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E-LEARNING SUPPORT FOR ECONOMIC-MATHEMATICAL METHODS

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Abstract

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Article is describing process of creating and using of e-learning program for graphical solution of linear programming problems that is used in the Economic mathematical methods course on Faculty of Business and Economics, MZLU. The program was created within FRVŠ 788/2008 grant and is intended for practicing of graphical solution of LP problems and allows better understanding of the linear programming problems. In the article is on one hand described the way, how does the program work, it means how were the algorithms implemented, and on the other hand there is described way of use of that program. The program is constructed for working with integer and rational numbers. At the end of the article are shown basic statistics of programs use of students in the present form and the part-time form of study. It is mainly the number of programs downloads and comparison to another programs and students opinion on the e-learning support.

mathematical modeling, linear programming, simplex method, graphical method, Borland Delphi, TeX, e-learning

This paper describes process of creation and use of "Graphical solution" program that was created as part of outputs that arose within FRVŠ 788/2008 grant. Program is intended for use of Economic mathematical methods students, mainly part-time form of study. Program is able to solve arbitrary linear programming problem with 2 structural variables and is intended mainly for educational purposes. In this paper will be described behavior and service of the program, input-output data format, program ability to make outputs and compatibility with other programs that arose within this grant. In the end of this paper will be mentioned basic information of program use by Economic mathematical methods students, obtained from University Information System MUAF.

MATERIALS AND METHODS

The graphical method of linear programming is used mainly as the educational method, from didactic reasons. With this method is it possible to solve only linear programming problems with 2 variables, but the main advantages of the method is, that it allows us clearly to show meaning of basic expressions used in mathematical modeling like a feasible solution, basic solution, optimal solution, etc. In the graphical solution there is also chance clearly to see important relations between those expressions and their real meaning. Those specifications can be generalized for larger problems (i.e. problems with more structural variables), that is impossible to solve graphically, but the relations and other requirements are remaining.

In the mathematical model there are two variables x_1 and x_2. The mathematical model always contains 3 parts that are:

- objective function,
- system of constraints,
- non-negativity constraints.

The non-negativity constraints occur in the model not due to the mathematic, but from pragmatic reasons.

All those three parts must be converted from mathematical model to graphical imaging.

Individual constraints represent marginal values that must be during solving of the problem respected. Mathematically, it is system of linear equations or inequations with one or more variables.

The objective function is function of one or two variables. Aim of the problem solving is to find such x_1 and x_2 that would find extreme value of objective function, i.e. maximum or minimum according to the type of the problem.

The non-negativity constraints define area, where the area of feasibility will occur – it is in the first quadrant. There will be found all feasible solutions. By adding of constraints the area of feasibility will be modified. Individual constraints will be displayed as half-plains in case, that constraint is defined as inequation or lines in case that constraint is equation. The area of feasibility is displayed as intersection point of all the constraints including non-negativity constraints. It can be:

- empty set of solutions,
- convex polyhedron,
- unbound convex set.

In case that area of feasibility is empty it means that the problem is unsolvable, i.e. there is no such linear combination of x_1 and x_2 values, that satisfies all the constraints include non-negativity constraints.

If the area of feasibility not empty, then is convex (polyhedron or unbound convex set), then we can find there optimal solution. It will occur in one or more vertices within the area of feasibility (so called basic optimal solution), or the value of objective function is increasing (decreasing) without limitation. For determination, if LP problem:

- has one or more basic optimal solutions,
- has one basic optimal solution and non-basic optimal solutions,
- has no final optimal solution,

it is needed to display the objective function.

One way to display the objective function is use of so called "streamlining vector". We may construct it in arbitrarily point. Direction of streamlining vector is determined by objective function coefficients c_1 and c_2 of variables x_1 and x_2. If we construct normal to the streamlining vector, we obtain so called "isocost line". Each point of that line inside the area of feasibility satisfies all the constraints and has same objective function value. The extreme value inside the area of feasibility will be then found as last intersection point(s) between such isocost line and area of feasibility. It can be:

- one point – vertex of the feasibility area – then the problem has 1 basic optimal solution,
- stroke – edge of the feasibility area – then the problem has 2 basic optimal solutions in vertices of the stroke and infinitely many non-basic optimal solutions in all the remaining points of the stroke
- half-line – edge of unbound feasibility area – in this case has LP problem 1 basic optimal solution in vertex of the half-line and infinitely many non-basic solutions in all the remaining points.

In case, that area of feasibility is unbound and streamlining vector direction is into such unbound area, then the LP problem does not have final optimal solution, because there is no last intersection point between normal to the streamlining vector and area of feasibility proposed by Holoubek (2006).

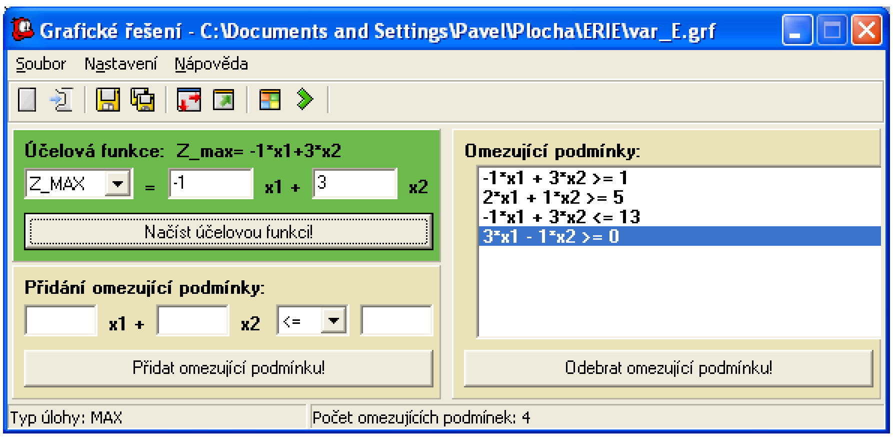
The objective function value of optimal solution we obtain by substituting of optimal solution coordinates into objective function.

RESULTS

For practicing of graphical solution of LP problem was made program, created in Borland Delphi, that allows to students solve given LP problem accordingly to way used in practices. During the creation process did authors inspired by some algorithms from Kadlec (2003). Program allows to students in case of wrong calculation immediately find the mistake, they are making, what may cause increase of study effectiveness. The program is able to work with integers and fractions.

Service of program

Work with the program is very simple. All needed control buttons are shown at fig. 1. After run of the program is needed to make following steps:



1: *Program interface*

1. With button show the canvas, where the graphical solution will be realized. On the canvas there are coordinate axis and non-negativity constraints for x_1 and x_2 variables. Canvas proportion is possible to change with button in required range.
2. Choose the objective function (maximization or minimization) type. It is realized by choosing required item from listbox in green or red highlighted "objective function" panel and putting the objective function coefficients into relevant cells.
3. Confirm by "Load objective function" button. Clicking this item will set the Objective function panel color to green in case of maximization type problem and red in minimization type problem and write objective function to top of the panel and on canvas (in case, that canvas is shown).
4. Add individual constraints. It is realized in "Add constraint" panel, where is needed to put structural coefficients, relation and right-hand side of individual constraint into relevant cells of that panel. After filling of those cells is needed to confirm it with "Add constraint" button. That will cause adding relevant constraint into list of constraints in the right part of window (see on fig. 1) and on canvas it will paint that constraint and repaint area of feasibility. Area of feasibility is highlighted with color and has changed by adding of this constraint.
5. After load of objective function and all the constraints is on the canvas shown area of feasibility (empty set, bound or unbound convex set) and we may start finding of optimal solution there.
6. Next step is to press button that will solve the LP problem and on canvas will paint streamlining vector with arrow and 3 normals to the streamlining vector. In case, that LP problem has optimal solution is the last normal highlighted and hits the area of feasibility in point(s), where the optimal solution occurs. It is the last intersection point of area of feasibility with normal to the streamlining vector. Optimal solution(s) set is highlighted and it may be point, stroke or half-line (in case that area of feasibility is empty streamlining vectors are not shown). At the same time will be written into canvas the verbal answer (see on fig. 2).

Input/output

Each LP problem with 2 variables can be saved into file, respectively load from a file. It is made by clicking on button. The file with LP problem has following text format. Numbers are separated with tabulators, texts (comments) in square brackets are obligatory, but may be arbitrary (e.g. in different languages):

```
[Number of constraints and variables]
   4         2
[Objective function]
Z_MAX  -1     -3
[Constraints]
  -1    3     >=      1
   2    1     >=      5
  -1    3     <=     13
   3   -1     >=      0
```

In the problem above, first two rows contain number of constraints and variables. In the first row there is only text and will not be thereinafter used. In the second row there are numbers 4 and 2, separated by tabulator. It means that LP problem has 4 constraints and 2 variables.

Next follows 2 rows with LP problem type and objective function coefficients. As in above, row with text will not be used and next row contains Z_MAX, i.e. maximization problem type and –1 and –3, what are objective function coefficients. In the following row there are beginning constraints. There is as above text in the square brackets followed by 4 rows (because there are 4 constraints in LP problem). Let's have a look at that row: first number is x_1 structural coefficient, followed by x_2 structural coefficient. Then there is relation, that may be <=, = or >= type and the last part of each row contains limitation of the constraint (also called as right-hand side).

The main advantage of this file format is in universality and edit-ability. The data may be also edited manually. Another advantage is full two-way compatibility with Simplex method program, where is also possible to solve the given problem.

After loading the LP problem is it possible to solve it.

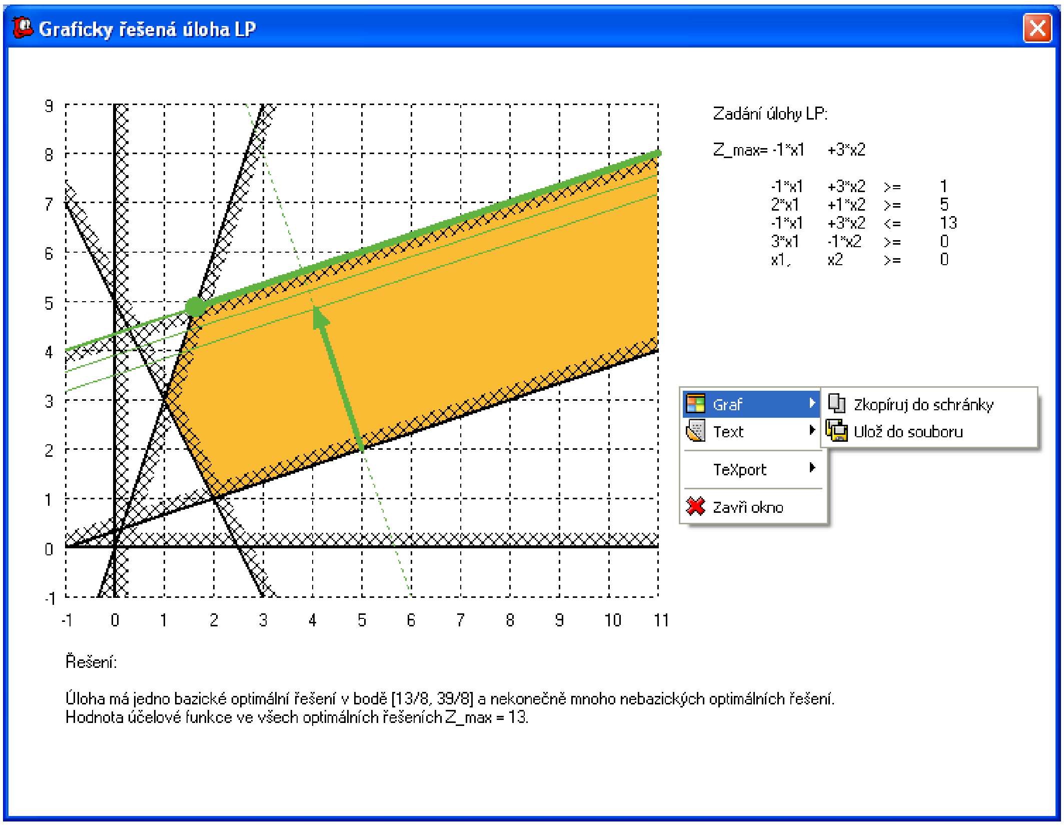
Export of results

Solved LP problem is possible to export. This option is useful not only for students, but also for teachers that get mighty tool for preparation of educational materials – into practices, for self-study, exam and studying materials. Export is realized by right-clicking on canvas with graph (see fig. 2) and choosing export option. Solved LP problem is possible to export as a bitmap or text. The whole LP problem is also possible to export into TeX (program generates TeX source code), what brings to users of program mighty tool for creation of studying materials. All the generated text is possible export to clipboard or save to file. During the creation of TeX generating procedures were used rules described in Rybička (2003).

Program use in e-learning application of UIS

In this paragraph will be described use of the program in e-learning and will be mentioned also statistics of download and use of the program and its comparison to another programs included in e-learning. The Economic mathematical methods course is educated in both semesters in present and part-time form of study. Let's have a look at statistics from summer semester of 2009 school year.

Within e-learning did students have 3 main programs: program for graphical solution of LP prob-



2: *Solved LP problem that is exported (within the context menu)*

lems, program for simplex method and program for transportation problems. Use of those 3 programs is in the table 1. Furthermore, they did have for each practice PDF slides with examples they will be done in the practice and practicing examples for individual chapters.

Now, let's compare use of the program globally and in comparison to another two programs. In summer semester 2009, EMM course had 295 students. From those 295 students at least once worked with e-learning 224, remaining 74 did not work. It means that with e-learning had worked 75% of all EMM students of present and part-time form study.

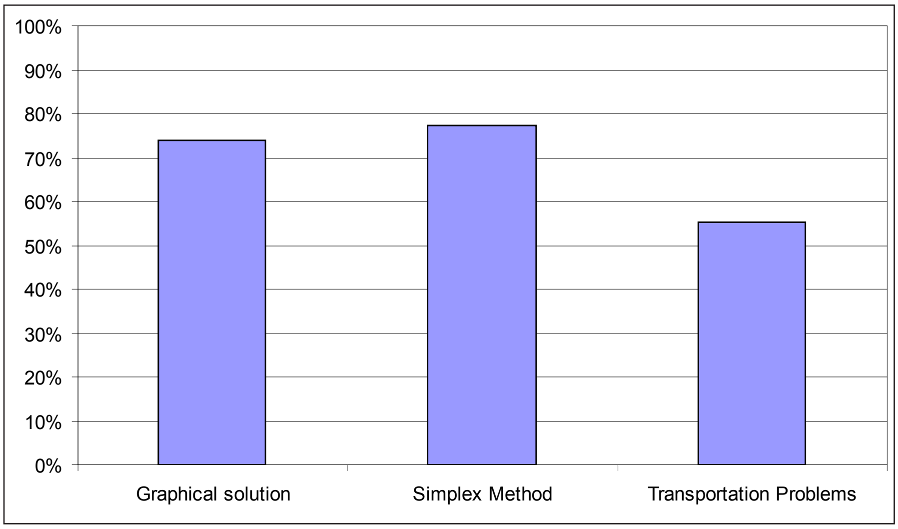
In the present form subscribed EMM totally 212 students. From them, exactly 157 downloaded at least once program "Graphical solution", i.e. with the program has worked 74% of present form students. Approximately 2/3 of them has downloaded that program more than once, the remaining 1/3 exactly once. The mean is 2.84 downloads (calculated from students, they have worked with program at least once). From the data above we know, that more than 50% of all present form students did download the program more times. From those data indirectly results, that if students have tried the program, they also have used it (not only "tried"). Direct proof of program use is in evaluations, where some students comment e-learning support (and concrete those programs) as very good and also that some of students attended to mistakes of program. Those comments were accepted and programs errors were eliminated.

In the part-time form, the course had 77 students. From them program "Graphical solution" downloaded 68 student. From the first point of view, it is a very good result, even better than in present form. But, there is one problem that disabled to make more accurate statistics for part-time form of study. E-learning course was accessible also for present form students they had consulting form of course. Because the information system cannot separate those two groups of students, are the statistics not available.

Regarding to number of e-learning users and feedback from students they successfully finished the course (from evaluations and interviews after passing the exam) we can suppose, that decision to make e-learning support was the right way. From that reason, e-learning support is improved and extended within the course, and into another courses. The way, how to extend the e-learning support is through bachelor or diploma thesis or grants.

DISCUSSION

The program for graphical solution arose on the base of experience obtained in the EMM courses. Regarding to experience in practical use in winter semester 2008 and on base of students feedback and opposition procedure, the program was dramatically improved. Authors of program are convinced, that



3: *Programs user ratio (all EMM students)*

in today's form the program may very good serve to its primary goal: self-studying of the linear programming problematic and increasing of self-study effectiveness. The program is among others very useful tool for teachers, when there is a need to prepare materials for teaching.

SUMMARY

This article describes process of creation and experience with "Graphical solution" program designed for Economic-mathematical methods course and its use within e-learning.

The first part of this article describes methods of graphical solution of linear programming problems. The methods knowledge is important mainly because of correct program behavior, because the program is mainly used for educational purposes. There are analyzed cases, when the linear programming problem is solvable, when not and how can be this solution verified. In case, that the LP problem is solvable, was described the way, how to find the optimal solution (one or more, if it is possible to find the optimal solution).

Next part of that article describes the "Graphical solution" program. Process of program creation and used algorithms are not described. In this article is described program service and behavior. Program itself is able to solve arbitrary LP problem with two structural variables. Requirements on input data are to be integers or fractions. There is described, how to add (or subtract) individual constraints and how adding or subtracting of constraint will change the feasibility area, how to set objective function type, its coefficients and its loading. Description follows with solving of the problem and answer dump.

Next part of the article describes input/output data format. The data format was made with respect to easy readability and edit-ability also off the main program. The data format are text file, single data are in the rows, separated with tabulators. Data format is both-way compatible with "Simplex method", i.e. arbitrary LP problem solvable graphically is possible to solve with "Simplex method" program, conversely[1].

The program results are possible to export, so that were further usable in study or education. It is possible to export the problem, respectively problem solution as a bitmap or in the TeX source code. Both possibilities are possible to save into clipboard or into file.

The last part describes use of the program within e-learning. Because the program was part of eLearning syllabus within UIS MUAF, the authors did have access into statistics of use of the program. There is described number of students subscribed in EMM course, number of program users and comparison to another programs, build in the grant.

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1 In case of another way conversion, the LP problem will be graphically solved only in case, that it has structural variables. Otherwise there will appear warning, that this LP problem is graphically unsolvable.

SOUHRN

E-learningová podpora v předmětu Ekonomicko matematické metody

Článek se zabývá tvorbou podpůrného programu „Grafické řešení" do předmětu Ekonomicko-matematické metody a zkušenosti s tímto programem a jeho využití v rámci e-learningu.

První část článku se zabývá metodikou grafického řešení úloh lineárního programování. Znalost metodiky je podstatná zejména s ohledem na korektní chování programu, neboť program bude sloužit jako pomůcka při výuce. Jsou v ní rozebrány případy, kdy je úloha lineárního programování řešitelná, kdy ne, a jakým způsobem lze tato fakta ověřit. V případě, že se jedná o řešitelnou úlohu, byl v metodice popsán způsob, jak lze nalézt optimální řešení (a jestli je jedno nebo více, pokud nalézt lze).

Další část se zabývá vytvořeným programem „Grafické řešení". Tvorba programu a algoritmy v programu použité nejsou v článku popsány, je v ní popsán způsob obsluhy a chování programu. Program jako takový je schopen graficky vyřešit libovolnou úlohu lineárního programování se dvěma strukturními proměnnými. Požadavek na vstupní data je, že se bude jednat buď o celočíselné hodnoty, resp. zlomky. Popisuje, jakým způsobem lze přidat (či odebrat) jednotlivé omezující podmínky, jak se přidání omezující podmínky projeví na množině přípustných řešení, dále jak nastavit (změnit) typ účelové funkce a její koeficienty a následně její načtení. Následuje popis, jak nalézt v programu výsledné optimální řešení a způsob výpisu odpovědi.

Následuje část, zabývající se formátem vstupně/výstupních dat. Ten byl záměrně vytvořen tak, aby byl jednoduše čitelný, resp. editovatelný i mimo hlavní program. Proto se jedná o čistě textový soubor, kde jsou jednotlivé údaje zapsány na řádcích a odděleny tabulátory. Datový formát je oboustranně kompatibilní i s programem pro řešení úloh lineárního programování simplexovou metodou, tj. libovolný problém řešitelný graficky je možné otevřít a spočítat programem „Simplexová metoda" a naopak[2].

Výsledky řešení programu lze dále exportovat tak, aby byly nadále použitelné, ať již ve výuce nebo při studiu. Program umožňuje exportovat zadání, resp. řešení úlohy a to buď jako klasickou bitmapu, anebo jako zdrojový formát pro TeX. Pro obojí možnosti je možné zvolit export buď do schránky, anebo do souboru.

Poslední část článku se týká využití programu v rámci e-learningu. Jelikož program byl umístěn v eLearningových osnovách v UIS MZLU, měli autoři programu přístup ke statistikám jeho využití. Jsou zde proto popsány jednak počty studentů předmětu, dále počty uživatelů a opakovaných uživatelů programu a srovnání s ostatními programy, vytvořenými v rámci daného grantu.

matematické modelování, lineární programování, simplexová metoda, grafické řešení, Borland Delphi, TeX, e-learning

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2 V opačném případě dojde ke grafickému řešení jen v případě, že úloha má dvě strukturní proměnné, jinak se objeví upozornění, že úlohu těchto rozměrů nelze řešit.