MODELS AND ALGORITHMS OF ADAPTIVE ANIMAL FLOW CONTROL IN ROTARY MILKING PARLORS

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Abstract

The study addresses the influence of milking duration of individual cows on the performance of conveyor-like rotary milking parlors and seeks to optimize their operation parameters and operating modes.

The observational experiment was conducted in the Zhdanovsky Farm in Nizhny Novgorod Region, Russia. The dairy farm had a herd of 600 cows, divided into 10 groups by physiological characters and milk yield, and operated a 36 point milking parlor. Distribution of milking time of individual cows was studied using statistical analysis methods. The cyclogram of parlor operation and the functional connection of main parameters were analyzed using Maple analytical computing system, including its standard libraries and functions.

The trends in idle time, which occurs due to undermilking of animals in one turn of the parlor, were studied. The idle time can result in overestimation of the number of stalls or decrease in the nameplate performance of the milking parlor by 30–40% from 120 to 93 cows per hour. Mathematical models, taking into account the influence of the milking time of individual animals (2 to 17 minutes) on the parameters of parlor operation, were developed. The algorithms of adaptive control over the rotational speed were proposed to minimize idle time in parlor operation and maintain the nameplate performance.

The mathematical models, control algorithms and developed software can serve as a scientific basis for new designs of high-performance rotary milking parlors.

Keywords: milking parlor performance, idle time, operation cycle, milking time, rotation speed

INTRODUCTION

For many years researchers and engineers have been searching for organizational and technological solutions in order to improve the overall performance of dairy farms and the labor productivity of milking machine operators (Teslenko, 2009, 2010, 2002; Tsekuliya et al., 1981; Fubbeker, 2002; Rodens, 2001; Alstrup et al., 2015). For this purpose, they tried to make milking process similar to a sequence of manufacturing operations. The sought similarity was achieved with the development of conveyor-like rotary milking parlors. In such parlors, animals as the object of labor continuously move from one operation to another in the milking process...
Parlor milking efficiency is strongly influenced by the milking time of individual cows. To address this issue, parlor producers offer remedies that, in effect, can be narrowed down only to two solutions. The first one is to set a constant parlor rotation speed (often, it is one rotation per 15 or 20 mins which corresponds to the milking time of the slowest milker). As the undermilked cow approaches the exit, the operator stops the platform from the control panel.

The second one is that the rotation speed is regulated from the control panel in combination with a herd management system, which analyzes the data for the previous day, such as entrance-exit time, milking time, quantity of milk produced by individual cows, etc. The speed is set automatically so that milking is completed in one turn of the platform, i.e., the rotation speed is chosen according to the data of the slowest milker. If any cow is still undermilked in the last third of the turn, the milking system detects it and automatically reduces the speed or even stops the parlor immediately before exit to complete the milking.

However, our previous studies show that the operation of milking parlor by such algorithm can be inefficient, and its productivity, compromised.

The optimum rotation speed \( v \) should be between the minimum \( v_{\text{min}} \) and the maximum \( v_{\text{max}} \). The rotation speed may vary greatly and depends on the individual characteristics of animals, but on average it can be taken as 0.08–0.16 m/sec. Yet, the use of crowd gates in modern milking parlors can improve entrance time at the average and even the use of crowd gates in modern milking parlors on average it can be taken as 0.08–0.16 m/sec. Yet, the performance equation for the path length takes the form:

\[
L_p = N_{st}l + B_1 + B_2,
\]

where \( L_p \) is the path length, \( m; N_{st} \) is the number of stalls on the carousel, pcs.

The section, where cows are prepared for milking and teat cups are attached should be equal to the length of one stall plus one stall for entrance. Completion of milking, machine stripping, removal of teat cups, and post-treatment of udder should be performed immediately before the cow leaves the milking parlor in a section also equal to the length of one stall plus one or two stalls for exit. Thus, the expression for the path length takes the form:

\[
L_p = N_{st} \times l + 2l + 3l.
\]

Obviously, each cow must go through this cycle \( t_{\text{op}} \) to be milked dry at the end. The time required for this includes the time of milking machine operation, which is taken equal to the milking time \( t_{\text{mil}} \) and the time of manual operations \( t_{\text{m.op}} \):

\[
t_{\text{op}} = t_{\text{mil}} + t_{\text{m.op}}.
\]
By dividing (7) by (8), we obtain the expression for optimal speed of the continuous milking process:

\[ \nu_{opt} = \frac{N_{nl} + 2l + 3l}{t_{m} + t_{m,opt}}. \]  

(9)

As can be seen from expression (9), the speed depends on the structural dimensions of the milking area, the milking time of individual cows depending on the physiological characteristics of the animals, and the technical characteristics of the milking apparatus. In addition, the speed is influenced by the time of manual operations performed by the personnel.

The average milking time is 4–6 minutes, but any group of cows can also have slow-milkers. According to Kormanovsky’s calculations, in order to determine the time of one full cycle, the average milking time of one cow should be taken as 8 min. In practice, an even longer milking time of 14–18 mins is taken when milking large herds. At that, the radius of milking parlor and the number of stalls are increased to maintain the regularity of pace. The peripheral velocity of rotation also increases accordingly and is usually regulated using the control panel and the heard management system. The latter analyzes the milking data for the previous days, such as average milk yield and milking time of individual cows. The rotation speed is set automatically so that milking of one cow is completed in one turn of the carousel. If a cow remains undermilked in the last third of the turn, the system detects it and automatically slows the movement of the carousel.

It has been experimentally shown that the milking parlor operation is significantly affected by high variability of milking time, which in individual animals can exceed the effective time of one turn of the carousel. For this reason, idle time reaches 20–30% of the total operation time of the milking parlor (Bilibin, 1977; Zvinyatkovsky, 1975). Some researchers proposed corrective coefficients that take the idle time into account, but there are no well-founded recommendations on the choice of their numerical values. The probabilistic characteristics of a milking parlor can be calculated using the milking time of the slowest milker. However, such approach can hardly find a wide application since it gives a significantly reduced capacity of the milking parlor and the number of slow-milkers in a herd is never very high (Ignatkin, 2009). Nevertheless, some producers resort to this approach to calculate the capacity and maintain the reasonable productivity of milking parlors by increasing the radius of the carousel, the corresponding number of stalls and the peripheral velocity of rotation.

This paper proposes a method of theoretical calculations of the throughput capacity of milking parlors. We observed the operation of the SAC milking parlor in the Zhdanovsky Agricultural Production Cooperative in Kostovsky district of Nizhny Novgorod Region (Russia). According to the obtained data, most cows were milked in 1.5 to 15 mins. A 15-minute turn provided enough time for pre-milking preparation routine, milk letdown and post-treatment of each cow before exit and gave 4 turns per hour. The carousel had 36 stalls, 3 stalls for entrance and exit, 3 stalls for preparation, and 30 stalls for milking. By multiplying 30 stalls by 4 turns per hour, we obtain the throughput capacity of 120 cows per hour. This calculation is based on the assumption that the milking parlor operates continuously without stopping or idle time; however, in reality, it is not always so. Stops and idle time can occur because of cows that are not yet trained to enter the parlor, fresh calved cows that should be milked separately, or if some stalls on the parlor remain empty (DeLaval, 2012, 2013).

The actual throughput capacity of the studied parlor was 93 cows on average. Taking into account the lognormal distribution of milking time (Tsoi, 2010), which corresponds to the facts reported in professional literature, the emphasis should be made on the algorithm of changing the angular velocity of parlor rotation and the corresponding duration of the milking cycle.

Obviously, the duration of milking cycle for one animal will vary in the range:

\[ t_{m} \in \{t_{min} \ldots t_{max}\} \]  

(10)

or, according to experimental data, from 1.5 to 17 mins.

Therefore, there arises a question how to determine the angular velocity of parlor rotation that would ensure an adequate milking cycle for each animal. At that, the main contradiction between theory and practice is that statistical models conflict with the logic of attending to individual animals. The purpose of adaptive management is to remove this contradiction, using specific data for each animal instead of averaged estimates.

**MATERIALS AND METHODS**

Let us consider possible variants:

If \( t_{m} = \bar{m}_{m} \), i.e. the time of one turn is equal to the expected milking time (in our case, 6 mins), all the cows on the left side of the graph (Fig. 1) will be milked dry in one turn of the parlor, and all the cows of the right side, i.e. having \( t_{m} > \bar{m}_{m} \), will create idle time as it will be necessary to stop the parlor.

The total idle time of the milking parlor can be expressed in the form of an integral:

\[ T_{idle} = \int_{t_{m}}^{\bar{m}_{m}} f(t)dx \]  

(11)

or, taking into account the lognormal distribution, in the following form:
the radius of the carousel should be increased, too, and this affects the capital investment into the milking parlor (Fig. 2).

In the general case, this relationship is quite easy to determine:

\[ 2\pi R_{crl} = N_a \times l_a, \]  \hspace{1cm} (13)

where \( l_a \) is the projection of the stall length to the middle circumference of the carousel.

It is \( l_a^{\text{hl}} = l_{\text{cow}} \) in case of tandem design, \( l_a^{\text{hl}} = l_{\text{cow}} \cos \alpha \) in case of herringbone design, and \( l_a^{\text{pp}} = b \) in case of parallel design.

In the latter case, the density of animals and the flow rhythm are maximal. Given the above, it is obvious that:

\[ r_{\text{min}} \leq r \leq r_{\text{max}} = \frac{t_{\text{n}}[l_{\text{cow}} - l_{\text{cow}} \cos \alpha ; b]}{2\pi R_{crl}}, \]  \hspace{1cm} (14)

Thus, the necessary condition of maintaining a constant rhythm \( r \) is \( N_a \rightarrow \max \).

By analyzing the expression

\[ N_a = \frac{2\pi R_{crl}}{l_a}, \]  \hspace{1cm} (15)

it becomes obvious that in order to maintain an acceptable value of \( R_{crl} \), it is necessary to reduce \( l_a \), then we get the optimal number of stalls \( N_a \) and the corresponding value of the flow rhythm \( r \) for a smaller radius of the carousel. At that, the fastest flow rhythm \( r \) will be obtained at \( l_a = b \), i.e. in case of parallel design of the milking parlor.

When analyzing this option of reducing idle time by increasing the duration of \( t_{\text{ct}} \), it should be noted that it inevitably leads to an increase in the number of stalls and the carousel radius, while the capital investments into the milking parlor rise accordingly. Thus, this option has a shortfall that if the time of one turn corresponds to the maximum milking time \( t_m = t_{m,\max} \), it conflicts with the principles of continuous flow of production and the regularity of pace. That is to say that to keep the flow rhythm \( r \) constant, the following condition must be met (Kormanovsky, 1982):

\[ t_1 = t_2 = t_3 = \ldots = t_n = r, \]  \hspace{1cm} (2)

\[ \tau_{\text{ct}} = t_1 N = t_2 N = \ldots = t_n N = r N, \]  \hspace{1cm} (16)

where \( t_1 \ldots t_n \) is the time of individual operations in the milking cycle, \( \tau_{\text{ct}} \) is the milking cycle (in our case \( \tau_{\text{ct}} \)), \( N \) is the number of animals on the carousel (in our case \( N = N_a \)).

Inevitably, as \( \tau_{\text{ct}} = \tau_{\text{ct}} \) rises, \( N = N_a \) must be increased as well in order to keep the given flow rhythm \( r \). This means that by slowing down the rotation speed, we must simultaneously increase the radius of the carousel and the number of stalls to provide a longer milking time per one turn.
When solving the problem of stabilizing the cow flow rhythm
\[ r_1 = r_2 = r = \frac{t_m}{N_{st}} \]
one should bear in mind that if \( N_{st} \) remains the same, \( r \) will increase:
\[ r_2 = \frac{t_{in}}{N_{st}} = \frac{m_{st}}{N_{st}} K \]
where \( K \) is the magnification coefficient for milking duration that is equal to
\[ K = \frac{t_{m,\text{max}}}{m_{st}} \]
in our case
\[ K = \frac{17}{6} \approx 2.8. \]

Consequently, the new value of the flow rhythm \( r_2 \) also increases 2.8 times, which will slow down the entrance and exit time by 2.8 times. Obviously, such condition cannot be acceptable, therefore, it is necessary to increase the number of stalls \( N_{st} \) also by 2.8 times, say from 24 to 68, but this may prove to be disadvantageous financially.

Consider the variation of coefficient \( K \) for various designs of milking parlors (such as tandem, herringbone and parallel). Taking into account the condition that the value of \( r \) should remain constant, let us write:
\[ r_1 = r_2 = \frac{t_{in}}{N_{st}} = \frac{m_{st}}{N_{st}}, \]
\[ r_2 = \frac{t_{in}}{N_{st}} = \frac{t_{m,\text{max}}}{N_{st}}. \]

By making the right-hand members equal, we obtain:
\[ r_1 = r_2 = \frac{m_{st}}{N_{st}} = \frac{t_{m,\text{max}}}{N_{st}} \]
from which
\[ \frac{N_{st}}{m_{st}} = \frac{t_{m,\text{max}}}{N_{st}}. \]

Let us analyze this expression against \( l_c \), having in mind various parlor designs (tandem, herringbone and parallel).

For different values
\[ \frac{l_c}{l_{int}} = 1, \]
\[ \frac{l_{int}}{l_{int}} = 1, \]
\[ \frac{l_{int}}{l_{int}} = 1, \]
we get:
\[ \frac{l_c}{l_{int}} = 1, \]
\[ \frac{l_{int}}{l_{int}} = 1, \]
\[ \frac{l_{int}}{l_{int}} = 1. \]

\[ N_{st} = 2\pi R_{rcri} \]

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\[ N_{st} = 2\pi R_{rcri} \]

Having made \( R_{rcri} = \text{const} \), let us consider changing the number of stalls \( N_{st} \) by taking series relationships and introducing a new constant \( I_{cri} = 2\pi R_{rcri} \):
\[ N_{st}^0 + N_{st}^0 + N_{st}^0 = \frac{l_{int}}{l_{int}} + \frac{l_{int}}{l_{int}} + \frac{l_{int}}{l_{int}} = N_{st}^0 + N_{st}^0 + \frac{l_{int}}{l_{int}} + \frac{l_{int}}{l_{int}} + \frac{l_{int}}{l_{int}} - K_1, \]
\[ N_{st}^0 + N_{st}^0 = \frac{l_{int}}{l_{int}} + \frac{l_{int}}{l_{int}} + \frac{l_{int}}{l_{int}} = K_2. \]

The overall coefficient of changing the number of stalls shall be:
\[ N_{st}^0 + N_{st}^0 = \frac{l_{int}}{l_{int}} + \frac{l_{int}}{l_{int}} + \frac{l_{int}}{l_{int}} = K_2. \]

Consequently, the overall coefficient of increasing the number of stalls in tandem design shall be:
\[ K = K_1 \times K_2 = \frac{l_{int}}{l_{int}} + \frac{l_{int}}{l_{int}} + \frac{l_{int}}{l_{int}} = 2.5 \]
given the size of a cow \( l_c \approx 2 \text{ m}; b \approx 0.8 \text{ m}. \)

Therefore, the flow rhythm can be normalized by only changing the design from tandem to parallel, and thus increasing the number of stalls by 2.5 times without changing the radius, which is very important when choosing the type of parlor and the milking time.

Another condition of optimization is modernization of the technological process for the specific parlor design. Depending on the chosen milking time of one animal, it is obvious that the condition \( t_{in} = \frac{t_{m,\text{max}}}{m_{st}} \) is hardly acceptable, because it will be necessary to significantly reduce the rotation speed of the carousel and set the maximum time of one turn. In this case the values of \( R_{a} \) and \( N_a \) will also be maximal. To maintain a high throughput capacity of the parlor, it is advisable to introduce adaptive control of the parlor rotation from \( t_{min} \).
to \( t_{\text{max}} \). In this case, the maximum throughput capacity can serve as a criterion of optimization.

If milking in a milking parlor is categorized as a single-flow process (Koshkin, 1986), it should be taken into consideration that the maximum milking time \( t_{\text{max}} \) is its weakest point, hence it is hardly possible to maintain the rhythm flow \( r \) constant and it is necessary to vary both \( t_{\text{m}} \) and \( N_{\text{st}} \). But, unlike the time of turn, the number of stalls cannot be changed once the milking parlor is installed. What is a solution here is adaptive control of the parlor rotation speed.

For this, it is necessary to solve the problem of \( t_{\text{m}} \) optimization, which is a rather complicated function and should be solved using the criterion of optimization, the square deviation and expected milking time of one cow.

The duration of one milking cycle can also be taken in the view of the previously examined conditions that

\[
\begin{align*}
\Delta r &= t_{\text{cy, max}} - t_{\text{cy, min}}, \\
\tau_{\text{m}, \text{av}} &= t_{\text{cy, max}} + n_1, \\
\tau_{\text{m}, \text{opt}} &= t_{\text{cy, max}} + n_1, \\
\end{align*}
\]

Similarly, for \( \tau_{\text{m}, \text{av}} \):

\[
\begin{align*}
\tau_{\text{m}, \text{av}} &= t_{\text{cy, max}} + n_1, \\
\tau_{\text{m}, \text{opt}} &= t_{\text{cy, max}} + n_1, \\
\end{align*}
\]

Having analyzed the last expression and having adopted \( t_{\text{cy, max}} = \tau_{\text{m}, \text{opt}} \), we assumed that the parameter varies greatly from 2 to 14 min according to the longnormal estimate of milking time, as proposed by Tsoi. Then, when evaluating the criterion of optimization, the following condition should be adopted:

\[
\tau_{\text{m}, \text{av}} - \tau_{\text{m}, \text{opt}} = \min, \\
\Delta r = |\tau_{\text{max}} - \tau_{\text{min}}| \to \min, \\
T_{\text{cyl}} = \min.
\]

Analyzing specific cases of milking groups of animals, it is possible to approximately determine the idle time in milking parlor operation for different values of \( \tau_{\text{m}} \):

\[
\begin{align*}
T_{\text{cyl1}} &= \sum_{i=1}^{n}(t_{\text{m}, \text{cy}} - t_{\text{m}, \text{av}}); t_{\text{m}, \text{cy}} = t_{\text{m}, \text{av}}; t_{\text{m}, \text{cy}} > t_{\text{m}, \text{av}}, \\
T_{\text{cyl2}} &= \sum_{i=1}^{n}(t_{\text{m}, \text{cy}, \text{max}} - t_{\text{m}, \text{cy}}); t_{\text{m}, \text{cy}, \text{max}} > t_{\text{m}, \text{av}}; t_{\text{m}, \text{cy}} < t_{\text{m}, \text{cy}, \text{max}}.
\end{align*}
\]

where \( T_{\text{cyl1}} \) and \( T_{\text{cyl2}} \) are the idle time with stopping the parlor and idle time without stopping the parlor, respectively.

For efficient operation of the parlor, the conditions and the limitations can be specified in the general form:

\[
\begin{align*}
T_{\text{cyl1}} &\to \min, \\
T_{\text{cyl2}} &\to \min, \\
T_{\text{m, max}} &\neq t_{\text{m, max}}; \\
T_{\text{m, min}} &\neq t_{\text{m, min}}; \\
T_{\text{m, av}} &\neq \frac{t_{\text{m, max}}}{n}; \\
T_{\text{m, av}} &\neq \frac{t_{\text{m, max}}}{n}
\end{align*}
\]

The first two conditions logically follow from the previous analysis. The third condition \( T_{\text{m, max}} \neq t_{\text{m, max}} \) follows from the inexpediency of taking \( T_{\text{m, max}} = t_{\text{m, max}} \) because deacceleration of the carousel becomes abrupt and the throughput capacity of the milking parlor reduces. The fourth condition (limitation) is accepted by the condition of considerable idle time of the first type \( T_{\text{cyl1}} \). The fifth condition, in the general case, will be valid with the adaptive control of the rotation speed, when \( t_{\text{cy, max}} \) will tend to \( t_{\text{m, max}}/2 \).

The sixth condition appears advisable due to the above assumption that animals having abnormally long milking time should be milked dry in two (not one) turns of the carousel, or the following condition must be fulfilled:

\[
t_{\text{m, max}} = n \times t_{\text{m, av}},
\]

where \( n \) is an integer (1, 2, ...).

Thus, when considering the optimal rotation speed of the parlor or the optimal time of one turn, it is important to remember the condition for minimizing the variation in the flow rhythm, i.e. \( \Delta r = |\tau_{\text{max}} - \tau_{\text{min}}| \to \min \).

This is possible given an average estimate of the variation of parameter \( \tau_{\text{m, av}} \), which can be written in the general form:
The expression for $T_{\Sigma idl_2}$ can be written as:

$$T_{\Sigma idl_2} = \tau_{t_{n,max}} \cdot N_c \cdot \sum_{i=1}^{n} t_{m,i}.$$  

(38)

If $\tau_{t_{n,max}} = t_{m,max}$, only the in-cycle idle time of some stalls will occur, without stopping the milking parlor, or the following condition will be met:

$$\begin{cases} 
\tau_{t_{n,max}} \rightarrow t_{m,max}, \\
T_{\Sigma idl_2} \rightarrow 0, \\
T_{\Sigma idl_2} \rightarrow \max. 
\end{cases}$$  

(37)

In this case, the flow pace sprawls and the parlor performance is reduced, unless $R_{\tau_{t}}$ and $N_{a_{r}}$ are increased, which is unrealistic under operating conditions.

Considering a specific implementation of milking process ($N_c$), the expression for $T_{\Sigma idl_2}$ can be written as:

$$T_{\Sigma idl_2} = t_{m,max} \cdot N_c \cdot \sum_{i=1}^{n} t_{m,i}. $$

(39)

If $\tau_{t_{n,min}} = t_{n,max}$, the first-type idle time occurs and the parlor is stopped, the expression for $T_{\Sigma idl_1}$ can be written as:

$$T_{\Sigma idl_1} = \sum_{i=1}^{n} (t_{m,i} - t_{m,min}).$$

(40)

Let us solve this system of equations if each parameter tends to minimum and, thus, find mean correction of $\tau_{t_{n}}$:

$$\begin{cases} 
T_{\Sigma idl_2} = t_{m,max} \cdot N_c \cdot \sum_{i=1}^{n} t_{m,i} \rightarrow \min \\
T_{\Sigma idl_2} = \sum_{i=1}^{n} (t_{m,i} - t_{m,min}) \rightarrow \min. 
\end{cases}$$

If $\tau_{t_{n,max}} = t_{m,max}$, the following condition should be met:

$$T_{\Sigma idl_2} + T_{\Sigma idl_1} = T_{\Sigma idl}. $$

(41)

since the first member of equation $T_{\Sigma idl}$ becomes zero.

In a similar way, if $\tau_{t_{n,min}} = t_{m,min}$:

$$T_{\Sigma idl_2} + T_{\Sigma idl_1} = T_{\Sigma idl}.$$  

(42)

This equation gives a maximum estimate of $t_{m,max}$. However, consistently excluding abnormally high realizations, corresponding to the milking times of the slowest milkers in the herd, equation (42) can be rewritten as follows:

$$T_{\Sigma idl} = \sum_{i=1}^{N} (t_{m,i} - t_{m,min}) = N_c \cdot \sum_{i=1}^{n} t_{m,i}. $$

(43)

Solving this equation in relation to $t_{m,min}$, we get:

$$t_{m,min} = \frac{N_c \cdot \sum_{i=1}^{n} t_{m,i}}{N_c - N_{n_{min}}}. $$

(44)

This expression is applicable for estimating the parameter $t_{m,min}$ and establishing the optimal range of its variation. There follows an important conclusion that when estimating the parameter $t_{m,min}$ in each specific example of milking process, all the values of numerical series from 0 to $N_c$ should be excluded from the variation range when

$$t_{m,min} = \frac{\pi t_{m,opt}}{n}, $$

(45)

where $n > 1$ and $n$ is an integer.

For preliminary estimation of parameter $t_{m,opt}$, one can use its approximate evaluation:

$$t_{m,opt} = \frac{t_{m,max} \cdot t_{m,min}}{2}. $$

(46)

This value should be preliminarily determined for specific milking schedules and specific group of animals that include slow-milkers. Therefore, the general algorithm for controlling the milking process in a rotary milking parlor can be implemented as follows:

1. Create an array of data on the groups of animals that are milked at the milking parlor, and estimate the process paramecters $t_{m,max}$, $t_{m,min}$, $\sigma$, etc.;
2. Estimate the preliminary value of the parameter: $t_{m,opt} = f(t_{m,max}, t_{m,min})$;
3. Analyze the distribution and the number of slow-milkers in each group by the condition $t_{m,min} = \pi t_{m,opt}$;
4. Determine the idle time $T_{\Sigma idl_1}$, $T_{\Sigma idl_2}$ of the milking parlor operation;
5. Correct the parameter $t_{m,opt}$ by the condition: $T_{\Sigma idl} = T_{\Sigma idl}$;
6. Determine the tolerance for variation of the flow rhythm: $\Delta r = t_{m,max} - t_{m,min}$;
7. Set the optimum rotation speed of the milking parlor and the law of adaptive control;
8. Calculate the actual throughput capacity of the milking parlor: \( Q_e = f(t_{\text{tn}}, t_{\text{tn, max}}, t_{\text{T on}}, t_{\text{T off}}) \);
9. Calculate the difference between the actual capacity and the nameplate capacity of the milking parlor:
   \[ \Delta Q = \frac{Q_e - Q_{\text{max}}}{Q_e} \times 100\%; \Delta Q \leq 5\% \].

This work seek the way to increase the productivity of milking process (reducing the total milking time \( \Phi \)) for a herd of \( N \) cows. Modern milking parlors are equipped with processors that record the milking time, the milk yield, the milk flow rate, etc. of individual cows. At that, the processor accumulates data on the above parameters for the previous periods and the specific features of animals, such as age and individual productivity. These factors play an important role in setting the stop time and the rotation speed of the milking parlor. They allow developing an algorithm and an adaptive control program that can take into account an increasing number of process parameters.

The following process parameters were chosen for the purposes of this study:
- \( N \) — total number of cows in the herd, animals;
- \( t_{\text{tn}} \) — milking time of the \( i \)-th cow, sec;
- \( t_{\text{pre}} \) — time of pre-milking preparation routine performed by operators, sec;
- \( t_{\text{post}} \) — time of post-milking treatment performed by operators, sec;
- \( n \) — number of stalls in the milking parlor, pcs;
- \( n_0 \) — number of vacant stalls for entrance and exit to/from the milking parlor, pcs;
- \( N_s \) — number of slow-milkers in the herd, animals;
- \( \alpha \) — coefficient that corrects for the excessive milking time as compared to the average milking time of individual cows in the herd, \( \alpha \geq 2 \);
- \( t_{\text{ini}} \) — idle time (stop) for entrance and exit, sec;
- \( t_{\text{m, max}} \) — maximum milking time of the slowest milker in the herd, sec;
- \( t_{\text{m, min}} \) — minimum milking time of the fastest milker in the herd, sec;
- \( t_{\text{m, av}} \) — average milking time of cows in the herd, sec;
- \( \omega_{\text{ini}} \) — initial angular velocity of parlor rotation, rad/s;
- \( \beta \) — coefficient of angular velocity variation;
- \( \gamma_0 \) — group of slow-milkers, animals;
- \( \gamma_1 \) — group of fast-milkers, animals;
- \( t_{\text{tn, max}} \) — time of one turn of the carousel, sec;
- \( \gamma \) — coefficient of angular velocity variation, taking into account walking speed of the cow.

Thus, it is possible to form the vector of input parameters: \( \mathbf{X} = (x_1, x_2, x_3, \ldots, x_{18}) \).

The mathematical model of total milking time was based on systems of algebraic equations and inequalities given in (Tareeva, 2016; Kirsanov et al., 2012a, 2012b, 2012c, 2014). The methods of automatic control theory and parameter sensitivity analysis were used to build the mathematical model of total milking cycle and find the optimal rotation speed of the milking parlor.

The function of total milking time depends on all the parameters given above: \( \Phi = \Phi(x_1, x_2, x_3, \ldots, x_{18}) \). This function should adequately reflect the process of milking a herd of cows in the studied milking parlor (Obolensky et al., 2016). Further, in order to find the minimum value of this function in the eighteen-dimensional space of all parameters, we used conventional hill-climbing methods to examine the multivariable function for extremum. The input parameters can be narrowed down to the sensitive parameters that have the greatest effect on the decrease in \( \Phi \), for example, the milking time of the slowest milkers.

To obtain an adequate expression for function \( \Phi(x_1, x_{18}) \), let us consider some statistical data.
Fig. 3 shows the milking time of three cows, having average milking time in the studied herd, for three consecutive days (morning and afternoon milking).

The experimental observation over the milking process was based on the step-by-step timing technique, which involves recording the duration of each operation in the milking cycle, idle time, downtime of the milking parlor and other factors in accordance with the Industrial Standard OST 70.20.2.80 “Testing of agricultural machinery. Milking parlors for cows. Test program and methods” [OST 70. 20.2. 80]. The experimental observation included the following phases: preparation for observation, observation, processing of observation data, analysis of results, and development of process improvement proposals. During the observations, manual time tracking, photographic recording and computer analysis of the data on milking time were used. The duration of individual operations of the milking cycle was measured continuously, operation by operation (Tareeva, 2016).

Fig. 4 shows the milking time of three slow-milkers for three consecutive days (morning and afternoon milking). Figs. 3 and 4 indicate that it is impossible to predict the variation of milking time of the slowest milkers. According to [1], a ‘slow-milker’ is an animal whose milking time is twice as long as the average milking time. The optimal time of turn for the previous day is taken to calculate the initial time of the turn. Then, the optimal time of turn is calculated without taking into account the milking time of the slowest milkers. According to [1], a ‘slow-milker’ is an animal whose milking time is twice as much as $t_{m,av}$ or longer. The optimal time of turn is calculated by formula (44):

$$\tau_{m, opt} = \frac{\sum_{i=1}^{N} N_{i} N_{m,i} (t_{m,i} - t_{m,ini}) + \sum_{i=1}^{N} N_{i} N_{m,ini}}{N_{i} - N_{m,ini}}.$$

Therefore, when evaluating the parameter $\tau_{m, opt}$ in each specific milking process, one should exclude from the variation range all values of the numerical series from 0 to N animals that have $t_{m,max} = n \tau_{m, opt}$, where $n > 1$ and n is an integer.

Next, the initial angular velocity of the rotation is calculated by the formula:

$$\omega_{ini} = \frac{2\pi}{\tau_{m,ini}}.$$

(47)
Start

Data input
\[ t_{m,i}, t_{pre}, t_{poez}, n, N_c, \alpha, t_{ee} \]

Set input parameters

I

Calculate maximum, minimum and average milking time
\[ t_{m,max} = \max\{t_{m,i}\}, t_{m,min} = \min\{t_{m,i}\}, i \in I, N_c \]

Calculation
\[ t_{m,av} = \frac{t_{m,max} + t_{m,min}}{2} \]

Calculate the preliminary time of on turn
\[ \tau_{tn,ini} = \frac{\sum_{i=1}^{N_c-N_{sm}} (t_{m,i} - t_{m,min}) + \sum_{i=1}^{N_c-N_{sm}} t_{m,i}}{N_c - N_{sm}} \]

Calculate the initial angular velocity
\[ \omega_{ini} = \frac{2\pi}{\tau_{tn,ini}} \]

II

Calculations

5: Block diagram of the algorithm, taking into account stops for entrance to the milking parlor
Source: Kirsanov et al., 2016
III

Analysis of speed limitation with account for preparation time

\[ \omega_{\text{ini}} \leq \frac{6\pi}{n(t_u - 3t_{ee})} \]

Setting the initial rotation speed with account for limitations

\[ \omega_{\text{ini}} = \beta \omega_{\text{ini}}, \quad 0 < \beta < 1 \]

Set the rotation speed \( \omega = \omega_{\text{ini}} \)

Calculate the time of one turn \( t_{tm} = \frac{2\pi}{\omega_{\text{ini}}} \)

\( j = 1 \)

\( t_{m,j} > \alpha t_{tm} \)

Form the array of slow-milker numbers \( j \in j_{sm} \)

Form the array of fast-milker numbers \( j \in j_{fm} \)

5: Block diagram of the algorithm, taking into account stops for entrance to the milking parlor – continued
Figure 6 – Block diagram of the algorithm, taking into account stops for entrance to the milking parlor (continued) [21]

IV

Start of milking; the \( k \)-th cow steps on the carousel

\( k = 1 \)

\( k \in j_{sm} \)

\[
t_{m,k} \leq \left( t_{ee} + \frac{2\pi}{n\omega_{ini}} \right) (n - 3) - t_{pre} - t_{post}
\]

\( \omega = \omega_{ini} \)

\[
\omega_{ini} = \frac{2\pi}{n \left( \frac{t_{m,k} + t_{pre} + t_{post}}{n - 3} - t_{ee} \right)}
\]

\( k = k + 1 \)

\( k > N_c \)

STOP

5: Block diagram of the algorithm, taking into account stops for entrance to the milking parlor – continued
At the next stage, the initial speed of rotation is corrected, taking into account the time of pre-milking treatment (Fig. 5, part III). According to the specified value of \( \omega_{ini} \), the time of one turn is calculated, starting with the moment the first cow enters the milking parlor. Further, according to the data on milking for the previous period and the criterion specified in (Kormanovsky, 1982), cows are grouped into slow-milkers and fast-milkers. This is necessary for further analysis and setting of rotation speed upon each cow enters the parlor.

The principal stage, i.e. milking, is given in Fig. 5. The cycle counter takes values from 1 to \( N \) cows. At that, it is first determined if the cow that has just stepped on the carousel is a slow-milker. Depending on that, the rotation speed remains the same or is recalculated by the conditions and formulas in Fig. 5. The time of one turn for non- slow-milkers is recalculated by formulas that provide for the stop time, the time of pre- and post-milking treatment, the number of stalls and the initial rotation speed. Thus, the array of rotation speeds is formed on the basis of the data on individual milking time for previous periods.

The above block diagram served as a basis for software for calculating the function of total milking time \( \Phi(x_1, ..., x_{18}) \). The software was developed in Maple, version 15, and has a blocked structure (Strebulyayev et al., 2007). Maple was chosen because of its advanced capabilities in symbolic computing and processing large arrays of information. Maple combines a powerful programming language, an editor for writing and editing documents and programs, a modern multi-window user interface, a core of algorithms and rules for converting mathematical expressions, etc. Fig. 6 shows a fragment of computation.

**RESULTS**

In this study we analyzed the biotechnical system ‘human-machine-animal’ using the theory of algorithms, factor analysis, experimental observations and information technologies. This made it possible to develop a mathematical model of the cyclogram (Fig. 7) of milking parlor operation. The prerequisites for developing an adaptive control algorithm for the studied process were obtained (Fig. 8).

Given the above, let us propose the algorithm of adaptive control of the duration of milking carousel rotation (\( \tau_{tn} \)):

1. Create an array of data on groups of animal that are milked in the milking parlor and estimate the parameters \( t_{m, max}, t_{m, min}, \sigma \), etc.;
2. Analyze the distribution and the number of slow-milkers in each group by the condition of milking them dry in a whole number of cycles: \( t_{m, max} = n \tau_{tn, opt} \), where \( n \geq 2 \);
3. Determine the preliminary duration of one turn of the carousel:

\[
\tau_{tn, opt} = \frac{\sum_{i=1}^{N} N_{c} t_{m, max}}{N_{c} N_{m, opt}} + \tau_{cy} + \tau_{cy, max}
\]

4. Assign \( t_{m,1} = \tau_{cy, max} \) to the first cow;

The above block diagram served as a basis for software for calculating the function of total milking time \( \Phi(x_1, ..., x_{18}) \). The software was developed in Maple, version 15, and has a blocked structure. Maple was chosen because of its advanced capabilities in symbolic computing and processing large arrays of information. Maple combines a powerful programming language, an editor for writing and editing documents and programs, a modern multi-window user interface, a core of algorithms and rules for converting mathematical expressions, etc. Figure 8 shows a fragment of computation.

**Discussion**

A milking parlor cyclically repeats a set of operations, such as animal's entrance to the carousel, pre-milking preparation of the udder, attachment of teat cups, automatic milking of the animal, post- milking treatment of the udder, and animal's exit from the parlor. The number of occupied stalls and the overall performance of the milking parlor are directly affected by the total milking time.
5. Compare \( t_{cy, \Sigma_{\text{max}}} \) and \( \tau_{tn, \text{opt}} \):
   - if \( t_{cy, \Sigma_{\text{max}}} \leq \tau_{tn, \text{opt}} \), then the parameter \( \tau_{tn, \text{opt}} \) remains the same;
   - if \( t_{cy, \Sigma_{\text{max}}} \geq \tau_{tn, \text{opt}} \), then \( \tau_{tn, \text{opt}} = t_{cy, \Sigma_{\text{max}}} \).
6. Refer to the cycle counter to compare \( t_{cy, \Sigma_{\text{max}}} \) and \( \tau_{tn, \text{opt}} \);
7. If \( t_{cy, \Sigma_{\text{max}}} > \tau_{tn, \text{opt}} \), then the parameter \( \tau_{tn, \text{opt}} \) remains the same; otherwise, if \( t_{cy, \Sigma_{\text{max}}} \leq \tau_{tn, \text{opt}} \), then assign new value to \( \tau_{tn, \text{opt}} \);
8. Verify the ‘abnormality’ condition for parameter \( t_{cy} \), as compared to the average milking time of the group \( (t_{\text{cy,}i}) \); if the condition is fulfilled, then the parameter is considered abnormal;
9. Verify the condition of compensation for the abnormal milking cycle;
10. Switch on the cycle counter for comparison of the abnormal \( t_{cy, \Sigma_{\text{max}}} \) and the current \( t_{cy,i} \); Value of the parameter. If the compared parameters become equal after 5...6 consecutive milking cycles, then the condition of compensation is fulfilled and \( \tau_{tn} \) = \( t_{cy, \Sigma_{\text{max}}} \) can be assigned. Otherwise, the abnormal parameter of the cycle should be excluded, and this cow should be milked for one more turn;
11. Determine the calculated idle time:
    \[
    T_{\Sigma_{\text{idl}}} = t_{\Sigma_{\text{max}}} \times \sum_{i} t_{\text{tn,}i};
    \]
12. Correct the parameter \( \tau_{tn, \text{opt}} \) by the condition \( T_{\Sigma_{\text{idl}}} = \min; \)
13. Calculate the allowance for variation of the flow rhythm \( \Delta r = r_{\text{max}} - r_{\text{min}} \):
    \[
    r_{\text{max}} \times r_{\text{min}} = \frac{t_{m,i} \times \left[1; 1; 1; \cos a; b\right]}{2\pi R_{\text{crl}}^2};
    \]
14. Calculate the optimal linear and circumferential velocity of the carousel:
    \[
    V_{\text{opt}} = \frac{L_{\text{crl}}}{\tau_{tn, \text{opt}}}; \quad \omega = \frac{V_{\text{opt}}}{R_{\text{crl}}};
    \]
15. Calculate the actual throughput capacity of the milking parlor:
    \[
    Q_{\text{act}} = \frac{Q_{\text{p}} - Q_{\text{m,i}}}{T_{\Sigma_{\text{opt}}}});
    \]
16. Calculate the difference between the actual and the nameplate performance of the milking parlor:
    \[
    \Delta Q = \frac{Q_{\text{act}} \times 100\%}{Q_{\text{p}}}; \quad \Delta Q \leq 5\%.
    \]

Notation to Figs. 5 and 9:
- \( t_{\text{pre,}i} \) — time of pre-milking preparation routine performed by operators, sec;
- \( t_{\text{post},i} \) — time of post-milking treatment performed by operators, sec;
- \( n \) — number of stalls in the milking parlor, pcs;
- \( N_{\text{c}} \) — total number of cows in the herd, animals;
- \( N_{\text{sm}} \) — number of slow-milkers in the herd, animals;
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I

Data input
$t_{m,i}, t_{pre}, t_{post}, n, N_c, l_c, V, \alpha, \beta, \gamma$

Set input parameters

II

Calculations

Calculate maximum, minimum and average milking time
\[ t_{m,\text{max}} = \max \{ t_{m,i} \}, \quad t_{m,\text{min}} = \min \{ t_{m,i} \}, \quad i \in 1, N_c \]

\[ t_{m,\text{av}} = \frac{t_{m,\text{max}} + t_{m,\text{min}}}{2} \]

Calculate the preliminary time of on turn
\[ \tau_{\text{tn,ini}} = \frac{\sum_{i=1}^{N_c-N_{sm}} (t_{m,i} - t_{m,\text{min}}) + \sum_{i=1}^{N_c-N_{sm}} t_{m,i}}{N_c - N_{sm}} \]

Calculate the initial angular velocity
\[ \omega_{\text{ini}} = \frac{2\pi}{\tau_{\text{tn,ini}}} \]

8: Block diagram of the algorithm without stops for entrance to the milking parlor (see notation)
Analysis of speed limitation with account for preparation time

\[ \omega_{ini} \leq \frac{6\pi}{n\ell_n} \]

- **yes**
  - \( \omega_{ini} < \frac{2\pi V}{ni} \)
    - **no**
      - \( \omega_{ini} = \beta \omega_{ini} \), \( 0 < \beta < 1 \)
    - **yes**
      - \( \omega_{ini} = \gamma \omega_{ini} \), \( 0 < \gamma < 1 \)
  - **no**

Set the rotation speed \( \omega = \omega_{ini} \)

Calculate the time of one turn \( t_{tri} = \frac{2\pi}{\omega_{ini}} \)

\[ j = I \]

- **yes**
  - \( t_{m,j} > \alpha t_{tri} \)
    - **no**
      - Form the array of fast-milker numbers \( j \in j_{fm} \)
    - **yes**
      - Form the array of slow-milker numbers \( j \in j_{sm} \)

8: Block diagram of the algorithm without stops for entrance to the milking parlor (see notation) – continued
IV

Start of milking: the \( k \)-th cow steps on the carousel

\( k = 1 \)

\( k \in j_{sm} \)

no

\( t_{m,k} \leq \left( \frac{2\pi}{n\omega_{ini}} \right) (n - 3) - t_{pre} - t_{post} \)

no

\( \omega_{ini} = \frac{2\pi}{n \left( \frac{t_{m,k} + t_{pre} + t_{post}}{n - 3} \right)} \)

no

\( k = k + 1 \)

\( k > N_c \)

yes

STOP

Calculation of angular velocity of parlor rotation

\[ \omega = \omega_{ini} \]

yes

\[ \omega = \omega_{ini} \]

\[ \omega_{ini} = \frac{2\pi}{n \left( \frac{t_{m,k} + t_{pre} + t_{post}}{n - 3} \right)} \]

\[ k = k + 1 \]

\[ k > N_c \]

yes

STOP

8: Block diagram of the algorithm without stops for entrance to the milking parlor (see notation) – continued
α — coefficient that corrects for the excessive milking time as compared to the average milking time of individual cows in the herd, \( \alpha \geq 2; \)

\( t_{\text{r}} \) — idle time (stop) for entrance and exit, sec;

\( t_{\text{m,\max}} \) — maximum milking time of the slowest milker in the herd, sec;

\( t_{\text{m,\min}} \) — minimum milking time of the fastest milker in the herd, sec;

\( t_{\text{m,av}} \) — average milking time of cows in the herd, sec;

\( n_v \) — number of vacant stalls, pcs;

\( \beta \) — coefficient of angular velocity variation;

\( j_{\text{s}} \) — group of slow-milkers, animals;

\( j_{\text{f}} \) — group of fast-milkers, animals;

\( \tau_{\text{tn}} \) — time of one turn of the carousel, sec;

\( \gamma \) — coefficient of angular velocity variation, taking into account walking speed of the cow.

\( V \) — walking speed of the cow, m/sec.

**DISCUSSION**

A milking parlor cyclically repeats a set of operations, such as animal’s entrance to the carousel, pre-milking preparation of the udder, attachment of teat cups, automatic milking of the animal, post-milking treatment of the udder, and animal’s exit from the parlor. The number of occupied stalls and the overall performance of the milking parlor are directly affected by the total time of milking individual animals and, in particular, the maximum milking time of the slowest milker in the herd.

Most researchers assume that the parameters of distribution of milking time remain constant during entire milking, while both the parameters and the sample size, which they relate to, always vary during the milking process.

The number of slowest milkers on a dairy farm is never high under the conditions of limited size of animals group. However, the slow-milkers influence the full cycle of the parlor operation (one turn of the carousel), which causes idle time (stops). The number of such compromised cycles is always limited, by their influence is significant. Therefore, setting the rotation speed to the minimum to suit the slowest milkers gives a significant error in the throughput capacity of the conveyor-like milking parlor.

Taking into account the final values of the studied parameters, it seems expedient to take full or partial sums of milking time of individual cows as calculated values and exclude abnormally high milking time of slow-milkers. This can optimize the duration of the milking cycle and the throughput capacity of the milking parlor. The calculation of parameters presents no difficulties in practice, given modern computerized herd management systems that recording the milking time of individual cows for previous periods. If available, such information can reduce the milking time by redistributing the slow-milkers in the process flow or by milking them dry in two or more turns of the carousel. It should be noted that the functioning of the milking parlor also depends on the efficiency of the milking unit and animal’s reaction to it.

**CONCLUSION**

The presented mathematical models, adaptive algorithms and software can serve as a foundation for further development of high-capacity rotary milking parlors. The results of this study can also be adopted for milking ewe and goats.

**REFERENCES**


