form of numeric series. Besides, it also could be too complicated.

CONCLUSIONS

The analytical shape of the cam orbit represents a base for mathematic modeling of kinematics and dynamic properties of a whole category of agricultural machines – i.e. of rotary rakes. All these mechanisms are based on the same principle and for that reason it is possible to describe them by means of an identical mathematical model. For any
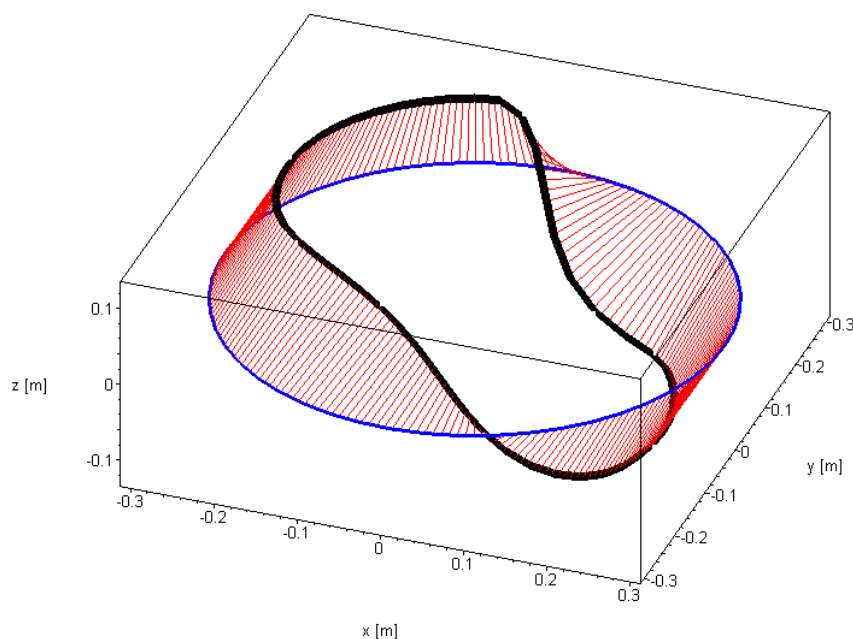
4: *Cam orbit (in black), trajectory of the cam join (in blue), and 200 positions of the cam arm (in red)*

concrete mechanism, it is only necessary to insert numeric values expressing concrete structural dimensions. This means that the calculation itself is not specially formulated for a concrete cam orbit but that it can be applied to any cam orbit of any arbitrary model of hay aggregators manufactured by various companies. The analytical form of this mathematical model can be then applied for the optimization of the whole machine, both from the viewpoint of design optimization and operational reliability. The aforesaid model of cam orbit enables a significant simplification, acceleration, cost-reduction and above all quality improvement of developing and designing of corresponding mechanisms.

SUMMARY

In the majority of hay aggregators/rakes, the cam mechanism is an important structural element because it determines the beginning and the end of swath. The shape of cam orbit is therefore dependent on structural and operational parameters of these mechanisms. In dependence on these parameters it is possible to derive an analytical shape of the spatial curve of cam orbit. The mathematical description of cam orbit is based on an optimized transition function. The authors derived it in detail and described also its application when calculating the cam orbit. The final mathematical description of the cam orbit is an explicit mathematical function of basic structural parameters of this mechanism and can be used as a core element of the subsequent optimization of the whole rotary rake.

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