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COMPARISON OF POWER OF MODIFIED JARQUE-BERA NORMALITY TESTS AND SELECTED TESTS OF NORMALITY

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Abstract

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The aim of this paper is to modify the classical Jarque-Bera test and the robust Jarque-Bera test of normality. We use the median as an estimator instead of the mean in the classical Jarque-Bera test and in the robust Jarque-Bera test. This leads to the modified Jarque-Bera test and the modified robust Jarque-Bera test. Paper also demonstrates results of simulation studies of power of such tests with the various alternatives – light tailed alternatives as exponential, lognormal and gamma distribution, heavy tailed alternatives as Cauchy, Laplace, t_3, t_5 and logistic distributions and short tailed alternatives as beta and uniform distributions. These tests of normality are also used for normality testing of selected datasets of financial time series. Source data include logarithmic returns of monthly average prices of Prague stock exchange index PX and monthly average prices of CZK/EUR exchange rate in the period from 2000 to 2007.

tests of normality, Monte Carlo simulation, power comparison, financial time series, logarithmic returns

The topic of this paper is the problem of testing normality, i.e. problem of testing whether a sample of observations comes from a normal distribution. Normality is one of the most common assumptions made in the development and use of statistical procedures. For example in the majority of cases of relevant analysis of financial time series is expected that returns derived from financial time series is Gaussian normal distributed random variable with constant expected value and constant variance.

There exists a vast literature on tests of normality and their statistical properties (for example Anderson and Darling (1952), Shapiro and Wilk (1965), Lilliefors (1967), Jarque and Bera (1980), D`Agostino (1986), Thode (2002), Gel, Miao and Gastwirth (2006), Gel and Gastwirth (2007) etc.). Therefore, we can used formal testing procedures that have been proposed to test of normality, as well as general goodness of fit tests, plotting methods, outliers tests and other tests that are useful in detecting nonnormality in specialized situations. For example, today the most commonly used omnibus test against general alternatives is the Shapiro-Wilk test which is based on expected values of the normal order statistics. However, the most popular omnibus test in economic and finance is the Jarque-Bera test which is based on the third and fourth sample moments. Nevertheless we can used the other tests of normality – the D`Agostino test, the Anderson-Darling test, the Cramer-von Mises test, the Shapiro-Francia test, the Pearson chi-square test, the directed SJ test, the robust Jarque-Bera test or the Lilliefors (Kolmogorov-Smirnov) test which is the best known omnibus test based on empirical distribution function.

This paper is divided into four parts. In part 1, the Jarque-Bera normality test and the robust Jarque-Bera normality test are described as well as the new robust normality tests are proposed. Part 2 deals with the gist of this paper – this part presents results of comparative power study of selected tests of normality. Consequently, we used these normality tests for testing of selected datasets of financial time series. In part 3, results are discussed and confronted with previously published papers and results. The final part of this paper is the summary.

MATERIAL AND METHODS

The most widely used test against general alternatives is the Shapiro-Wilk test (Shapiro and Wilk (1965)). In contrast, the tests of normality based on the third and fourth sample moments are not consistent against all alternatives. The most popular test based on the third and fourth sample moments in economics and finance is the Jarque-Bera test (Jarque and Bera (1980)). The Jarque-Bera test is a goodness-of-fit measure of departure from normality, based on the sample kurtosis and skewness. The Jarque-Bera test statistic (JB) is defined as

$$JB = \frac{n}{6}\left(\frac{\hat{\mu}_3}{\hat{\mu}_2^{3/2}}\right)^2 + \frac{n}{24}\left(\frac{\hat{\mu}_4}{\hat{\mu}_2^2} - 3\right)^2,$$

where $\hat{\mu}_k = \frac{1}{n}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^k$ is a sample estimate of the k-th population central moment for any positive integer k, X_i for $i = 1, \ldots, n$ is a sample of independent and identically distributed random variables, $\overline{X}$ is the sample estimate of first population moment (sample mean), $\hat{\mu}_3 / \hat{\mu}_2^{3/2}$ is the sample estimate of the skewness and $\hat{\mu}_4 / \hat{\mu}_2^2$ is the sample estimate of the kurtosis.

The Jarque-Bera test statistic follows asymptotically the chi-square distribution with two degrees of freedom and can be used to test the null hypothesis that the data are from a normal distribution. The null hypothesis is a joint hypothesis of the skewness being 0 and the excess kurtosis being 0. Under the null hypothesis of normality, the sample skewness and kurtosis are asymptotically normally distributed with covariance matrix defined as

$$\sqrt{n}\begin{bmatrix}\hat{\mu}_3/\hat{\mu}_2^{3/2} \\ \hat{\mu}_4/\hat{\mu}_2^2 - 3\end{bmatrix} \Rightarrow N\left(\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}6 & 0\\ 0 & 24\end{bmatrix}\right).$$

Now we can propose modification of the Jarque-Bera test. The one possible approach is to use the median as an estimator instead of the mean in the classical JB test statistic. This leads to the modified Jarque-Bera test (MJB). This is a special case of the class of MJB(i, j, k, l) test for $i, j, k, l \in \{0,1\}$ proposed by Střelec and Stehlík (2008). For instance MJB(1, 1, 1, 1) denotes the test where in all places of $\hat{\mu}_k$ of the Jarque-Bera test statistic the median is used instead of arithmetic mean. Consequently, the modified Jarque-Bera test statistic is defined as

$$MJB = \frac{n}{C_1}\left(\frac{\hat{m}_3}{\hat{m}_2^{3/2}}\right)^2 + \frac{n}{C_2}\left(\frac{\hat{m}_4}{\hat{m}_2^2} - 3\right)^2,$$

where $\hat{m}_k = \frac{1}{n}\sum_{i=1}^{n}\left(X_i - M\right)^k$ is a sample estimate of the k-th population central moment for any positive integer k, X_i for $i = 1, \ldots, n$ is a sample of independent and identically distributed random variables, M is the sample median, $\hat{m}_3 / \hat{m}_2^{3/2}$ is the sample estimate of the skewness and $\hat{m}_4 / \hat{m}_2^2$ is the sample estimate of the kurtosis. C_1 and C_2 are positive constants. We can obtain these constants from the Monte Carlo simulations. Therefore, we recommend $C_1 = 18$ and $C_2 = 24$. Consequently, the modified Jarque-Bera test statistic follows asymptotically the chi-square distribution with two degrees of freedom, because sample skewness and kurtosis are asymptotically normally distributed.

Similarly, we can modify the robust Jarque-Bera test proposed by Gel and Gastwirth (2007). The robust Jarque-Bera test statistic (RJB) is defined as

$$RJB = \frac{n}{C_1}\left(\frac{\hat{\mu}_3}{J_n^3}\right)^2 + \frac{n}{C_2}\left(\frac{\hat{\mu}_4}{J_n^4} - 3\right)^2,$$

where $\hat{\mu}_k = \frac{1}{n}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^k$ is a sample estimate of the k-th population central moment for any positive integer k, X_i for $i = 1, \ldots, n$ is a sample of independent and identically distributed random variables, $\overline{X}$ is the sample estimate of first population moment (sample mean), $J_n = \frac{C}{n}\sum_{i=1}^{n}\left|X_i - M\right|$ for $C = \sqrt{\pi/2}$ is the average absolute deviation from the sample median (MAAD) proposed by Gastwirth (1982), $\hat{\mu}_3 / J_n^3$ is the robust sample estimate of the skewness and $\hat{\mu}_4 / J_n^4$ is the robust sample estimate of the kurtosis. C_1 and C_2 are positive constants. Gel and Gastwirth (2007) recommend on the basis of the Monte Carlo simulations $C_1 = 6$ and $C_2 = 64$. Consequently, the robust Jarque-Bera test statistic follows asymptotically the chi-square distribution with two degrees of freedom.

The modified robust Jarque-Bera test (MRJB) used the median as an estimator instead of the mean in the classical RJB test statistics. This leads to the modified robust Jarque-Bera test. This is a special case of the class of MRJB(i, j, k, l) test for $i, j, k, l \in \{0, 1\}$ proposed by Střelec and Stehlík (2008). For instance MRJB(1, 1, 1, 1) denotes the test where in all places of $\hat{\mu}_k$ of the robust Jarque-Bera test statistic the median is used instead of arithmetic mean. Consequently, the modified robust Jarque-Bera test statistic is defined as

$$MRJB = \frac{n}{C_1}\left(\frac{\hat{m}_3}{J_n^3}\right)^2 + \frac{n}{C_2}\left(\frac{\hat{m}_4}{J_n^4} - 3\right)^2,$$

where $\hat{m}_k = \frac{1}{n}\sum_{i=1}^{n}\left(X_i - M\right)^k$ is a sample estimate of the k-th population central moment for any positive integer k, X_i for $i = 1, \ldots, n$ is a sample of independent and identically distributed random variables, M is the sample median, J_n is the average absolute deviation from the sample median (MAAD), $\hat{\mu}_3 / J_n^3$ is the robust sample estimate of the skewness and $\hat{\mu}_4 / J_n^4$ is the robust sample estimate of the kurtosis. C_1 and C_2 are positive constants. We can obtain these constants from the Monte Carlo simulations. Therefore, we recommend $C_1 = 18$ and $C_2 = 58$. Consequently, the modified robust Jarque-Bera test statistic follows asymptotically the chi-square distribution with two degrees of freedom, because sample skewness and kurtosis are asymptotically normally distributed.

To compare performance of the modified Jarque-Bera and the modified robust Jarque-Bera tests to existing tests of normality, in particular, the Jarque-Bera test (JB), the robust Jarque-Bera test (RJB), the Shapiro-Wilk test (SW), the Shapiro-Francia test (SF), the D`Agostino test (DT), the Anderson-Darling test (AD), the Cramer-von Mises test (CM), the Pearson chi-square test (PT), the directed SJ test (SJ) and the Lilliefors (Kolmogorov-Smirnov) test (L(KS)), we carry out a simulation study. Number of Monte Carlo simulations is 10,000.

RESULTS

Tab. I shows the size of the JB, MJB, RJB and MRJB tests for various sample sizes and $\alpha = 0.05$ based on 10,000 Monte Carlo simulations.

I: *Size of the Jarque-Bera test (JB), the modified Jarque-Bera test (MJB), the robust Jarque-Bera (RJB) and the modified robust Jarque-Bera test (MRJB) for $\alpha = 0.05$ based on 10,000 simulations.*

	sample size (*n*)							
	25	**50**	**75**	**100**	**125**	**150**	**175**	**200**
JB	0.0503	0.0471	0.0546	0.0439	0.0498	0.0508	0.0516	0.0484
MJB	0.0505	0.0466	0.0523	0.0443	0.0471	0.0500	0.0534	0.0453
RJB	0.0522	0.0491	0.0539	0.0446	0.0497	0.0518	0.0525	0.0504
MRJB	0.0525	0.0501	0.0537	0.0451	0.0512	0.0556	0.0572	0.0465

Tab. II–V show power of the selected tests of normality for sample sizes $n = 25$, 50, 100 and 200, respectively. Consequently, Tab. VI and Fig. 1 contain comparison of power of selected normality tests. As can been seen from the tables II–VI, the ranging of considered completing tests depends heavily on the type of tails. In the case of light tailed alternatives (e.g. exponential, lognormal, gamma distributions) the best test is the SW test and the worst is the directed SJ test, while the heavy tailed alternatives (Cauchy, Laplace, t_3, t_5 and logistic distributions) give two situation: the best one test for the most heavy tailed distributions (e.g. Cauchy, Laplace and t_3) is the SJ test and for a slightly less heavy tailed alternatives (e.g. logistic and t_5 distributions) is the best one the RJB test. The worst one for heavy tailed alternatives is the PT test. For (very) short tailed alternative distributions as beta distribution and uniform distribution is the best one the DT test and the worst one is the SJ test.

II: *Power of the Shapiro-Wilk test (SW), the Shapiro-Francia test (SF), the Lilliefors (Kolmogorov-Smirnov) test (L(KS)), the Anderson-Darling test (AD), the Cramer-von Mises test (CM), the Pearson chi-square test (PT), the directed SJ test (SJ), the D`Agostino test (DT), the Jarque-Bera test (JB), the modified Jarque-Bera test (MJB), the robust Jarque-Bera (RJB) and the modified robust Jarque-Bera test (MRJB) for $\alpha = 0.05$ and sample size $n = 25$.*

	Cauchy	**Laplace**	**t_3**	**t_5**	**logistic**	**exp**	**log-normal**	**gamma (2,2)**	**beta (2,2)**	**uniform**
SW	0.92	0.31	0.41	0.22	0.13	0.92	0.98	0.65	0.07	0.29
SF	0.94	0.38	0.47	0.26	0.16	0.89	0.97	0.61	0.03	0.12
$L(KS)_{emp}$	0.90	0.26	0.31	0.15	0.09	0.70	0.88	0.41	0.05	0.12
AD_{emp}	0.93	0.32	0.38	0.20	0.12	0.87	0.96	0.58	0.07	0.24
CM_{emp}	0.93	0.31	0.36	0.19	0.11	0.82	0.94	0.52	0.07	0.19
PT_{emp}	0.84	0.16	0.20	0.09	0.06	0.76	0.89	0.35	0.06	0.10
SJ_{emp}	0.95	0.45	0.48	0.28	0.17	0.47	0.77	0.28	0.00	0.00
DT_{emp}	0.90	0.32	0.44	0.25	0.16	0.69	0.88	0.47	0.05	0.24
JB_{emp}	0.92	0.35	0.46	0.27	0.17	0.75	0.91	0.50	0.00	0.00
MJB_{emp}	0.91	0.34	0.45	0.26	0.16	0.77	0.93	0.52	0.00	0.00
RJB_{emp}	0.95	0.42	0.49	0.29	0.18	0.65	0.87	0.45	0.00	0.00
$MRJB_{emp}$	0.94	0.42	0.48	0.27	0.17	0.72	0.91	0.49	0.01	0.00

Number of Monte Carlo simulations is 10,000.

Now we focus to comparison of power of the JB, MJB, RJB and MRJB tests. The MJB and MRJB tests outperform the classical JB and RJB tests for light tailed skewed alternative distributions (e.g. exponential, lognormal and gamma distributions), especially in small sample sizes. The MJB test shows higher power than the JB test in detecting the exponential, lognormal and gamma distributions in samples of size 25–50. In contrast, the MRJB test shows higher power than the RJB test in detecting the exponential, lognormal and gamma distributions in samples of size 25–75. On the other hand, the JB and RJB tests show equal or higher power than the MJB and MRJB tests for these alternatives in large sample sizes.

For heavy tailed symmetric distributions (e.g. Cauchy, Laplace, t_3, t_5 and logistic distributions) the MJB and MRJB tests are equal or slightly less powerful than the JB and RJB test, especially in small and moderate sample sizes ($25 \leq n \leq 100$).

III: *Power of the Shapiro-Wilk test (SW), the Shapiro-Francia test (SF), the Lilliefors (Kolmogorov-Smirnov) test (L(KS)), the Anderson-Darling test (AD), the Cramer-von Mises test (CM), the Pearson chi-square test (PT), the directed SJ test (SJ), the D`Agostino test (DT), the Jarque-Bera test (JB), the modified Jarque-Bera test (MJB), the robust Jarque-Bera (RJB) and the modified robust Jarque-Bera test (MRJB) for* $\alpha = 0.05$ *and sample size* $n = 50$.

	Cauchy	**Laplace**	t_3	t_5	**logistic**	**exp**	**log-normal**	**gamma (2,2)**	**beta (2,2)**	**uniform**
SW	1.00	0.53	0.63	0.35	0.19	1.00	1.00	0.95	0.15	0.75
SF	1.00	0.60	0.69	0.41	0.24	1.00	1.00	0.92	0.05	0.48
$L(KS)_{emp}$	0.99	0.44	0.48	0.21	0.11	0.96	1.00	0.68	0.08	0.26
AD_{emp}	1.00	0.55	0.61	0.29	0.16	1.00	1.00	0.88	0.14	0.58
CM_{emp}	1.00	0.55	0.58	0.26	0.14	0.99	1.00	0.82	0.12	0.43
PT_{emp}	0.98	0.27	0.31	0.12	0.07	0.98	1.00	0.63	0.07	0.18
SJ_{emp}	1.00	0.72	0.73	0.43	0.25	0.72	0.94	0.42	0.00	0.00
DT_{emp}	0.99	0.49	0.64	0.39	0.22	0.96	1.00	0.78	0.23	0.79
JB_{emp}	1.00	0.56	0.69	0.43	0.25	0.98	1.00	0.83	0.00	0.02
MJB_{emp}	1.00	0.56	0.68	0.42	0.24	0.98	1.00	0.83	0.00	0.03
RJB_{emp}	1.00	0.66	0.72	0.45	0.26	0.94	0.99	0.74	0.00	0.00
$MRJB_{emp}$	1.00	0.66	0.71	0.44	0.25	0.96	0.99	0.75	0.00	0.00

Number of Monte Carlo simulations is 10,000.

IV: *Power of the Shapiro-Wilk test (SW), the Shapiro-Francia test (SF), the Lilliefors (Kolmogorov-Smirnov) test (L(KS)), the Anderson-Darling test (AD), the Cramer-von Mises test (CM), the Pearson chi-square test (PT), the directed SJ test (SJ), the D`Agostino test (DT), the Jarque-Bera test (JB), the modified Jarque-Bera test (MJB), the robust Jarque-Bera (RJB) and the modified robust Jarque-Bera test (MRJB) for* $\alpha = 0.05$ *and sample size* $n = 100$.

	Cauchy	**Laplace**	t_3	t_5	**logistic**	**exp**	**log-normal**	**gamma (2,2)**	**beta (2,2)**	**uniform**
SW	1.00	0.79	0.88	0.56	0.30	1.00	1.00	1.00	0.46	1.00
SF	1.00	0.84	0.91	0.63	0.36	1.00	1.00	1.00	0.22	0.97
$L(KS)_{emp}$	1.00	0.70	0.73	0.32	0.15	1.00	1.00	0.95	0.15	0.59
AD_{emp}	1.00	0.82	0.85	0.46	0.24	1.00	1.00	1.00	0.32	0.95
CM_{emp}	1.00	0.81	0.83	0.41	0.21	1.00	1.00	0.99	0.24	0.85
PT_{emp}	1.00	0.48	0.53	0.18	0.09	1.00	1.00	0.95	0.11	0.45
SJ_{emp}	1.00	0.94	0.93	0.65	0.41	0.90	1.00	0.62	0.00	0.00
DT_{emp}	1.00	0.72	0.87	0.58	0.32	1.00	1.00	0.99	0.62	1.00
JB_{emp}	1.00	0.79	0.91	0.64	0.39	1.00	1.00	1.00	0.03	0.72
MJB_{emp}	1.00	0.79	0.91	0.63	0.38	1.00	1.00	0.99	0.06	0.75
RJB_{emp}	1.00	0.88	0.93	0.67	0.42	1.00	1.00	0.98	0.00	0.01
$MRJB_{emp}$	1.00	0.88	0.93	0.66	0.42	1.00	1.00	0.97	0.00	0.01

Number of Monte Carlo simulations is 10,000.

For short tailed alternative distributions (e.g. beta and uniform distributions) the MJB test shows higher or equal power than the JB test in all sample sizes. In contrast, the MRJB test shows equal power than the RJB test in small and moderate sample sizes and higher power than the RJB test in large sample sizes ($125 \leq n \leq 200$).

V: *Power of the Shapiro-Wilk test (SW), the Shapiro-Francia test (SF), the Lilliefors (Kolmogorov-Smirnov) test (L(KS)), the Anderson-Darling test (AD), the Cramer-von Mises test (CM), the Pearson chi-square test (PT), the directed SJ test (SJ), the D`Agostino test (DT), the Jarque-Bera test (JB), the modified Jarque-Bera test (MJB), the robust Jarque-Bera (RJB) and the modified robust Jarque-Bera test (MRJB) for* $\alpha = 0.05$ *and sample size* $n = 200$.

	Cauchy	**Laplace**	t_3	t_5	**logistic**	**exp**	**log-normal**	**gamma (2,2)**	**beta (2,2)**	**uniform**
SW	1.00	0.98	0.99	0.81	0.50	1.00	1.00	1.00	0.92	1.00
SF	1.00	0.98	0.99	0.85	0.57	1.00	1.00	1.00	0.74	1.00
$L(KS)_{emp}$	1.00	0.95	0.94	0.53	0.25	1.00	1.00	1.00	0.34	0.94
AD_{emp}	1.00	0.98	0.98	0.74	0.41	1.00	1.00	1.00	0.71	1.00
CM_{emp}	1.00	0.98	0.98	0.68	0.36	1.00	1.00	1.00	0.55	1.00
PT_{emp}	1.00	0.78	0.79	0.28	0.11	1.00	1.00	1.00	0.22	0.90
SJ_{emp}	1.00	1.00	1.00	0.88	0.64	0.99	1.00	0.82	0.00	0.00
DT_{emp}	1.00	0.94	0.99	0.82	0.51	1.00	1.00	1.00	0.97	1.00
JB_{emp}	1.00	0.97	0.99	0.87	0.59	1.00	1.00	1.00	0.67	1.00
MJB_{emp}	1.00	0.96	0.99	0.86	0.58	1.00	1.00	1.00	0.71	1.00
RJB_{emp}	1.00	0.99	1.00	0.88	0.63	1.00	1.00	1.00	0.20	0.99
$MRJB_{emp}$	1.00	0.99	1.00	0.89	0.63	1.00	1.00	1.00	0.31	0.99

Number of Monte Carlo simulations is 10,000.

VIa: *Comparison of power of the Shapiro-Wilk test (SW), the Shapiro-Francia test (SF), the Lilliefors (Kolmogorov-Smirnov) test (L(KS)), the Anderson-Darling test (AD), the Cramer-von Mises test (CM), the Pearson chi-square test (PT), the directed SJ test (SJ), the D`Agostino test (DT), the Jarque-Bera test (JB), the modified Jarque-Bera test (MJB), the robust Jarque-Bera (RJB) and the modified robust Jarque-Bera test (MRJB) for* $\alpha = 0.05$ *and sample sizes* $n = 25$ *and 50.*

	Cauchy	**Laplace**	t_3	t_5	**logistic**	**exp**	**log-normal**	**gamma (2,2)**	**beta (2,2)**	**uniform**
25	SJ_{emp}	SJ_{emp}	RJB_{emp}	RJB_{emp}	RJB_{emp}	SW	SW	SW	AD_{emp}	SW
	RJB_{emp}	$MRJB_{emp}$	SJ_{emp}	SJ_{emp}	SJ_{emp}	SF	SF	SF	CM_{emp}	DT_{emp}
	$MRJB_{emp}$	RJB_{emp}	$MRJB_{emp}$	$MRJB_{emp}$	JB_{emp}	AD_{emp}	AD_{emp}	AD_{emp}	SW	AD_{emp}
	SF	SF	SF	JB_{emp}	$MRJB_{emp}$	CM_{emp}	CM_{emp}	MJB_{emp}	PT_{emp}	CM_{emp}
	AD_{emp}	JB_{emp}	JB_{emp}	SF	SF	MJB_{emp}	MJB_{emp}	CM_{emp}	$L(KS)_{emp}$	SF
	CM_{emp}	MJB_{emp}	MJB_{emp}	MJB_{emp}	MJB_{emp}	PT_{emp}	$MRJB_{emp}$	JB_{emp}	DT_{emp}	$L(KS)_{emp}$
	SW	DT_{emp}	DT_{emp}	DT_{emp}	DT_{emp}	JB_{emp}	JB_{emp}	$MRJB_{emp}$	SF	PT_{emp}
	JB_{emp}	AD_{emp}	SW	SW	SW	$MRJB_{emp}$	PT_{emp}	DT_{emp}	$MRJB_{emp}$	$MRJB_{emp}$
	MJB_{emp}	CM_{emp}	AD_{emp}	AD_{emp}	AD_{emp}	$L(KS)_{emp}$	$L(KS)_{emp}$	RJB_{emp}	MJB_{emp}	MJB_{emp}
	$L(KS)_{emp}$	SW	CM_{emp}	CM_{emp}	CM_{emp}	DT_{emp}	DT_{emp}	$L(KS)_{emp}$	SJ_{emp}	JB_{emp}
	DT_{emp}	$L(KS)_{emp}$	$L(KS)_{emp}$	$L(KS)_{emp}$	$L(KS)_{emp}$	RJB_{emp}	RJB_{emp}	PT_{emp}	RJB_{emp}	RJB_{emp}
	PT_{emp}	PT_{emp}	PT_{emp}	PT_{emp}	PT_{emp}	SJ_{emp}	SJ_{emp}	SJ_{emp}	JB_{emp}	SJ_{emp}

	Cauchy	Laplace	t_3	t_5	logistic	exp	log-normal	gamma (2,2)	beta (2,2)	uniform
50	SJ_{emp}	SJ_{emp}	SJ_{emp}	RJB_{emp}	RJB_{emp}	SW	SW	SW	DT_{emp}	DT_{emp}
	RJB_{emp}	RJB_{emp}	RJB_{emp}	$MRJB_{emp}$	$MRJB_{emp}$	SF	SF	SF	SW	SW
	$MRJB_{emp}$	$MRJB_{emp}$	$MRJB_{emp}$	SJ_{emp}	SJ_{emp}	AD_{emp}	AD_{emp}	AD_{emp}	AD_{emp}	AD_{emp}
	SF	SF	SF	JB_{emp}	JB_{emp}	CM_{emp}	CM_{emp}	JB_{emp}	CM_{emp}	SF
	AD_{emp}	JB_{emp}	JB_{emp}	MJB_{emp}	SF	PT_{emp}	JB_{emp}	MJB_{emp}	$L(KS)_{emp}$	CM_{emp}
	CM_{emp}	MJB_{emp}	MJB_{emp}	SF	MJB_{emp}	JB_{emp}	MJB_{emp}	CM_{emp}	PT_{emp}	$L(KS)_{emp}$
	SW	AD_{emp}	DT_{emp}	DT_{emp}	DT_{emp}	MJB_{emp}	PT_{emp}	DT_{emp}	SF	PT_{emp}
	JB_{emp}	CM_{emp}	SW	SW	SW	$L(KS)_{emp}$	$L(KS)_{emp}$	$MRJB_{emp}$	MJB_{emp}	MJB_{emp}
	MJB_{emp}	SW	AD_{emp}	AD_{emp}	AD_{emp}	$MRJB_{emp}$	DT_{emp}	RJB_{emp}	$MRJB_{emp}$	JB_{emp}
	$L(KS)_{emp}$	DT_{emp}	CM_{emp}	CM_{emp}	CM_{emp}	DT_{emp}	$MRJB_{emp}$	$L(KS)_{emp}$	SJ_{emp}	$MRJB_{emp}$
	DT_{emp}	$L(KS)_{emp}$	$L(KS)_{emp}$	$L(KS)_{emp}$	$L(KS)_{emp}$	RJB_{emp}	RJB_{emp}	PT_{emp}	JB_{emp}	RJB_{emp}
	PT_{emp}	PT_{emp}	PT_{emp}	PT_{emp}	PT_{emp}	SJ_{emp}	SJ_{emp}	SJ_{emp}	RJB_{emp}	SJ_{emp}

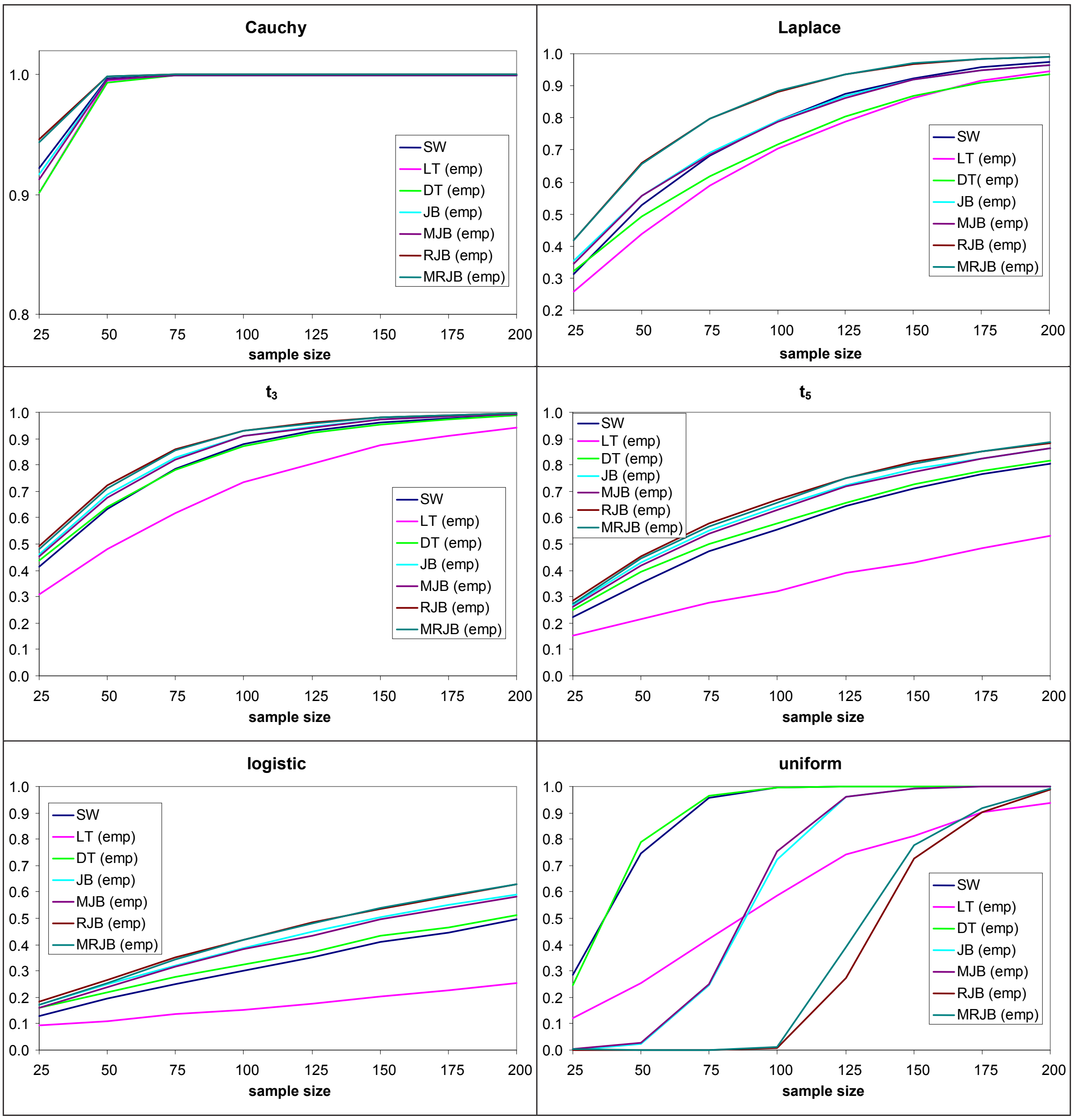
VIb: *Comparison of power of the Shapiro-Wilk test (SW), the Shapiro-Francia test (SF), the Lilliefors (Kolmogorov-Smirnov) test (L(KS)), the Anderson-Darling test (AD), the Cramer-von Mises test (CM), the Pearson chi-square test (PT), the directed SJ test (SJ), the D`Agostino test (DT), the Jarque-Bera test (JB), the modified Jarque-Bera test (MJB), the robust Jarque-Bera (RJB) and the modified robust Jarque-Bera test (MRJB) for* $\alpha = 0.05$ *and sample sizes* $n = 100$ *and 200.*

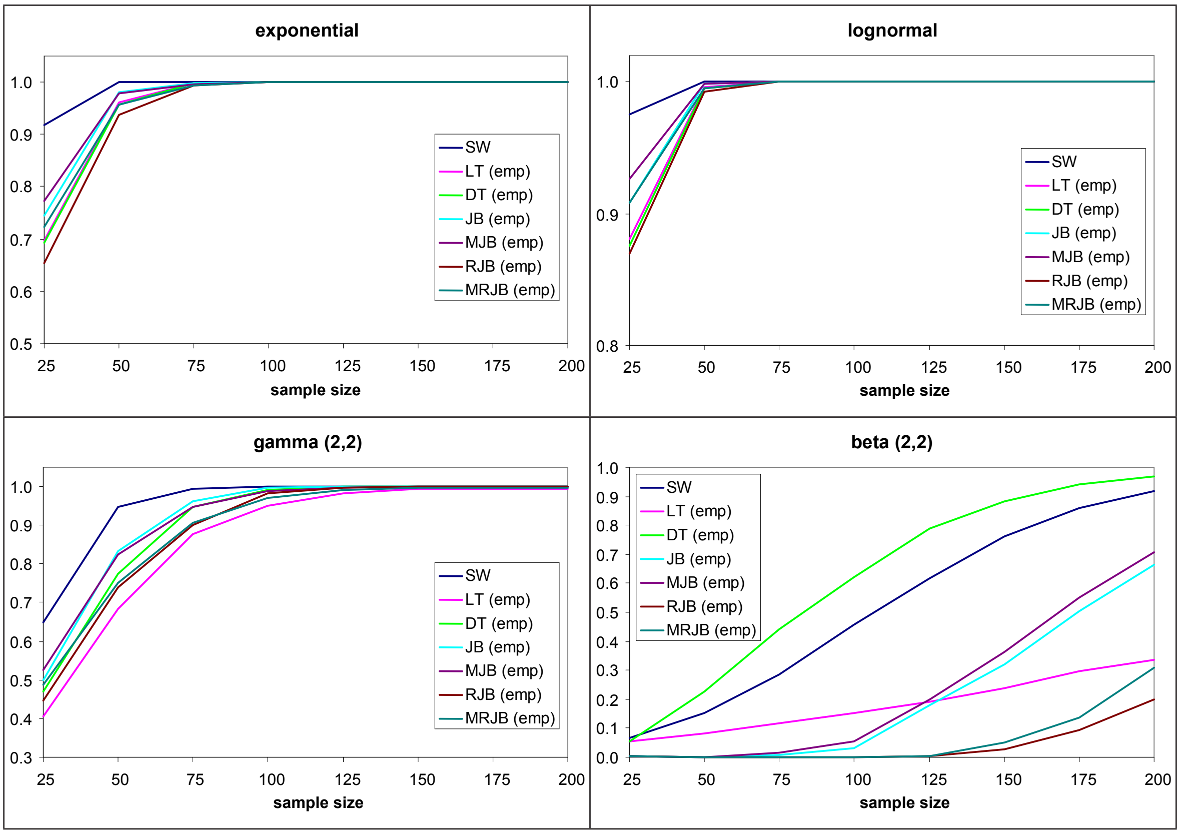
	Cauchy	Laplace	t_3	t_5	logistic	exp	log-normal	gamma (2,2)	beta (2,2)	uniform
100	SJ_{emp}	SJ_{emp}	SJ_{emp}	RJB_{emp}	$MRJB_{emp}$	SW	SW	SW	DT_{emp}	DT_{emp}
	RJB_{emp}	$MRJB_{emp}$	$MRJB_{emp}$	$MRJB_{emp}$	RJB_{emp}	SF	SF	SF	SW	SW
	$MRJB_{emp}$	RJB_{emp}	RJB_{emp}	SJ_{emp}	SJ_{emp}	AD_{emp}	AD_{emp}	AD_{emp}	AD_{emp}	SF
	SF	SF	SF	JB_{emp}	JB_{emp}	CM_{emp}	CM_{emp}	JB_{emp}	CM_{emp}	AD_{emp}
	AD_{emp}	AD_{emp}	JB_{emp}	MJB_{emp}	MJB_{emp}	PT_{emp}	JB_{emp}	DT_{emp}	SF	CM_{emp}
	CM_{emp}	CM_{emp}	MJB_{emp}	SF	SF	JB_{emp}	MJB_{emp}	CM_{emp}	$L(KS)_{emp}$	MJB_{emp}
	SW	JB_{emp}	SW	DT_{emp}	DT_{emp}	MJB_{emp}	PT_{emp}	MJB_{emp}	PT_{emp}	JB_{emp}
	JB_{emp}	SW	DT_{emp}	SW	SW	$L(KS)_{emp}$	$L(KS)_{emp}$	RJB_{emp}	MJB_{emp}	$L(KS)_{emp}$
	MJB_{emp}	MJB_{emp}	AD_{emp}	AD_{emp}	AD_{emp}	$MRJB_{emp}$	DT_{emp}	$MRJB_{emp}$	JB_{emp}	PT_{emp}
	$L(KS)_{emp}$	DT_{emp}	CM_{emp}	CM_{emp}	CM_{emp}	DT_{emp}	$MRJB_{emp}$	$L(KS)_{emp}$	$MRJB_{emp}$	$MRJB_{emp}$
	DT_{emp}	$L(KS)_{emp}$	$L(KS)_{emp}$	$L(KS)_{emp}$	$L(KS)_{emp}$	RJB_{emp}	RJB_{emp}	PT_{emp}	SJ_{emp}	RJB_{emp}
	PT_{emp}	PT_{emp}	PT_{emp}	PT_{emp}	PT_{emp}	SJ_{emp}	SJ_{emp}	SJ_{emp}	RJB_{emp}	SJ_{emp}
200	SJ_{emp}	SJ_{emp}	SJ_{emp}	$MRJB_{emp}$	SJ_{emp}	SW	SW	SW	DT_{emp}	DT_{emp}
	RJB_{emp}	$MRJB_{emp}$	RJB_{emp}	RJB_{emp}	RJB_{emp}	SF	SF	SF	SW	SW
	$MRJB_{emp}$	RJB_{emp}	$MRJB_{emp}$	SJ_{emp}	$MRJB_{emp}$	AD_{emp}	AD_{emp}	AD_{emp}	SF	SF
	SF	SF	SF	JB_{emp}	JB_{emp}	CM_{emp}	CM_{emp}	JB_{emp}	AD_{emp}	JB_{emp}
	AD_{emp}	AD_{emp}	JB_{emp}	MJB_{emp}	MJB_{emp}	PT_{emp}	JB_{emp}	DT_{emp}	MJB_{emp}	MJB_{emp}
	CM_{emp}	CM_{emp}	MJB_{emp}	SF	SF	JB_{emp}	MJB_{emp}	CM_{emp}	JB_{emp}	AD_{emp}
	SW	SW	SW	DT_{emp}	DT_{emp}	MJB_{emp}	PT_{emp}	MJB_{emp}	CM_{emp}	CM_{emp}
	JB_{emp}	JB_{emp}	DT_{emp}	SW	SW	$L(KS)_{emp}$	$L(KS)_{emp}$	RJB_{emp}	$L(KS)_{emp}$	$MRJB_{emp}$
	MJB_{emp}	MJB_{emp}	AD_{emp}	AD_{emp}	AD_{emp}	$MRJB_{emp}$	DT_{emp}	$MRJB_{emp}$	$MRJB_{emp}$	RJB_{emp}
	$L(KS)_{emp}$	$L(KS)_{emp}$	CM_{emp}	CM_{emp}	CM_{emp}	DT_{emp}	$MRJB_{emp}$	$L(KS)_{emp}$	PT_{emp}	$L(KS)_{emp}$
	DT_{emp}	DT_{emp}	$L(KS)_{emp}$	$L(KS)_{emp}$	$L(KS)_{emp}$	RJB_{emp}	RJB_{emp}	PT_{emp}	RJB_{emp}	PT_{emp}
	PT_{emp}	PT_{emp}	PT_{emp}	PT_{emp}	PT_{emp}	SJ_{emp}	SJ_{emp}	SJ_{emp}	SJ_{emp}	SJ_{emp}

Number of Monte Carlo simulations is 10,000.

Now we focus our attention to comparison of power of the JB and MJB tests vs. the RJB and MRJB tests. The JB and MJB tests show higher power than the RJB and MRJB tests for short tailed and skewed alternative distributions (e.g. exponential, lognormal, gamma, beta and uniform distributions). In contrast, the JB and MJB tests are less powerful than the RJB and MRJB tests for heavy tailed alternative distributions (e.g. Cauchy, Laplace, t_3, t_5 and logistic distributions). Finally, the RJB and MRJB tests should be useful tests of normality for heavy tailed alternatives, but the directed SJ test shows higher power than the RJB and MRJB tests and might be preferred.

Next the MJB and MRJB tests will be used for normality testing of several datasets of financial time series. Source data include logarithmic returns of monthly average prices of Prague stock exchange index PX and monthly average prices of CZK/EUR exchange rate in the period from 2000 to 2007. Tab. VII contains the basic sample characteristics and Fig. 2 presents histogram and QQ-plot of these financial time series.



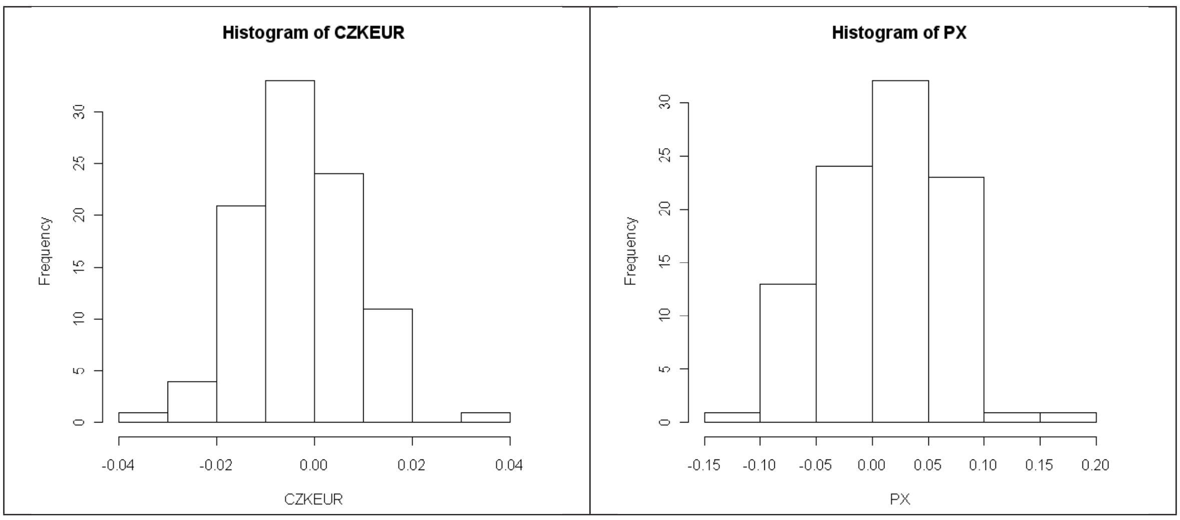


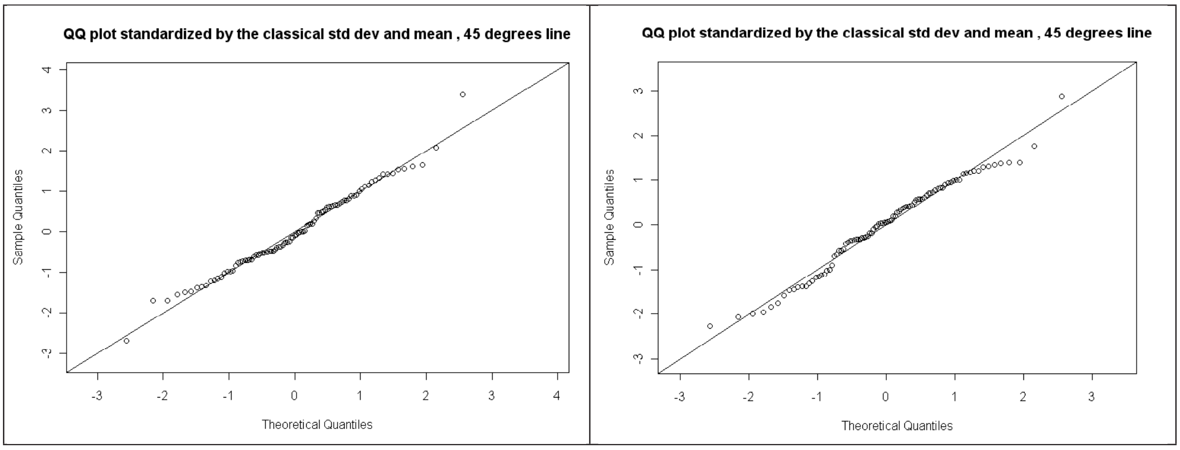
1: *Comparison of power of the Shapiro-Wilk test (SW), the Lilliefors (Kolmogorov-Smirnov) test (L(KS)), the D`Agostino test (DT), the Jarque-Bera test (JB), the modified Jarque-Bera test (MJB), the robust Jarque-Bera (RJB) and the modified robust Jarque-Bera test (MRJB) for* $\alpha = 0.05$ *and sample sizes* $n = 25, 50, 75, 100, 125, 150, 175$ *and 200.*
Number of Monte Carlo simulations is 10,000.

VII: *Basic sample characteristic of logarithmic returns of monthly average prices of Prague stock exchange index PX and monthly average prices of CZK/EUR exchange rate*

	n	$\bar{r}$	$\tilde{r}$	r_{max}	r_{min}	R_r	s_r	SK_r	$K_r - 3$
PX	95	0.0130	0.0160	–0.1028	0.1609	0.2637	0.0513	–0.1807	–0.2988
CZK/EUR	95	–0.0033	–0.0043	–0.0333	0.0346	0.0679	0.0112	0.2976	0.3282

Note: n = number of observations; $\bar{r}$ = average return; $\tilde{r}$ = median return; r_{max} = maximal value; r_{min} = minimal value; R_r = spread, s_r = standard deviation; SK_r = skewness; K_r = kurtosis.





2: *Histogram and QQ plot of logarithmic returns of monthly average prices of exchange rate CZK/EUR and monthly average prices of Prague stock exchange PX in period from 2000 to 2007.*

Tab. VIII contains values of test statistics and p-values of used normality tests – the Shapiro-Wilk test (SW), the Shapiro-Francia test (SF), the Lilliefors (Kolmogorov-Smirnov) test (L(KS)), the Anderson-Darling test (AD), the Cramer-von Mises test (CM), the Pearson chi-square test (PT), the directed SJ test (SJ), the D`Agostino test (DT), the Jarque-Bera test (JB), the modified Jarque-Bera test (MJB), the robust Jarque-Bera (RJB) and the modified robust Jarque-Bera test (MRJB).

VIII: *Test statistics and p-values of the Shapiro-Wilk test (SW), the Shapiro-Francia test (SF), the Lilliefors (Kolmogorov-Smirnov) test (L(KS)), the Anderson-Darling test (AD), the Cramer-von Mises test (CM), the Pearson chi-square test (PT), the directed SJ test (SJ), the D`Agostino test (DT), the Jarque-Bera test (JB), the modified Jarque-Bera test (MJB), the robust Jarque-Bera (RJB) and the modified robust Jarque-Bera test (MRJB) of logarithmic returns of monthly average prices of Prague stock exchange index PX and monthly average prices of CZK/EUR exchange rate.*

	CZK/EUR			**PX**		
	statistic	***p*-value**		**statistic**	***p*-value**	
		asympt.	**emp.**		**asympt.**	**emp.**
SW	0.9850	0.3542	.	0.9794	0.1392	.
SF	0.9810	0.1606	.	0.9797	0.1293	.
$L(KS)_{emp}$	0.0680	0.3452	0.3388	0.0650	0.4166	0.4114
AD_{emp}	0.4009	0.3545	0.3560	0.6293	0.0982	0.0974
CM_{emp}	0.0688	0.2870	0.2864	0.0901	0.1513	0.1483
PT_{emp}	5.5789	0.9358	0.8806	15.9790	0.1922	0.1074
SJ_{emp}	–0.2947	0.6112	0.6941	–0.1343	0.5514	0.6303
DT_{emp}	2.7394	0.2542	0.2447	0.6626	0.7180	0.7129
JB_{emp}	2.0788	0.3537	0.2655	0.7637	0.6826	0.6410
MJB_{emp}	2.5869	0.2754	0.1840	0.8316	0.6614	0.6109
RJB_{emp}	1.4093	0.4943	0.4065	0.6756	0.7133	0.6628
$MRJB_{emp}$	1.7386	0.4195	0.3431	0.7751	0.6795	0.6387

We can see that the null hypothesis of logarithmic returns of monthly average prices of Prague stock exchange index PX and CZK/EUR exchange rate are not rejected at the 5% significance level. We can also see that the RJB and MRJB tests show higher p-values than the JB and MJB tests. It may be given by outlying observations (see Fig. 2 – QQ plot). The JB test shows higher p-values than the MJB test and the RJB test shows higher p-values than the MRJB test, respectively. Logarithmic returns of monthly average prices of PX and CZK/EUR may be heavy tailed distributed, because for heavy tailed distributions (e.g. Cauchy, Laplace, t_3, t_5 and logistic distributions) the MJB test is equal or slightly less powerful than the JB test and the MRJB test is equal or slightly less powerful than the RJB test, respectively, especially in small and moderate sample sizes (see above).

DISCUSSION

In this paper we modified the classical Jarque-Bera test proposed by Jarque and Bera (1980) and the robust Jarque-Bera test proposed by Gel and Gastwirth (2007).

We can compare our simulation results with the results of the other studies. For example Gel and Gastwirth (2007) present the results of the simulation study to compare power of the JB, RJB, SJ and SW tests of normality. They found that the new RJB test performs better than the other tests in detecting heavy tailed alternatives such as *t*-distribution with three and five degrees of freedom and the logistic distribution, especially in small and moderate sample sizes ($n \leq 70$). By contrast, for the double exponential (Laplace) and Cauchy distributions, the directed SJ test performs better than the other tests, especially in small and moderate sample sizes, again. In all these cases the new RJB test is noticeably more powerful than the JB and SW tests. For the contaminated normal distribution, the JB and RJB tests show similar performance results. On the other hand, the SW and SJ tests are less powerful than the JB and RJB tests. For the exponential distribution the best performance is shown by the SW test followed by the JB and RJB tests. The SJ test does not perform well for exponential distribution. Consequently, JB and SW tests perform better than the SJ and RJB tests in detecting exponential distribution. Gel and Gastwirth (2007) also conclude that the loss in power in detecting the exponential distribution is less than the gain in power for other deviations from normality.

Our results of the JB, MJB, RJB, MRJB, SJ and SW tests correspond with these results. The JB and MJB tests show higher power than the RJB and MRJB tests for short tailed and skewed alternative distributions (e.g. exponential, lognormal, gamma, beta and uniform distributions) and, in contrast, the JB and MJB tests are less powerful than the RJB and MRJB tests for heavy tailed alternative distributions (e.g. Cauchy, Laplace, t_3, t_5 and logistic distributions).

SUMMARY

This paper deals with comparison of power of modified Jarque-Bera normality tests and selected tests of normality. We modify the classical Jarque-Bera test (JB) and the robust Jarque-Bera test (RJB) – we use the median as an estimator instead of the mean in the classical Jarque-Bera test statistic and in the robust Jarque-Bera test statistic. This leads to the modified Jarque-Bera test (MJB) and the modified robust Jarque-Bera test (MRJB).

On the basis of Monte Carlo simulations we can concluded that the MJB and MRJB tests outperform the classical JB and RJB tests for light tailed skewed alternative distributions (e.g. exponential, lognormal and gamma distributions), especially in small sample sizes. For heavy tailed symmetric distributions (e.g. Cauchy, Laplace, t_3, t_5 and logistic distributions) the MJB test is equal or slightly less powerful than the JB test and the MRJB test is equal or slightly less powerful than the RJB test, respectively, especially in small and moderate sample sizes.

The JB and MJB tests show higher power than the RJB and MRJB tests for short tailed and skewed alternative distributions (e.g. exponential, lognormal, gamma, beta and uniform distributions) and, in contrast, the JB and MJB tests are less powerful than the RJB and MRJB tests for heavy tailed alternative distributions (e.g. Cauchy, Laplace, t_3, t_5 and logistic distributions).

Finally, the RJB and MRJB tests should be useful tests of normality for heavy tailed alternative distributions, but the directed SJ test shows higher power than the RJB and MRJB tests and might be preferred for heavy tailed symmetric alternatives. Similarly, the JB and MJB tests should be useful normality tests for light tailed alternative distributions, but some tests, namely the Shapiro-Wilk test, the Shapiro-Francia test, the Anderson-Darling test and the Cramer-von Mises test, are more powerful than the JB and MJB tests and might be preferred for light tailed alternatives. For very short tailed alternative distributions as beta distribution and uniform distribution is the best one the D`Agostino test.

The paper also deals with applications of selected normality tests on datasets of selected of financial time series (e.g. logarithmic returns derived from average monthly prices of Praque stock exchange index PX and CZK/EUR exchange rate). Results show that hypothesis of normality was not rejected for these financial time series, for the nominal level $\alpha = 0.05$. The JB test shows higher *p*-values than the MJB test and the RJB test shows higher *p*-values than the MRJB test, respectively.

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SOUHRN

Srovnání síly modifikovaných Jarque-Bera testů a vybraných testů normality

Článek se zabývá srovnáním síly modifikovaných Jarque-Bera testů a vybraných testů normality. Jarque-Bera test (JB) normality a robustní Jarque-Bera test (RJB) normality byly modifikovány tak, že aritmetický průměr, jako výběrový odhad střední hodnoty, byl nahrazen výběrovým mediánem.

Touto úpravou tak byl získán modifikovaný Jarque-Bera test (MJB) a modifikovaný robustní Jarque-Bera test (MRJB).
Na základě Monte Carlo simulací bylo zjištěno, že MJB a MRJB testy vykazují lepší výsledky než klasické JB a RJB testy pro pravděpodobnostní zešikmená rozložení s lehkými konci (tj. pro exponenciální, lognormální či gamma rozložení) a to především pro malé rozsahy souborů. V případě symetrických pravděpodobnostních rozložení s těžkými konci (tj. Cauchyho, Laplaceovo, t_3, t_5 či logistické rozložení) vykazují MJB a MRJB testy stejnou nebo mírně nižší sílu než JB a RJB testy a to především pro malé a středně velké rozsahy souborů.
Dále JB a MJB testy vykazují vyšší sílu testu než RJB a MRJB test pro zešikmená pravděpodobnostní rozložení a rozložení s krátkými konci (tj. exponenciální, lognormální, gamma, beta či rovnoměrné rozložení). Naopak JB a MJB testy vykazují nižší sílu než RJB a MRJB testy pro alternativní rozložení s těžkými konci (tj. Cauchyho, Laplaceovo, t_3, t_5 či logistické rozložení).
Celkově lze tedy RJB a MRJB testy považovat za vhodné testy pro testování normality pro alternativní rozložení s těžkými konci. Pro tato rozložení však lze s úspěchem využít i SJ test, jelikož vykazuje ještě vyšší sílu než RJB a MRJB testy. Obdobně lze za vhodné testy pro testování normality pro alternativní rozložení s lehkými konci považovat JB a MJB testy. Avšak pro tyto alternativy je vhodnější využít Shapiro-Wilkova, Shapiro-Francia, Anderson-Darlingova či Cramer-von Misesova testu, které vykazují pro tyto alternativy vyšší sílu než JB a MJB testy. Pro alternativní rozložení s velmi krátkými konci jako např. beta rozložení či rovnoměrné rozložení je nejvhodnějším testem D`Agostinův test.
Článek také obsahuje aplikaci těchto testů normality pro testovaní vybraných finančních časových řad – logaritmických cenových změn odvozených z průměrných měsíčních cen burzovního indexu PX a devizového kurzu CZK/EUR. Výsledky testů ukazují, že hypotéza o normalitě rozložení uvedených souborů nebyla na 5% hladině významnosti zamítnuta a že JB a RJB testy vykazují vyšší *p*-hodnoty než MJB a MRJB testy.

testy normality, Monte Carlo simulace, komparace síly testů, finanční časové řady, logaritmické cenové změny

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