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DYNAMIC AGING CHAIN OF THE CZECH REPUBLIC POPULATION

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Abstract

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System dynamics is a method enabling simulation and subsequent analysis of various socio-economic problems. Even though it was founded about fifty years ago, it is relatively new and little used in the Czech Republic. It has a good practice to make use of molecules, standard partial modelling structures which make the modelling processes easier and more effective.
The objective of this article is to introduce and provide such molecule of an aging chain for the Czech Republic population. To increase its usefulness the aging chain is disaggregated and divided into two chains, one for each sex. The aging chain molecule consists of stock and flow diagram, a system of differential equations and parameters quantified on the basis of demographic data for Czech Republic. Proposed model of aging chain also capture a special phenomenon of the Czech population – the postponing of motherhood and thus the increase in average age of mothers. This fact led to special model structure that is uncommon for existing aging chains of different populations.
The model is constructed on the basis of official demographic data of the Czech Statistical Office and the results of the simulation are compared with the surveyed data. The intersection of data sources resulted into disaggregation of population into twelve age cohorts. The chain is created to serve as a molecule for more complex models. Therefore, variables functioning as interface for implementation into such models are indicated in the text.

system dynamics, population, aging chain, mortality rate, fertility rate, life expectancy

System dynamics models can be characterised by complex structure of feedbacks, delays and non-linear behaviour (Sterman, 2000). Population models using aging chains are a common part of many system dynamics models. When modelling city dynamics, Forrester (1969) uses three simple aging chains for human population (with respect to employment, not age), commercial sphere and housing development. Each chain is composed of three stages following one another. In the project "Limits to Growth", Meadows *et al.* (1972) enhance an older Forrester's model of the world population (1971) and use a population model with four age groups. More aging chains were tested for this model (with one, four and fifteen cohorts), four groups were the compromise between detail and computer capacity (Sterman, 2000). This significant model is probably the most well-known (and most criticised) system dynamics model. Its update also contains aging chain with four age cohorts (Meadows *et al.*, 2004).

Wang and Sterman (1985) propose a disaggregated model of the Chinese population. They use it for testing various programmes of population regulation. Applying an aging chain structure, Wang and Ma (1987) model the ageing of fixed capital in the framework of a dynamic model of large Chinese cities.

When analysing a company organizational structure, Kunc (2008) uses a five-stage promotion chain with a structure similar to an aging chain differing only in the fact that groups are characterised by qualification and not age (novice, intermediate, advanced, senior and managers). Using the proposed model he then searches for an ideal qualification structure from the point of view of both, organization performance and wage costs. Similar to Forrester (1969), Eskinasi *et al.* (2009) also used aging chains for two types of housing; the model was applied to projecting urban transformation in Haaglanden, the region in the Netherlands.

The system dynamics makes a good practice of the so-called molecules or System Zoo (Bossel, 2007a, 2007b). These are commonly existing partial structures of system dynamics models. The general diagram and the form of difference equations are necessary to modify for particular conditions and quantify parameters. The aging chain is a part of System Zoo (Bossel, 2007a: 54–57); among "system Zoo animals" belong even more complex models which also use the aging chain. It mainly concerns the whole world models from the "Limits to Growth" (Bossel, 2007b: 148–212).

The objective of the paper is to design and quantify an aging chain for the population of the Czech Republic, i.e. a basic part of a system dynamics population model. The development of the aging chain is initiated by impulses from many directions. Such aging chain should be part of several system dynamics models, for example to analyse the retirement reform, healthcare reform, to test some social impacts of financial crisis etc. The proposed aging chain is construed as a molecule for socio-economic system dynamics models focusing on the Czech Republic. Its free provision to other system dynamists will not only ease their work but at the same time it can increase their model accuracy because an exhausting and long process of its construction often leads to the use of a more aggregated form of a chain.

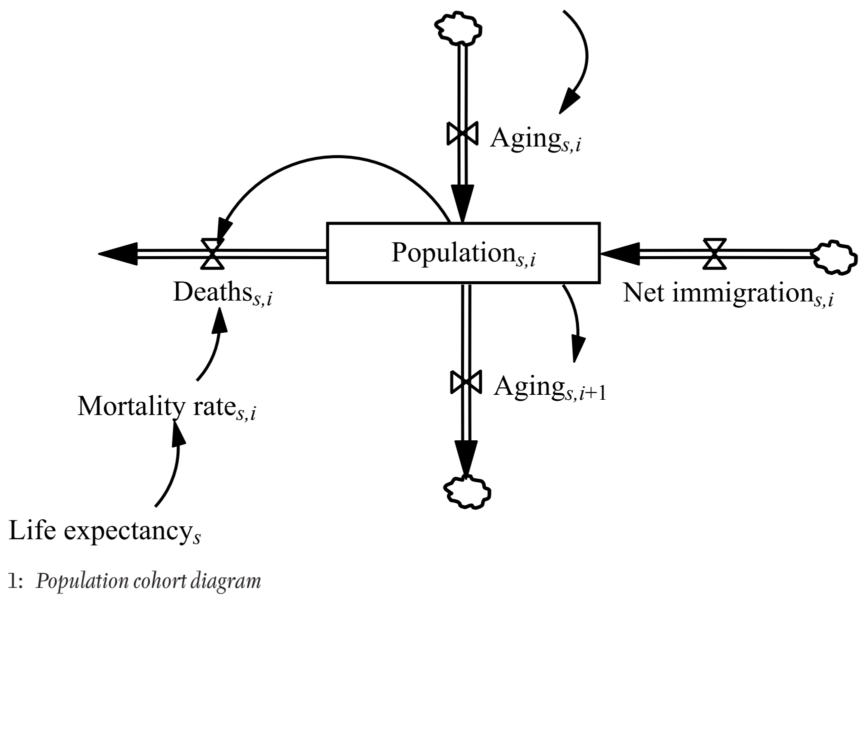
MATERIALS AND METHODS

For parameters quantification, data from the Demographic Yearbook of the Czech Republic 2009 (Czech Statistical Office, 2010a) and time series from the Czech Demographic Handbook 2009 (Czech Statistical Office, 2010b) are used, although it was very often necessary to convert them into the same age cohorts' structure. Further, for the comparison of the model behaviour and its estimates, extrapolations of Czech Statistical Office (2010c) and Eurostat (Giannakouris, 2008) were used.

The aging chain model of Czech Republic population was created in Vensim DSS 5.10e simulation programme, the quantification of selected parameters was done in SAS 9.1 programme. For graphical demonstration of dependencies within the model the so-called Stock and Flow Diagram is used, described in Coyle (1996: 19–47) or Sterman (2000: 135–229). Simulation was run with fourth order Runge-Kutta numerical integration method and time step $dt = 0.03125$. This method is one of the commonly used in system dynamics modelling. The computer capacity allows such precision simulation without long computation time of simulation. Use of less accurate method (e.g. Euler integration with bigger time step) could result in lower accuracy of simulation.

The model divides the population according to gender; that is why two aging chains are used. Each chain contains twelve age groups (the first age group is between 0–14 years of age being followed by a series of five-year groups, the last group is the population in the age of 65 and older). These groups are chosen because of compatibility with all data sources used for parameters quantification (Czech Statistical Office, 2010a, 2010b).

In sense of system dynamics, each age group presents a state (or stock) variable with two input and two output flows. This stock variable is represented by box $\text{Population}_{s,i}$ in Fig. 1. The subscript s denotes the sex differentiation, subscript i stands for population cohort ($i = 1, \ldots, 12$).



1: *Population cohort diagram*

In Fig. 1, the input flows for these groups are births (for the 0–14 group) or the aging from previous age groups (for all other groups). The output flows are the deaths and aging to the next age group. Thus, the aging of the population is modelled by the flow between individual stock variables. Obviously, the aging outflow variable is missing in the age group of 65+. Deaths flow is the function of the quantity of population cohort and the mortality rate of this group. Mortality rate of specific age cohort is function of life expectancy at birth, which is calculated for whole population of each gender. Net immigration can be inflow or outflow, for this aging chain immigration is taken as exogenous variable.

An alternative to Fig. 1 is equation (1), where index *s* stands for gender, index *i* stands for age group, T_0 is initial time, *T* is current time and *t* is a moment between T_0 and *T*. $P_{s,i}$ stands for population cohort *i* for gender *s*, $A_{s,i}$ is the flow of aging to population cohort *s,i*, $IM_{s,i}$ is flow presenting net immigration, $D_{s,i}$ is the flow of death from population cohort *s,i*. Thus the population in cohort is represented by continuous variable. This stock integrates flow variables that represent aging, immigration and deaths, *variable*(*t*) represents the value of the variable at any time *t*.

$$P_{s,i}(T) = \int_{T_0}^{T} \left[A_{s,i}(t) - A_{s,i+1}(t) + IM_{s,i}(t) - D_{s,i}(t) \right] dt + P_{s,i}(T_0) \quad (1)$$

Exit rate $E_{s,i}$ (aging plus deaths) in equation (2) is modelled as the first order delay (Sterman, 2000: 411–417), where Average Time In Cohort ($ATIC_{s,i}$) equals five years for all age groups except the first one where it equals fifteen years.

$$E_{s,i} = \frac{P_{s,i}}{ATIC_{s,i}} . \quad (2)$$

Net immigration is an exogenous variable. The deaths and aging for each age group are a function of $E_{s,i}$ and death rate ($DR_{s,i}$) for a particular gender and age group (equations (3) and (4)).

$$D_{s,i} = E_{s,i} \times DR_{s,i} , \quad (3)$$

$$A_{s,i} = E_{s,i} \times (1 - DR_{s,i}). \quad (4)$$

$DR_{s,i}$ is calculated on the basis of mortality rates $MR_{s,i}$ (a number of the deaths in a particular age group per 1,000 inhabitants of this age group per year). For the purposes of the continuous simulation DR must be calculated by equation (5). Finally, $DR_{s,i}$ is a function of life expectancy of a particular gender in a particular year for the age 0. The function is created using ordinary least square method (determination coefficient R^2 does not drop below 0.9 for any cohort) and transferred into a graphical function (LOOKUP function in SW Vensim) where extreme values were treated.

$$DR_{s,i} = 1 - e^{-MR_{s,i} \times ATIC_{s,i}}. \quad (5)$$

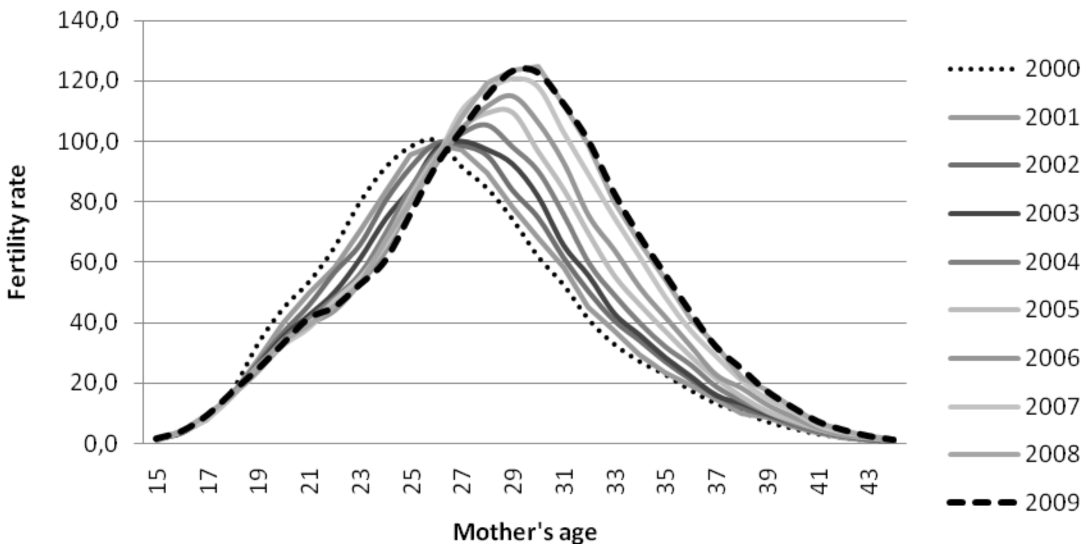
Similar to mortality rates fertility rates are modelled. In equation (6), live-born children (*LB* = A_1) are a function of the number of women in corresponding age groups multiplied by a fertility rate. The fertility rate for an age group is a function of the total fertility rate (*TF*) per total childbearing female population (15–49 years) and fertility rate index (FI_i) for a particular female age group ($P_{f,i}$). As the mortality rate, even the fertility index for an age group is modelled by a graphical function using a regression analysis (again, R^2 does not drop below 0.9).

$$LB = \sum_{i=2}^{8} \frac{P_{f,i}}{1000} \times FI_i \times TF . \quad (6)$$

In contrast to the models referenced in introduction, the created PopulCZ model demonstrates the dependence of fertility indexes on an average age of mothers. As Fig. 2 shows, the average age of mothers in the CR grew by more than two years in the last ten years. When using the fertility rates as weights, the weighted mean of mothers' ages changed from 26.67 in 2000 to 28.92 in 2009. For illustration, the weighted mean of mothers' ages changed from 26.35 to 29.34 for same years, when using the number of born children as weights (i.e. this calculation contain numbers of mothers in each age group). For both calculations, only rounded to integer ages of mothers were available (Czech Statistical Office, 2010a, 2010b). The increase is a result of an increasing age of first-child mothers but it is also supported by a return to a larger number of children. Nevertheless, the distribution of fertility with static parameters, as shown in the referred models or introduced by Sterman (2000), would be oversimplification for actual Czech Republic population.

Fertility is calculated only for age groups between 15–49 years of age (*i* = 2, …, 8), younger mothers are taken as 15 years old and older mothers are taken as 49 years old for this statistic. Live-born children determine an input flow to the age group between 0–14; 51.4% of these children are regarded as an input flow for male; the rest presents an input flow for female (a distribution is based on an arithmetic average of years 1990–2009).

After reaching the year 2010, the exogenous variables cannot be based on surveyed data. For the molecule purposes the male life expectancy linearly grows to 78.4 years in year 2030, female life expectancy grows to 83.7 and total fertility to 1.55 in same year. These are low estimates of Czech Statistical Office (2010c) for population projection. The net immigration is defined as constant (same from year 2009) in PopulCZ because the Czech Statistical Office (2010c) projection does not contain age and gender structure of immigration and its high estimate for 2009 and all subsequent years is only 56% of surveyed value for year 2008.



2: *Fertility rate distributed by mother's age*

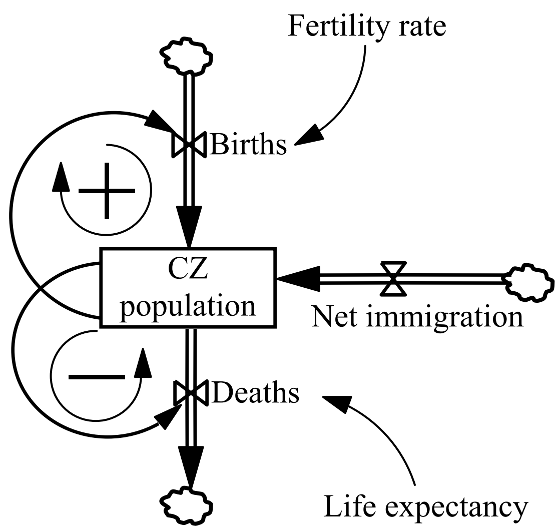
RESULTS AND DISCUSSION

Based on the above mentioned characteristics, a model PopulCZ was created consisting of more than 190 variables (24 stock variables, i.e. 12 age groups for each gender; 72 flow variables representing the aging, deaths, net immigration for each age group; other variables are parameters e.g. fertility rate indexes, life expectancy, mortality rates etc.) and the same number of equations, graphical functions and programming functions.

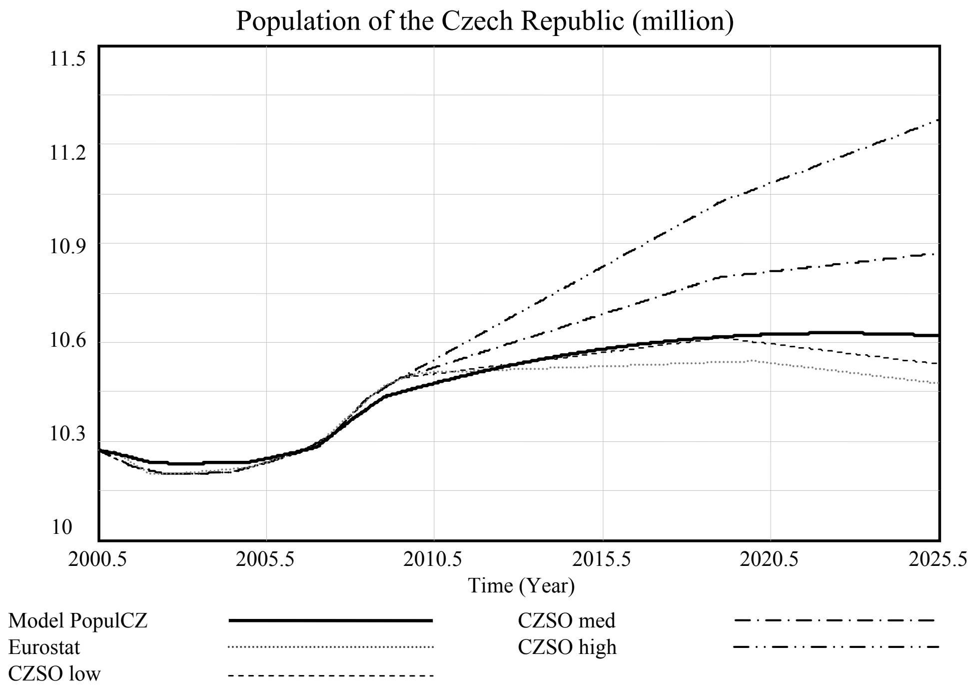
In comparison with other existing aging chains in system dynamics models, our presented chain is rather disaggregated. For instance for the project "Limits of Growth" (Forrester, 1971; Meadows *et al.* 1972, 2004) a chain with only four age groups was used. On the contrary, Wang and Sterman (1985) created 66 age groups of the Chinese population. However, they do not compose two chains to differentiate gender.

A simplified version of the model is presented in Fig. 3, the diagram shows two loops typical for population models: "reinforcing feedback loop of nascence" and "goal-seeking feedback loop of dying".

The whole stock and flow diagram is too extensive for the format of this article; however, it is available in its full form at *http://pef.czu.cz/~krejcii/*. To fulfil a molecule purposes, at the same address it is possible to obtain a complete model of PopulCZ aging chain for simulation software Vensim and use it for any system dynamics models. Although our molecule can be run only in Vensim, basic data, the stock and flow diagram and form of equations will be similar in any other system dynamics modelling



3: *A simplified diagram of the PopulCZ model*



4: *Comparing the PopulCZ model with the data of the Czech Statistical Office and Eurostat*

software (e.g. PowerSim, Stella) and thus our molecule can be still useful.

Year 2000.5 (1st of July, 2000) was established as a starting point for the model. The half of the year was selected because mortality and fertility rates are measured just up to this date. The year 2025.5 was appointed as final time (i.e. the 1st of July, 2025). Fig. 4 compares the surveyed data for Czech Republic population for period 2000–2010 (Czech Statistical Office, 2010a, 2010b, 2011) and extrapolations of Czech Statistical Office (Czech Statistical Office, 2010c) and Eurostat extrapolation (Giannakouris, 2008) with results of model PopulCZ. Eurostat represents values for years 2020 and 2030 in Fig. 4. For the Eurostat population curve, between the end of 2009 and extrapolation values, values were linearly imputed.

PopulCZ has exogenous parameters equal to low estimates of CZSO. Only net immigration is constant value from year 2009. Net immigration in PopulCZ is thus equal to 28,344. This value is close to medium estimate 25,000 of CZSO, the low variant is only 15,000 people (Czech Statistical Office, 2010c). Eurostat extrapolation uses lower values of life expectancy and total fertility than low estimation of CZSO, especially total fertility is lower for Eurostat estimation in 2060 (1.52) than low estimate of CZSO (1.53 in 2020, 1.55 in 2065).

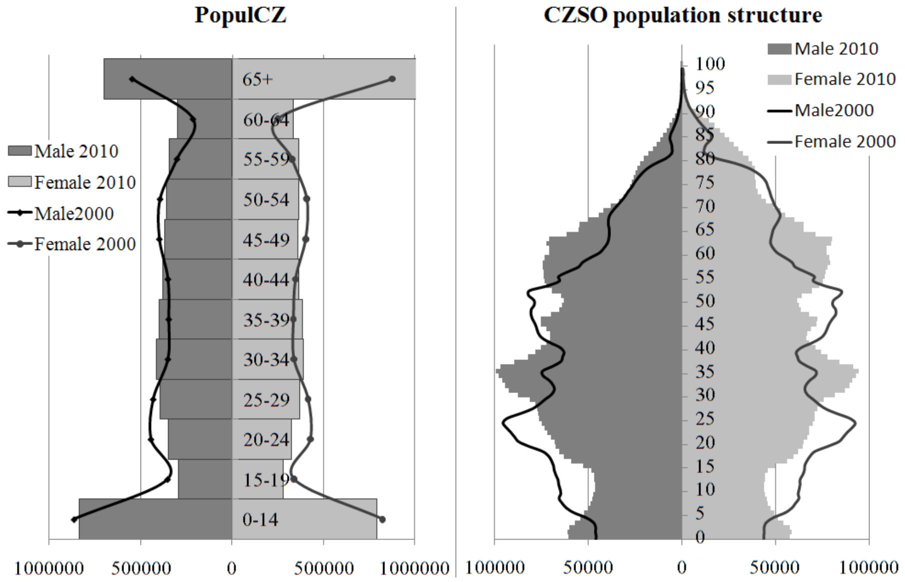
Table I shows values of the mean absolute percent error MAPE for whole population, each gender and age cohorts. The MAPE is calculated for 1st of July of years 2001–2010, for which the surveyed data are available. The year 2000 was excluded from that calculation because it is starting time for PopulCZ, therefore the values of cohorts are the surveyed values. Inclusion of the values of year 2000 would bias the MAPE to lower (seemingly better) results.

I: *Mean absolute percent error for PopulCZ (%)*

	Population	Male	Female
Total	0.24	0.28	0.24
0–14	6.11	6.04	6.17
15–19	6.54	6.47	6.61
20–24	2.12	2.03	2.21
25–29	4.58	4.52	4.65
30–34	8.19	8.11	8.28
35–39	3.79	3.83	3.86
40–44	4.49	4.07	4.93
45–49	5.67	5.3	6.05
50–54	2.61	2.72	2.52
55–59	6.84	7.11	6.59
60–64	5.89	6.01	5.78
65+	5.05	6.47	4.13

For total population MAPE shows good results of the PopulCZ (0.24%), the worst results the model reach for age cohort 30–34 for both genders. Up to the age cohort 45–49 the model provides better results for male population. For higher age cohorts the results are better for females, the biggest difference is for age cohort 65+.

Figure 5 compares the development of age distribution of PopulCZ with real population of Czech Republic (Czech Statistical office data, 2010a, 2011). Czech population is unevenly distributed,



5: *Comparison of Czech Republic and PopulCZ population structure*

with fast changes of peaks and bottoms in particular age cohorts. The figure hardly recalls a pyramid, which makes the Czech Republic population hard for modelling.

CONCLUSION

The proposed model of Czech Republic population is prepared as a molecule for other modelling work in the area of system dynamics which would otherwise use the more aggregated aging chains. In comparison with other foreign aging chains the PopulCZ counts among the more disaggregated aging chains. Moreover, the proposed aging chain is divided into two chains, one for each sex. Disaggregation and division to genders is precondition for aging chain flexibility and its usefulness for wide range of socio-economic problems focusing on different segments of population. Mean absolute percent error shows that the model PopulCZ provides very good results for whole population, for particular age cohorts this indicator range from 2.03% to 8.28%.

We found the static fertility distribution (used in referred aging chains) would be oversimplification. The molecule captures an important phenomenon of the Czech population – a shift in a mother's mean age. This fact was implemented into model as variable fertility distribution, which increases the quality of the model results.

The molecule is also provided to other authors for integration into their own system dynamics models or for inspiration. This should increase the model construction effectiveness and hopefully strengthen the communication and cooperation of small Czech system dynamics community.

Its concept allows adding other components; a mother's average age, life expectancy and total fertility serve as interface with other models. When implementing the PopulCZ model into larger models, with respect to the purpose of a particular model, it is recommended to enclose these variables to feedback loops in order to meet one of the basic principles of system dynamics, i.e. causally closed models.

Every model can be said to be "…imperfect, oversimplified, and unfinished" (Meadows, 1972: 21). The same also counts about the PopulCZ model. In the framework of future development, another disaggregation will be carried out in order to reach higher accuracy within age groups, which for some applications focusing on particular age groups may be vital. However, the referred models prove the fact that even more aggregated models will find their use.

SUMMARY

The paper presents an aging chain model, based on system dynamics principles. The purpose of the model is to meet the function of the molecule in more complex analytical models, which corresponds to good practice of system dynamics.

The objective of the paper is to construct a universal aging chain of Czech Republic population and also to provide that molecule to other authors. To ensure the usefulness of the aging chain the model is disaggregated and divided into two chains on the gender basis. Such disaggregation and division provides flexibility and applicability on wide range of socio-economic problems.

The aging chain is divided into twelve age cohorts according to gender, which fits the official data sources. The first cohort presents the population between 0 and 14 years of age, being followed by other ten five-year cohorts and the cohort of 65 years of age and older. As opposed to other common models, it captures the growth of the average age of mothers, which is typical for the Czech Republic. This fact is represented in aging chain model by variable fertility distribution.

The created simulation model is presented as the system of differential equations composed of approximately 190 variables and parameters (24 stock variables for gender-age cohorts, aging flow variables etc.) and a corresponding number of equations and graphical functions. Each cohort is a state variable represented by a definite integral. Basic flows for the age cohorts are aging, deaths and births. These flows present a function of the number of people in a given population cohort and their age as well as life expectancy for deaths flow and the mother's average age and total fertility for the births inflow. Mother's age, life expectancy and total fertility are identified as interface for the chain inclusion into other models.

The parameters are quantified on the basis of the demographic data of the Czech Statistical Office. The simulation results, using the forth order of Runge-Kutta's integration method, were tested on the official statistics for period 2001–2010. A calculated mean absolute percent error MAPE ranges between 2.03 and 8.28% for particular age groups. This indicator shows the value of 0.24% for the total value of the population and similar values for total male and female populations. Therefore, high ability of the model to simulate real population can be assumed.

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