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SEARCHING FOR LONG MEMORY EFFECTS IN TIME SERIES OF CENTRAL EUROPE STOCK MARKET INDICES

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Abstract

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This article deals with one of the important parts of applying chaos theory to financial and capital markets – namely searching for long memory effects in time series of financial instruments. Source data are daily closing prices of Central Europe stock market indices – Bratislava stock index (SAX), Budapest stock index (BUX), Prague stock index (PX) and Vienna stock index (ATX) – in the period from January 1998 to September 2007. For analysed data R/S analysis is used to calculate the Hurst exponent. On the basis of the Hurst exponent is characterized formation and behaviour of analysed financial time series. Computed Hurst exponent is also statistical compared with his expected value signalling independent process. It is also operated with 5-day returns (i.e. weekly returns) for the purposes of comparison and identification nonperiodic cycles.

Hurst exponent, R/S analysis, V-statistic, logarithmic ratio, stock market index

The financial market is a part of economic system. There are four parts of financial market: bond market, stock market, commodity market and exchange market. The basic information from financial markets is the price. The prices are monitored in certain time frequency and create time series.

In comparison with other economic time series, the financial time series have some characteristic properties – for example high frequency, leptokurtosis probability function with fat tails and a higher peak at the mean than the normal distribution, volatility clustering etc. One of the key properties of financial time series is long memory effect. Theoretically, what happens today impacts the future forever. In other words, current data is correlated with all future daily changes. In comparison short memory systems are characterized by using last i series values for making analysis. Long memory systems on the other hand are characterized by their ability to remember events in the long history of time series data and their ability to make decisions on the basic of such memories.

This paper is divided into four parts. In part 1, the Hurst process and R/S analysis is described. Part 2 deals with the gist of this paper – results of R/S analysis, Hurst exponent estimation and statistical comparison with his expected value signalling independent process. In part 3, result are discussed and confronted with previously published papers and results. The final part of this paper is the summary.

MATERIAL AND METHODS

Source data are daily closing prices of official stock market indices of Central Europe Stock Exchanges – Bratislava Stock Exchange (SAX), Budapest Stock Exchange (BUX), Prague Stock Exchange (PX) and Vienna Stock Exchange (ATX) – in the period from January 1998 to September 2007, or approximately 10 years of daily data, contains about 2400 data points.

According to the original theory by British hydrologist H. E. Hurst there are defined three processes (Hurst, 1951):

a) $H = 0.5$ would imply an indepedent process. This process would include the normal distribution and non-Gaussian independent processes like the Student t-distribution or gamma distribution etc.

b) *0.5 < H < 1* implies a persistent time series with long memory effects. Theoretically, what happens today impacts the future forever.
c) *0 < H < 0.5* implies an antipersistence time series. An antipersistent system covers less distance than a random one.

We can use R/S analysis for calculated empirical Hurst exponent. R/S analysis is a simple process with these sequential steps (Peters, 1994):

1. Begin with a time series of prices P_t registered in time points $t = 1, 2, \ldots, T$. Convert this into a time series of logarithmic ratios (returns) of length $N = T - 1$:

$$x_i = \log \frac{P_{i+1}}{P_i} \qquad i = 1, 2, \ldots, T - 1.$$

2. Divide this time period into m contiguous subperiods of length n, such that $m \cdot n = N$. For each subperiod of length n, the average value e_j is defined as:

$$e_j = \frac{1}{n} \sum_{i=1}^{n} x_{ij} \qquad j = 1, 2, \ldots, m.$$

3. The time series of accumulated departures z_{kj} from the mean value for each subperiod is defined as:

$$z_{kj} = \sum_{i=1}^{k} (x_{ij} - e_j) \qquad k = 1, 2, \ldots, n.$$

4. The range is defined as the maximum minus the minimum value of z_{kj} within each subperiod:

$$R_j = \max(z_{kj}) - \min(z_{kj}) \qquad .$$

5. The sample standard deviation calculated for each subperiod:

$$S_j = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_{ij} - e_j)^2}.$$

6. Each range is now normalized by dividing by the standard deviation corresponding to it.

$$(R/S)_j = \frac{R_j}{S_j}.$$

From step 2 we had m contiguous subperiods of length n. Therefore, the average R/S value for length n is defined as:

$$(R/S)_n = \frac{1}{m} \sum_{j=1}^{m} \frac{R_j}{S_j}.$$

7. The length n is increased to the next higher value, and $(T - 1)/n$ is an integer value. We use values of n that include the beginning and ending points of the time series, and steps 2 trough 6 are repeated until $n = (T - 1)/2$.
8. Hurst (1951) found that the distance that a random particle covers increases with the square root of time used to measure it, or:

$$(R/S)_n = c \cdot n^H,$$

where c is constant and H is the Hurst exponent. The Hurst exponent can be approximated by plotting the $\log(R/S)_n$ versus the log n and solving for the slope through an ordinary least squares regression. In particular, we are working from the following equation:

$$\log(R/S)_n = \log c + H \log n.$$

In general, run the regression over values of $n \geq 10$, because small values of n produce unstable estimates of R/S statistic when sample sizes are small.

Calculated value of the Hurst exponent (H) is now compared with expected value of the Hurst exponent (E(H)) derivate from expected R/S values ($E(R/S)_n$). Peters (1994) was able to derive an empirical correction $(n - 0.5)/n$ to earlier formula of $E(R/S)_n$ developed by Anis and Lloyd (1976) and $E(R/S)_n$ is now:
for $n \leq 300$, and

$$E(R/S)_n = \frac{n - 0.5}{n} \cdot [\Gamma\{0.5 \cdot (n - 1)\}/(\sqrt{\pi} \cdot \Gamma(0.5 \cdot n))] \cdot \sum_{r=1}^{n-1} \sqrt{\frac{(n - r)}{r}}$$

$$E(R/S)_n = \frac{n - 0.5}{n} \cdot \left(n \cdot \frac{\pi}{2}\right)^{-0.5} \cdot \sum_{r=1}^{n-1} \sqrt{\frac{(n - r)}{r}}$$

for $n > 300$.

Because the R/S values are normally distributed random variables, we would expect that the values of H would also be normally distributed with the expected variance of the Hurst exponent:

$$Var(H)_n = 1/T,$$

where T is the total number of observations in the sample.

In this paper, we will analyze time series of logarithmic returns and AR(1) residuals of logarithmic returns derived from selected stock market indices. The AR(1) residuals are used to minimize linear dependency or to reduce the result to insignificance level. First we begin with a series of logarithmic returns:

$$x_i = \log \frac{P_{i+1}}{P_i} \qquad i = 1, 2, \ldots, T - 1.$$

Then we regress x_i as the dependent variable against x_{i-1} as the independent variable, and obtain the intercept b_0, and the slope b_1. The AR(1) residuals of x_i are defined as:

$$y_i = x_i - (b_0 - b_1 \cdot x_{i-1}) \qquad i = 2, 3, \ldots, T.$$

For this cases (time series of logarithmic returns and AR(1) residuals of logarithmic returns) R/S analysis is performed, starting with step 2 of the step-by-step guide outlined above.

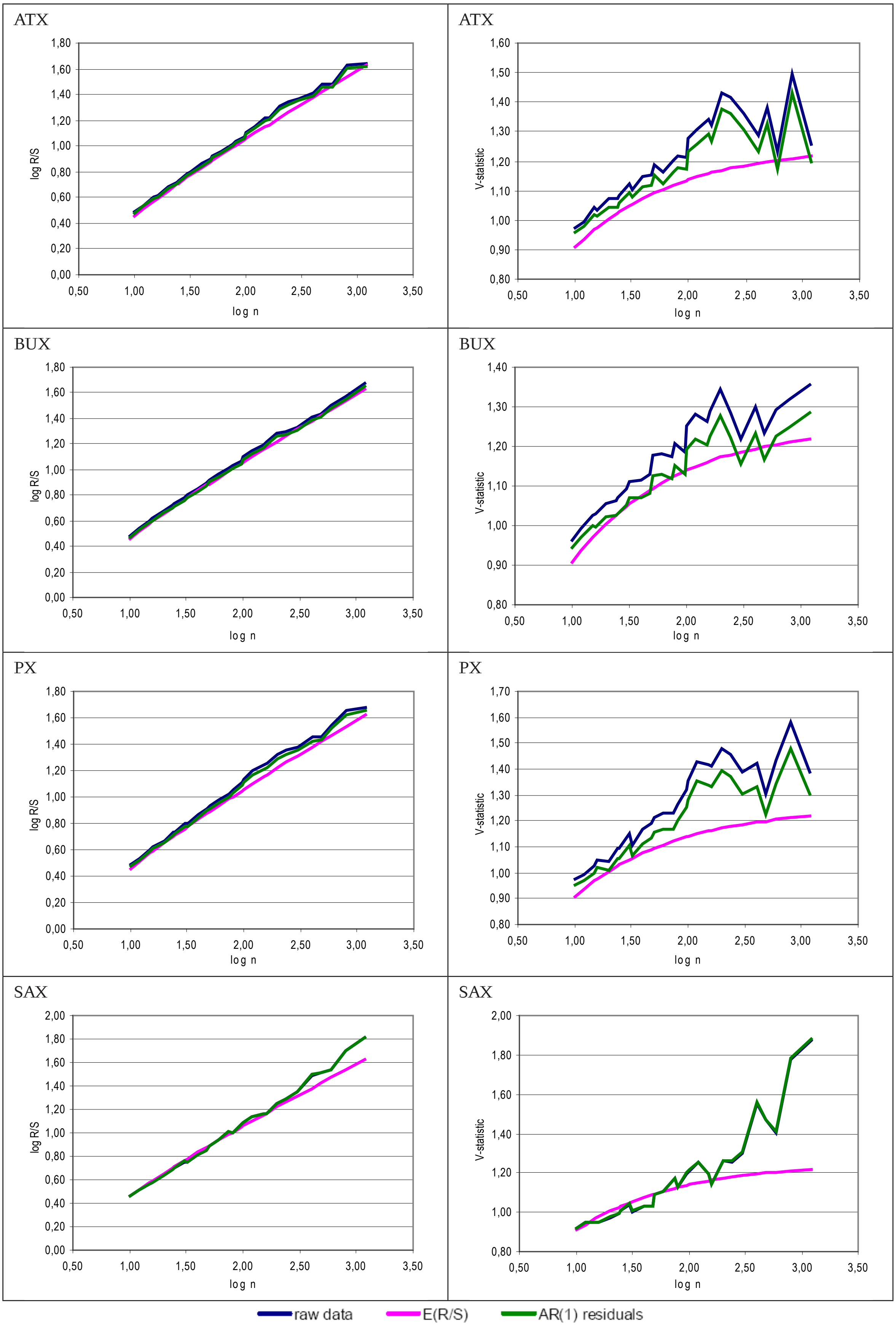
RESULTS

Analysis of daily logarithmic returns

Tab. I shows R/S_n and $E(R/S)_n$ values of daily logarithmic returns for ATX, BUX, PX and SAX series. Fig. 1 shows the $\log(R/S)_n$ plot for daily logarithmic returns for ATX, BUX, PX and SAX series. Also plotted is $E(R/S)_n$ as comparison against the null hypothesis that the system is an independent process (see above). Fig. 1 also shows the V-statistic and plot it versus $\log(n)$. V-statistic is the ratio of R/S_n to $\sqrt{n}$.

I: *R/S analysis of daily logarithmic returns*

n	log n	log(R/S)				
		ATX	BUX	PX	SAX	E(R/S)
10	1.0000	0.4896	0.4844	0.4902	0.4641	0.4582
12	1.0792	0.5382	0.5370	0.5371	0.5153	0.5113
15	1.1761	0.6062	0.5997	0.5990	0.5659	0.5742
16	1.2041	0.6175	0.6144	0.6231	0.5803	0.5921
20	1.3010	0.6810	0.6736	0.6698	0.6391	0.6528
24	1.3802	0.7210	0.7168	0.7283	0.6867	0.7013
25	1.3979	0.7347	0.7282	0.7387	0.7023	0.7120
30	1.4771	0.7907	0.7766	0.8005	0.7562	0.7594
32	1.5051	0.7969	0.7987	0.7969	0.7541	0.7760
40	1.6021	0.8612	0.8480	0.8677	0.8148	0.8327
48	1.6812	0.9024	0.8936	0.9155	0.8527	0.8783
50	1.6990	0.9254	0.9207	0.9331	0.8864	0.8885
60	1.7782	0.9547	0.9620	0.9788	0.9323	0.9334
75	1.8751	1.0188	1.0071	1.0274	1.0060	0.9877
80	1.9031	1.0384	1.0329	1.0536	1.0030	1.0033
96	1.9823	1.0758	1.0647	1.1119	1.0698	1.0470
100	2.0000	1.1054	1.0981	1.1321	1.0833	1.0568
120	2.0792	1.1547	1.1473	1.1947	1.1372	1.1000
150	2.1761	1.2152	1.1895	1.2393	1.1645	1.1525
160	2.2041	1.2220	1.2123	1.2521	1.1594	1.1676
200	2.3010	1.3061	1.2792	1.3202	1.2511	1.2196
240	2.3802	1.3413	1.2989	1.3533	1.2897	1.2617
300	2.4771	1.3719	1.3249	1.3809	1.3526	1.3130
400	2.6021	1.4099	1.4145	1.4537	1.4930	1.3779
480	2.6812	1.4810	1.4318	1.4562	1.5076	1.4194
600	2.7782	1.4783	1.5010	1.5457	1.5373	1.4700
800	2.9031	1.6257	1.5713	1.6506	1.7019	1.5348
1200	3.0792	1.6379	1.6721	1.6828	1.8121	1.6257



1: *R/S analysis and V-statistic – daily returns*

Tab. II shows regression results for daily logarithmic returns of analysed time series. The regression yielded $H = 0.57$ for ATX`s daily logarithmic returns, $H = 0.57$ for BUX`s daily logarithmic returns, $H = 0.59$ for PX`s daily logarithmic returns and $H = 0.63$ for SAX`s daily logarithmic returns, respectively. Expected value of the Hurst exponent ($E(H)$) is 0.56. The variance of $E(H)$ is 1/2400 and the standard deviation of $E(H)$ is 0.02. Therefore, H value is 0.79 (ATX), 0.46 (BUX), 1.73 (PX) and 3.72 (SAX) standard deviations, respectively, above its expected value – H value is insignificant, at the 95 percent level, for ATX, BUX and PX series, and significant, at the 99 percent level, for SAX series. Therefore, the SAX time series follows persistent process with long memory effect.

II: *Regression analysis – daily logarithmic returns*

	$10 \le n \le 1200$				
	ATX	BUX	PX	SAX	E(R/S)
Number of observation	28	28	28	28	28
R2	0.9974	0.9988	0.9975	0.9980	0.9993
Constant	–0.0618	–0.0596	–0.0832	–0.1904	–0.0701
standard error	0.0163	0.0108	0.0164	0.0157	0.0079
t-statistic	–3.7830	–5.5388	–5.0831	–12.1258	–8.8268
p-value	0.0008	0.0000	0.0000	0.0000	0.0000
Hurst exponent	**0.5741**	**0.5673**	**0.5933**	**0.6341**	**0.5581**
standard error	0.0082	0.0054	0.0082	0.0078	0.0040
significant H vs $E(H)$	0.7864	0.4548	1.7273	3.7227	x

Analysis of AR(1) residuals of daily logarithmic returns

Tab. III shows R/S_n and $E(R/S)_n$ values of AR(1) residuals of daily logarithmic returns for ATX, BUX, PX and SAX series. Fig. 1 also shows the $\log(R/S)_n$ plot for these series (see above).

III: *R/S analysis of AR(1) residuals of daily logarithmic returns*

n	log n	log(R/S) - AR(1) residuals				
		ATX	BUX	PX	SAX	E(R/S)
10	1.0000	0.4822	0.4748	0.4798	0.4645	0.4582
12	1.0792	0.5300	0.5260	0.5270	0.5158	0.5113
15	1.1761	0.5974	0.5879	0.5860	0.5664	0.5742
16	1.2041	0.6080	0.6012	0.6101	0.5809	0.5921
20	1.3010	0.6702	0.6594	0.6546	0.6397	0.6528
24	1.3802	0.7097	0.7012	0.7129	0.6874	0.7013
25	1.3979	0.7234	0.7123	0.7223	0.7029	0.7120
30	1.4771	0.7787	0.7606	0.7826	0.7569	0.7594
32	1.5051	0.7850	0.7815	0.7797	0.7548	0.7760
40	1.6021	0.8480	0.8310	0.8472	0.8156	0.8327
48	1.6812	0.8891	0.8752	0.8952	0.8535	0.8783
50	1.6990	0.9113	0.9010	0.9125	0.8873	0.8885
60	1.7782	0.9406	0.9425	0.9568	0.9332	0.9334
75	1.8751	1.0038	0.9868	1.0052	1.0069	0.9877
80	1.9031	1.0236	1.0126	1.0311	1.0039	1.0033
96	1.9823	1.0607	1.0442	1.0881	1.0708	1.0470
100	2.0000	1.0896	1.0769	1.1080	1.0842	1.0568

n	log n	log(R/S) - AR(1) residuals				
		ATX	BUX	PX	SAX	E(R/S)
120	2.0792	1.1381	1.1252	1.1707	1.1382	1.1000
150	2.1761	1.1983	1.1679	1.2142	1.1655	1.1525
160	2.2041	1.2048	1.1904	1.2268	1.1604	1.1676
200	2.3010	1.2884	1.2568	1.2944	1.2522	1.2196
240	2.3802	1.3235	1.2767	1.3269	1.2908	1.2617
300	2.4771	1.3537	1.3015	1.3542	1.3537	1.3130
400	2.6021	1.3917	1.3915	1.4263	1.4941	1.3779
480	2.6812	1.4629	1.4081	1.4289	1.5087	1.4194
600	2.7782	1.4598	1.4771	1.5175	1.5384	1.4700
800	2.9031	1.6069	1.5477	1.6222	1.7030	1.5348
1200	3.0792	1.6196	1.6484	1.6549	1.8133	1.6257

Tab. IV shows regression results for AR(1) residuals of daily logarithmic returns of analysed time series. The regression yielded $H = 0.57$ for ATX`s AR(1) residuals, $H = 0.56$ for BUX`s AR(1) residuals, $H = 0.58$ for PX`s AR(1) residual and $H = 0.63$ for SAX`s AR(1) residuals, respectively. Expected value of Hurst exponent ($E(H)$) is 0.56 and the standard deviation of $E(H)$ is 0.02, again. Therefore, H value is 0.51 (ATX), 0.12 (BUX), 1.28 (PX) and 3.74 (SAX) standard deviations, respectively, above its expected value – H value is insignificant, at the 95 percent level, for ATX, BUX and PX series, and significant, at the 99 percent level, for SAX series. Therefore, SAX time series contains "true Hurst process", i.e. persistent process with long memory effect.

IV: *Regression analysis – AR(1) residuals of daily logarithmic returns*

	$10 \leq n \leq 1200$				
	ATX	BUX	PX	SAX	E(R/S)
Number of observation	28	28	28	28	28
R2	0.9975	0.9990	0.9977	0.9980	0.9993
Constant	–0.0652	–0.0656	–0.0868	–0.1903	–0.0701
standard error	0.0158	0.0101	0.0156	0.0157	0.0079
t-statistic	–4.1179	–6.5253	–5.5778	–12.1388	–8.8268
p-value	0.0003	0.0000	0.0000	0.0000	0.0000
Hurst exponent	**0.5684**	**0.5606**	**0.5841**	**0.6344**	**0.5581**
standard error	0.0079	0.0050	0.0078	0.0078	0.0040
significant H vs $E(H)$	0.5050	0.1219	1.2746	3.7407	x

Subperiod analysis and searching for nonperiodic cycles

A break in the R/S graph and V-statistic graph appears to be at 200 observations (log(200) ≈ 2.30) for ATX and BUX series and at 120 observations (log(120) ≈ 2.08) for PX series (see Fig. 1). This breaks in the R/S graph may signal nonperiodic component in analysed time series. Therefore, ATX and BUX series may contain the nonperiodic cycles with frequency of approximately 200 trade days (about 10 months) and PX series may contain the nonperiodic cycles with frequency of approximately 120 trade days (about 6 months). By contrast SAX series contains long memory effect with infinity frequency of nonperiodic cycle.

Tab. V shows regression results for ATX series. A break in the R/S graph and V-statistic graph appears to be at 200 observations. Therefore, we will run regression to estimate H for $10 \leq n \leq 200$ and for $200 < n \leq 1200$. In subperiod $10 \leq n \leq 200$ the regression yielded $H = 0.61$ for daily logarithmic returns and $H = 0.60$ for AR(1) residuals of daily logarithmic returns. Expected value of the Hurst exponent is 0.58 with standard deviation of $E(H)$ 0.02, again. Therefore, H value is only 1.49 (for logarithmic returns) and 1.11 (for AR(1) residuals) standard deviations above its expected value, and is insignificant at the 95 percent level.

In subperiod 200 < *n* ≤ 1200 the regression yielded *H* = 0.47 for daily logarithmic returns and AR(1) residuals of daily logarithmic returns too (see Tab. V). *E*(*H*) is 0.52. Therefore, *H* value is 2.66 (for logarithmic returns) and 2.71 (for AR(1) residuals) standard deviations below its expected value. This is a significant result at the 99 percent level. There is an antipersistent process.

Consequently, ATX series follows an independent process for 10 ≤ *n* ≤ 200, and an antipersistent process for 200 < *n* ≤ 1200.

V: *Regression analysis – ATX daily returns*

	10 ≤ *n* ≤ 200			200 < *n* ≤ 1200		
	r_t	AR(1) residuals	E(R/S)	r_t	AR(1) residuals	E(R/S)
Number of observation	21	21	21	7	7	7
R^2	0.9995	0.9995	0.9995	0.9677	0.9678	1.0000
Constant	–0.1204	–0.1206	–0.1059	0.2189	0.2032	0.0232
standard error	0.0077	0.0077	0.0072	0.1471	0.1466	0.0022
t-statistic	–15.6399	–15.7471	–14.7925	1.4878	1.3862	10.6607
p-value	0.0000	0.0000	0.0000	0.1970	0.2243	0.0001
Hurst exponent	**0.6116**	**0.6038**	**0.5812**	**0.4663**	**0.4654**	**0.5206**
standard error	0.0045	0.0045	0.0042	0.0543	0.0541	0.0008
significant *H* vs *E*(*H*)	1.4869	1.1064	x	–2.6596	–2.7070	x

Tab. VI shows regression results for BUX series. A break in the R/S graph and V-statistic graph appears to be at 200 observations, again. In subperiod 10 ≤ *n* ≤ 200 the regression yielded *H* = 0.60 for daily logarithmic returns and *H* = 0.59 for AR(1) residuals of daily logarithmic returns. *E*(*H*) is 0.58, again. Therefore, *H* value is only 0.98 (for logarithmic returns) and 0.50 (for AR(1) residuals) standard deviations away from its expected value, and is insignificant, at the 95 percent level.

In subperiod 200 < *n* ≤ 1200 the regression yielded *H* = 0.55 for daily logarithmic returns and AR(1) residuals of daily logarithmic returns too (see Tab. VI). *E*(*H*) is 0.52. Therefore, *H* value is only 1.25 (for logarithmic returns) and 1.17 (for AR(1) residuals) standard deviations away from its expected value, and is insignificant at the 95 percent level, again. Regression results would imply an indepedent process.

VI: *Regression analysis – BUX daily returns*

	10 ≤ *n* ≤ 200			200 < *n* ≤ 1200		
	r_t	AR(1) residuals	E(R/S)	r_t	AR(1) residuals	E(R/S)
Number of observation	21	21	21	7	7	7
R^2	0.9996	0.9996	0.9995	0.9964	0.9961	1.0000
Constant	–0.1114	–0.1128	–0.1059	–0.0158	–0.0344	0.0232
standard error	0.0064	0.0062	0.0072	0.0567	0.0585	0.0022
t-statistic	–17.3387	–18.0718	–14.7925	–0.2784	–0.5887	10.6607
p-value	0.0000	0.0000	0.0000	0.7918	0.5816	0.0001
Hurst exponent	**0.6012**	**0.5915**	**0.5812**	**0.5463**	**0.5445**	**0.5206**
standard error	0.0038	0.0037	0.0042	0.0209	0.0216	0.0008
significant *H* vs *E*(*H*)	0.9796	0.5027	x	1.2562	1.1706	x

Tab. VII shows regression results for PX series. A break in the R/S graph and V-statistic graph appears to be at 120 observations. In subperiod 10 ≤ *n* ≤ 200 the regression yielded *H* = 0.64 for daily logarithmic returns and *H* = 0.63 for AR(1) residuals of daily logarithmic returns. *E*(*H*) is 0.59 this time. Therefore,

H value for logarithmic returns is 2.34 standard deviations above its expected value, and is significant result at the 95 percent level. For AR(1) residuals is H value only 1.67 standard deviations above its expected value, and is insignificant at the 95 percent level. Therefore, the time series of stock index PX does not contain long memory effect.

In subperiod $200 < n \leq 1200$ the regression yielded $H = 0.50$ around for both series, $E(H)$ is 0.52 this time (see Tab. VII). Therefore, H value is 0.97 (for logarithmic returns) and 1.15 (for AR(1) residuals) standard deviations below its expected value, and is insignificant at the 95 percent level.

VII: *Regression analysis – PX daily returns*

	$10 \leq n \leq 120$			$120 < n \leq 1200$		
	r_t	AR(1) residuals	E(R/S)	r_t	AR(1) residuals	E(R/S)
Number of observation	18	18	18	10	10	10
R^2	0.9992	0.9991	0.9996	0.9904	0.9904	1.0000
Constant	–0.1531	–0.1500	–0.1190	0.1437	0.1262	0.0136
standard error	0.0102	0.0104	0.0070	0.0639	0.0636	0.0030
t-statistic	–14.9967	–14.3635	–17.0703	2.2482	1.9838	4.5226
p-value	0.0000	0.0000	0.0000	0.0547	0.0826	0.0019
Hurst exponent	**0.6385**	**0.6248**	**0.5906**	**0.5042**	**0.5005**	**0.5240**
standard error	0.0064	0.0065	0.0044	0.0248	0.0247	0.0012
significant H vs $E(H)$	2.3479	1.6767	x	–0.9734	–1.1528	x

5-day returns

Tab. VIII and IX show R/S_n and $E(R/S)_n$ values of 5-day logarithmic returns and AR(1) residuals of 5-day logarithmic returns for ATX, BUX, PX and SAX series. Fig. 2 shows the $\log(R/S)_n$ plot and V-statistic plot for this series.

VIII: *R/S analysis of 5-day logarithmic returns*

n	log n	log(R/S)				
		ATX	BUX	PX	SAX	E(R/S)
10	1.0000	0.5011	0.4732	0.4925	0.4730	0.4582
12	1.0792	0.5209	0.5321	0.5427	0.5282	0.5113
15	1.1761	0.6113	0.5817	0.6021	0.6058	0.5742
16	1.2041	0.6204	0.6184	0.6332	0.6220	0.5921
20	1.3010	0.6896	0.6781	0.7075	0.7039	0.6528
24	1.3802	0.7523	0.7341	0.7572	0.7539	0.7013
30	1.4771	0.8022	0.7802	0.8055	0.7900	0.7594
32	1.5051	0.8570	0.8114	0.8320	0.8032	0.7760
40	1.6021	0.9035	0.8858	0.9040	0.8820	0.8327
48	1.6812	0.9422	0.8915	0.9348	0.9115	0.8783
60	1.7782	0.9685	0.9304	0.9498	0.9693	0.9334
80	1.9031	1.0126	1.0093	1.0483	1.1169	1.0033
96	1.9823	1.0844	1.0259	1.0336	1.1322	1.0470
120	2.0792	1.0903	1.1065	1.1288	1.1772	1.1000
160	2.2041	1.2305	1.1606	1.2400	1.3291	1.1676
240	2.3802	1.2513	1.2651	1.2733	1.4382	1.2617

Tab. X and XI show regression results for 5-day logarithmic returns and AR (1) residuals of logarithmic returns of ATX, BUX, PX and SAX series. These results of logarithmic returns full correspond with results of daily series mentioned above – *H* value is insignificant, at the 95 percent level, for ATX, BUX and PX series, and significant, at the 99 percent level, for SAX series, again.

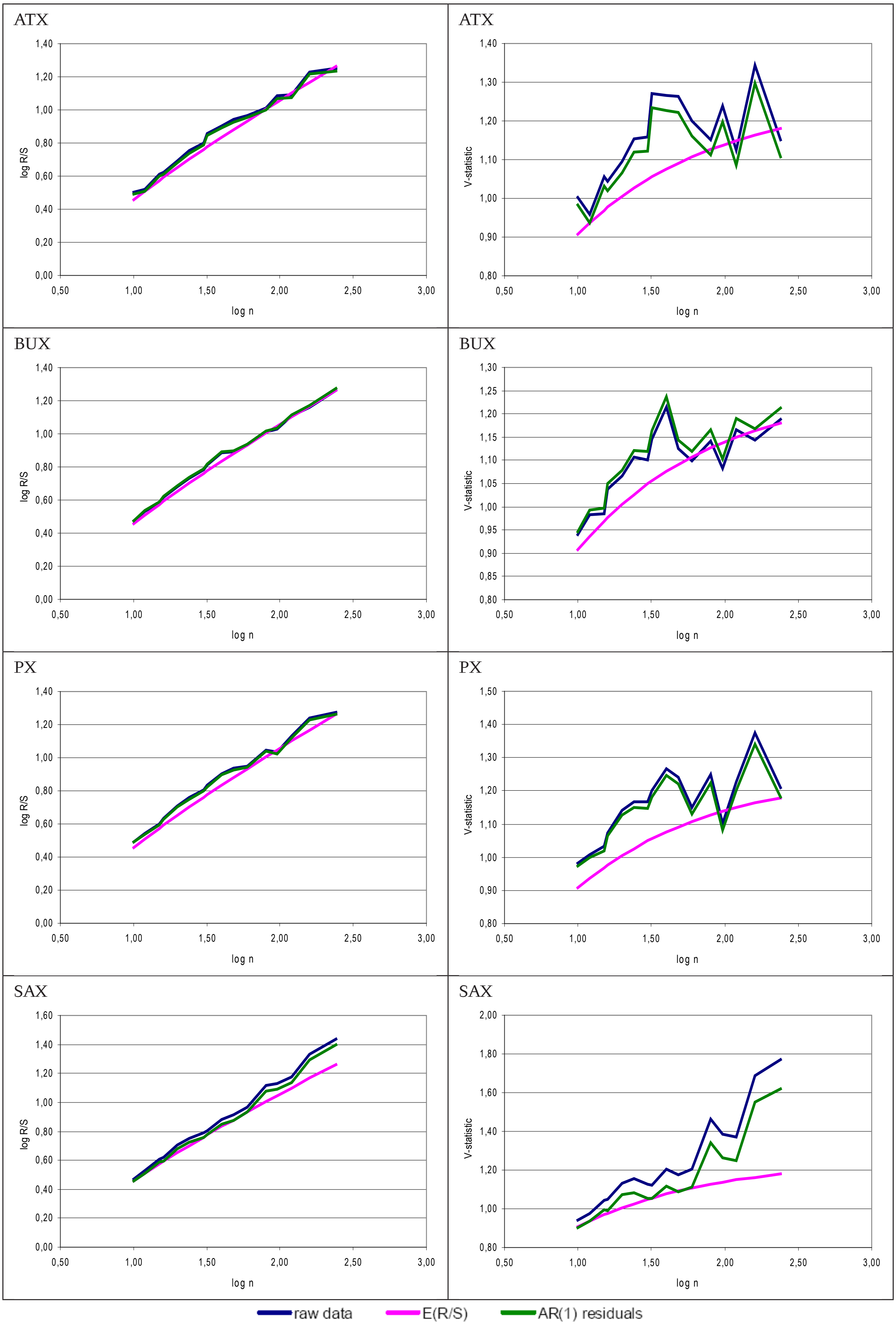
On the other hand all differences between *H* and *E*(*H*) of AR(1) residuals of logarithmic returns are insignificant, at the 95 percent level – AR(1) residuals of SAX`s logarithmic returns including (see Tab. XI). Regression results would imply an indepedent process.

IX: *R/S analysis of AR(1) residuals of 5-day logarithmic returns*

n	log n	log(R/S) – AR(1) residuals				
		ATX	BUX	PX	SAX	E(R/S)
10	1.0000	0.4927	0.4758	0.4887	0.4554	0.4582
12	1.0792	0.5109	0.5365	0.5388	0.5121	0.5113
15	1.1761	0.6012	0.5869	0.5965	0.5852	0.5742
16	1.2041	0.6101	0.6233	0.6287	0.5968	0.5921
20	1.3010	0.6783	0.6832	0.7025	0.6805	0.6528
24	1.3802	0.7388	0.7400	0.7505	0.7246	0.7013
30	1.4771	0.7888	0.7872	0.7984	0.7606	0.7594
32	1.5051	0.8440	0.8183	0.8252	0.7760	0.7760
40	1.6021	0.8894	0.8935	0.8968	0.8484	0.8327
48	1.6812	0.9274	0.8989	0.9270	0.8772	0.8783
60	1.7782	0.9538	0.9382	0.9417	0.9347	0.9334
80	1.9031	0.9979	1.0180	1.0397	1.0793	1.0033
96	1.9823	1.0694	1.0335	1.0249	1.0925	1.0470
120	2.0792	1.0751	1.1153	1.1194	1.1353	1.1000
160	2.2041	1.2150	1.1694	1.2294	1.2930	1.1676
240	2.3802	1.2345	1.2738	1.2629	1.3991	1.2617

X: *Regression analysis – 5-day logarithmic returns*

	$10 \leq n \leq 480$				
	ATX	BUX	PX	SAX	E(R/S)
Number of observation	16	16	16	16	16
R^2	0.9915	0.9964	0.9941	0.9971	0.9994
Constant	−0.0424	−0.0544	−0.0497	−0.2074	−0.1062
standard error	0.0327	0.0212	0.0277	0.0232	0.0086
t-statistic	−1.2941	−2.5705	−1.7929	−8.9325	−12.3048
p-value	0.2166	0.0222	0.0946	0.0000	0.0000
Hurst exponent	**0.5641**	**0.5578**	**0.5705**	**0.6822**	**0.5809**
standard error	0.0197	0.0128	0.0167	0.0140	0.0052
significant *H* vs *E*(*H*)	−0.3682	−0.5055	−0.2289	2.2184	x



2: R/S analysis and V-statistic – 5-day returns

XI: *Regression analysis – AR(1) residuals of 5-day logarithmic returns*

	$10 \leq n \leq 480$				
	ATX	BUX	PX	SAX	E(R/S)
Number of observation	16	16	16	16	16
R^2	0.9916	0.9962	0.9940	0.9964	0.9994
Constant	–0.0468	–0.0542	–0.0487	–0.2090	–0.1062
standard error	0.0322	0.0217	0.0275	0.0249	0.0086
t-statistic	–1.4526	–2.4988	–1.7743	–8.3984	–12.3048
p-value	0.1684	0.0255	0.0978	0.0000	0.0000
Hurst exponent	**0.5587**	**0.5619**	**0.5654**	**0.6643**	**0.5809**
standard error	0.0194	0.0131	0.0166	0.0150	0.0052
significant *H* vs *E*(*H*)	–0.4872	–0.4163	–0.3392	1.8268	x

Tab. XII, XIII and XIV show regression results for subperiods of ATX, BUX and PX series (SAX series may contain an infinity cycle). A break in the R/S graph and V-statistic graph appears to be at 32 observations for ATX (i.e. 160 trade days) and 40 observations for BUX and PX (i.e. 200 trade days). In this cases differences between *H* and *E*(*H*) are insignificant, at the 95 percent level. Regression results would imply an indepedent process, again.

XII: *Regression analysis – ATX 5-day returns*

	$10 \leq n \leq 32$			$32 < n \leq 480$		
	r_t	AR(1) residuals	E(R/S)	r_t	AR(1) residuals	E(R/S)
Number of observation	8	8	8	8	8	8
R^2	0.9942	0.9938	0.9998	0.9800	0.9797	0.9999
Constant	–0.2181	–0.2170	–0.1649	0.1316	0.1220	–0.0481
standard error	0.0396	0.0404	0.0066	0.0776	0.0777	0.0058
t-statistic	–5.5009	–5.3776	–25.0054	1.6948	1.5703	–8.2623
p-value	0.0015	0.0017	0.0000	0.1410	0.1674	0.0002
Hurst exponent	**0.7013**	**0.6916**	**0.6268**	**0.4760**	**0.4732**	**0.5515**
standard error	0.0310	0.0316	0.0052	0.0395	0.0395	0.0030
significant *H* vs *E*(*H*)	1.6329	1.4195	x	–1.6531	–1.7147	x

XIII: *Regression analysis – BUX 5-day returns*

	$10 \leq n \leq 40$			$40 < n \leq 480$		
	r_t	AR(1) residuals	E(R/S)	r_t	AR(1) residuals	E(R/S)
Number of observation	9	9	9	7	7	7
R^2	0.9984	0.9985	0.9998	0.9969	0.9967	0.9999
Constant	–0.1957	–0.1997	–0.1581	–0.0227	–0.0182	–0.0416
standard error	0.0187	0.0185	0.0069	0.0384	0.0394	0.0057
t-statistic	–10.4457	–10.8007	–23.0508	–0.5903	–0.4613	–7.2805
p-value	0.0000	0.0000	0.0000	0.5807	0.6639	0.0008
Hurst exponent	**0.6701**	**0.6774**	**0.6210**	**0.5388**	**0.5407**	**0.5484**
standard error	0.0142	0.0140	0.0052	0.0191	0.0195	0.0028
significant *H* vs *E*(*H*)	1.0757	1.2361	x	–0.2097	–0.1690	x

XIV: *Regression analysis – PX 5-day returns*

	$10 \leq n \leq 40$			$40 < n \leq 480$		
	r_t	AR(1) residuals	E(R/S)	r_t	AR(1) residuals	E(R/S)
Number of observation	9	9	9	7	7	7
R^2	0.9987	0.9986	0.9998	0.9762	0.9763	0.9999
Constant	–0.1892	–0.1867	–0.1581	0.0155	0.0149	–0.0416
standard error	0.0173	0.0180	0.0069	0.1070	0.1060	0.0057
t-statistic	–10.9202	–10.3621	–23.0508	0.1452	0.1405	–7.2805
p-value	0.0000	0.0000	0.0000	0.8902	0.8938	0.0008
Hurst exponent	**0.6805**	**0.6743**	**0.6210**	**0.5354**	**0.5312**	**0.5484**
standard error	0.0132	0.0137	0.0052	0.0531	0.0526	0.0028
significant *H* vs *E*(*H*)	1.3048	1.1686	x	–0.2852	–0.3779	x

DISCUSSION

Results presented above corresponding with results of the other studies. For example for subperiod $10 \leq n \leq 120$ we find the Hurst exponent of PX`s logarithmic returns $H = 0.63$. It corresponds with Střelec`s (2007a,b) findings value of the Hurst exponent for longer time series of PX stock index (from January 1995 to September 2007, i.e. about 3120 daily closing prices) – $H = 0.67$ for subperiod $10 \leq n \leq 120$.

These values of Hurst exponent for PX are also not far from Tran (2005), who found value of Hurst exponent for period of more than 11 years from September 1993 to October 2004 (i.e. 2607 daily closing prices) $H = 0.66$. Because the Hurst exponent is different from 0.5, he described this time series as phenomena of persistence. But, how significant is this result? In our paper we found significant difference, at the 95 percent level, between *H* and *E*(*H*) only for subperiod $10 \leq n \leq 120$ of PX`s logarithmic returns. But for AR(1) residuals of logarithmic returns is difference insignificant, at the 95 percent level. The time series PX contains only short memory effect. Long memory effect is not significant. Similar results were obtained by Peters (1994) for Dow Jones Industrial.

For the future research we can target the stability analysis of Hurst exponent. For example we can suppose this idea – if the Hurst exponent is stable, then the difference between *H* and *E*(*H*) will also be stable. Therefore, we can carry out R/S analysis for longer time series. Consequently, if the difference is stable, we will need about 5900 observations of PX`s daily closing prices or about 37,600 observations of ATX`s daily closing prices to reject null hypothesis. But firstly there is no guarantee that this will happen, and secondly there is not so long data history – for example today data history of daily closing prices of PX contains only about 3400 observations and it is too little for relevant analysis.

SUMMARY

This article deals with searching for long memory effects in financial time series. Source data are daily closing prices of Central Europe stock market indices – Bratislava stock index (SAX), Budapest stock index (BUX), Prague stock index (PX) and Vienna stock index (ATX) – in the period from January 1998 to September 2007. For analysed data R/S analysis is used to calculate Hurst exponent. On the basis of the Hurst exponent is characterized formation and behaviour of analysed time series. Computed Hurst exponent is also statistical compared with his expected value signalling independent process.

It was concluded that differences between the Hurst exponent (*H*) and expected value of the Hurst exponent (*E*(*H*)) of SAX`s daily logarithmic returns and AR(1) residuals of daily logarithmic returns are significant, at the 99 percent level, because *H* is about 3.7 standard deviations above its expected value. The other time series (e.g. ATX, BUX and PX) would imply an independent process, because differences between *H* and *E*(*H*) is insignificant at the 95 percent level. In conclusion, SAX time series follows a persistent process with long memory effect, whereas ATX, BUX and PX series follow an independent process.

For subperiod analysis was concluded that ATX series follows independent process for $10 \leq n \leq 200$ with Hurst exponent $H = 0.61$ (the difference from its expected value is insignificant, at the 95 percent level), and an antipersistent process for subperiod $200 < n \leq 1200$ with Hurst exponent $H = 0.47$ (this value is 2.7 standard deviations below its expected value, and is significant at the 99 percent level).

On the other hand for subperiod $10 \leq n \leq 120$ of PX`s logarithmic returns series was identified statistical significant difference, at the 95 percent level, between estimated Hurst exponent and its expected

value – this time series follows a persistent process with long memory effect. Secondly we analysed AR(1) residuals of PX`s logarithmic returns. The AR(1) process was used to minimize short memory effect and linear dependency. In this case was identify the statistical insignificant difference, at the 95 percent level, between estimated and expected values of the Hurst exponent. According to these results there is no long memory effect in PX time series – PX time series contains linear dependency and short memory effect.
Results of daily time series were verified by analysis of 5-day returns. These results imply analogical conclusions as results of analysis of daily returns mentioned above.

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SOUHRN

Hledání efektu dlouhodobé paměti v časových řadách středoevropských burzovních indexů

Tento článek se zabývá identifikací dlouhodobé paměti v procesu tvorby finančních časových řad. V práci analyzovanými časovými řadami jsou uzavírací kurzy a z nich odvozené logaritmické výnosy středoevropských burzovních indexů – Českého burzovního indexu PX, Maďarského burzovního indexu BUX, Rakouského burzovního indexu ATX a Slovenského burzovního indexu SAX – a to vše za období leden 1998 až září 2007. K identifikaci dlouhodobé paměti je využito R/S analýzy a z něj vycházejícího Hurstova exponentu charakterizující proces tvorby časové řady. Odhadnutý Hurstův exponent je v práci rovněž porovnáván s očekávanou hodnotou Hurstova exponentu vycházejícího z předpokladu nezávislého procesu.
Na základě v práci provedené analýzy byl identifikován persistentní proces, tj. proces s dlouhou pamětí, pouze v případě slovenského burzovního indexu SAX s hodnotou Hurstova exponentu $H = 0,63$, která je statisticky významně odlišná od očekávané hodnoty Hurstova exponentu. U ostatních sledovaných finančních časových řad nebyl persistentní proces identifikován – pouze v případě českého burzovního indexu PX byl u logaritmických výnosů pro interval $10 \leq n \leq 120$ identifikován statisticky významný rozdíl mezi odhadnutou a očekávanou hodnotou Hurstova exponentu naznačující persistentní proces s dlouhou pamětí. Následnou analýzou, kdy byla z časové řady odstraněna krátkodobá složka (v práci vyjádřená procesem AR(1)), však byla tato domněnka vyvrácena, neboť pro AR(1) residua nebyl statisticky významný rozdíl mezi odhadnutým a očekávaným Hurstovým exponentem na 5% hladině významnosti identifikován. Uvedené tak signalizuje pouze krátkodobou paměť procesu, po jejímž odstranění se časová řada chová jako nezávislý proces. Uvedené výsledky sledovaných denních logaritmických výnosů byly rovněž potvrzeny analýzou 5denních logaritmických výnosů sledovaných časových řad.

Hurstův exponent, R/S analýza, V-statistika, logaritmický výnos, burzovní index

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