DEFORMATION AND FRACTURE PROPERTIES OF DARK CHOCOLATE

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Abstract


Series of mechanical tests (uniaxial compression test, Brasil test, plate bending test) with cylindrical specimens made from dark chocolate have been performed. The stress-strain dependence has been plotted and modeled using data from compression test. The strain behaviour of the chocolate was found to be far from the linear elastic strain. The linear elasticity is limited for the very small strains. The Brasil test led to the development of the tensile stress in the specimen and its cracking. The dependence of the tensile stress on the strain rate has been confirmed. Also the linearity of the force vs. displacement data received during plate bending test has been limited to the very small value of the displacement. Impact behaviour of the chocolate has been also evaluated. Falling of the specimen from defined height led to its fracture. This experiment has been simulated using LS DYNA 3D finite element code.

Chocolate belongs to products that contain significant amounts of fats. Such products include e.g. butter, margarine, and many spreads such as cream, cheeses etc. The solid-like character of these materials is mainly due to an underlying network of weakly attractive polycrystalline fat particles. These fat particles arise from an aggregation process of primary triacylglycerol crystals into increasingly larger clusters until a three-dimensional network is formed (Narine and Marangoni, 1999). Many of the sensory attributes such as spreadability, mouthfeel, snap of chocolate, texture, etc., are dependent on the mechanical strength of the underlying fat crystal network. In addition to this obvious industrial importance, fat crystal networks form a particular class of soft materials, which demonstrate a yield stress and viscoelastic properties, rendering these materials plastic. From a materials sciences point of view, these materials are also extremely important. These properties are also very important for the evaluation of the extrusion processes. This is why we can report a significant efforts to model the mechanical strength (Kamphuis and Jongscaph, 1985; Kamphuis et al., 1984; Nederveen, 1963; Papenhuijzen, 1971, 1972; Payne, 1964; Van den Tempel, 1961, 1979) of these networks. A comprehensive model relating structural network characteristics and solid/liquid ratios of lipid networks to their mechanical strength is still missing.

In order to build up such model one must have available experimental data on the mechanical behaviour of these materials. The mechanical behaviour of the chocolate has been studied in many papers (Marangoni and Narine 2002). These studies were mostly limited to the evaluation of the visco-elastic properties under vibration test. There is a limited number of paper dealing with the evaluation of some basic parameters like the Young modulus, Poisson ratio, mechanical strength etc. The only two papers on the given topics have been reported (Minič 1989, Missaire et al., 1990).

In the given paper the following problems have been studied:

- evaluation of the elastic properties of the chocolate
- description of the stress strain behaviour under different loading rates
- evaluation of the tensile strength.

Obtained data have been used for the numerical simulation of the chocolate plate impact. The nu-
Numerical data have been compared with the experimental ones.

**MATERIAL AND EXPERIMENTAL PROCEDURE**

For the experiments a commercially available LINDT chocolate Excellence 85% Cocoa has been used. The specimens in form of a cylinder (12 mm in diameter and 6 mm in height) have been prepared from this chocolate.

The specimens were loaded using TIRA testing machine. Two different experiments have been performed:
1. The uniaxial compression test when specimens were compressed at the different crosshead speeds - 1, 10 and 100 mm/min. The integrated software converted the force crosshead displacement to stress-strain data.
2. The Brasil test (Hondros, 1959), originally developed to characterise the tensile strength of concrete (Talhurst, 1964). The schematic of this experiment is shown in Fig. 1.

![Schematic of the Brasil test](image1)

The specimen is loaded in pressure by the force $P$. The loading is converted to the tension. For the tensile stress in the middle of the specimen is given by the relation:

$$\sigma_t = \frac{2P}{\pi DL},$$

where $D$ is the specimen diameter and $L$ is its height.

3. In order to get the deeper insight on the chocolate behaviour the plate bending experiments have been performed. The schematic of this experiment is shown in Fig. 2.

![Schematic of the plate bending](image2)

A circular plate of the chocolate (65 mm in diameter, thickness $h = 3$ mm) is supported at the circle with the radius $r_1 = 60$ mm. The plate is loaded by the steel cylinder with the radius of 5 mm. Under assumption on the linear elastic behaviour of the plate material the governing equation has the form

$$\frac{1}{r} \frac{d}{dr} \left\{ r \frac{d}{dr} \left[ r \frac{d}{dr} \left( \frac{dw}{dr} \right) \right] \right\} = \frac{p(r)}{K},$$

where $w(r)$ is the plate deflection and $K$ is the plate rigidity.

$$K = \frac{Eh^3}{12(1-\nu^2)}$$

$E$ is the Young modulus and $\nu$ is the Poisson ratio, $p(r)$ is the pressure on the upper plate surface. For the loading shown in the Fig. 2 the pressure is given by:

$$p(r) = \frac{F}{\pi r_1^2}.$$

All experiments were performed at the room temperature.

**RESULTS AND DISCUSSION**

**COMPRESSION TEST**

The following procedure converted force $F(t)$ displacement $\Delta h(t)$ record to stress – strain data:
Deformation and fracture properties of dark chocolate

\[ \varepsilon = \frac{\Delta h}{h_0} \]

\[ \sigma = \frac{F}{A_0} , \]

where \( \varepsilon \) is the engineering strain, \( h_0 \) is the original height, \( \Delta h \) is the change in the height, \( \sigma(t) \) is the engineering stress at time \( t \), \( A_0 \) is the initial cross-section of the specimen. These data can be used for the evaluation of the true strain and true stress (Calzada and Peleg, 1978):

\[ \varepsilon_{true} = \ln(1 + \varepsilon) \]

\[ \sigma_{true} = \sigma e^{\varepsilon}. \]

The stress is calculated usually of the assumption that there is a little or no change in volume of chocolate during its compression. The compressibility given by \( \frac{dV}{V} \) can be expressed as (Ferry, 1980):

\[ \frac{1}{V} \frac{dV}{dc} = 1 - 2v. \]

The example of the stress – strain dependence is given in Fig. 3.

It has been found that this dependence can be fitted by the polynomial:

\[ \sigma (\text{MPa}) = a\varepsilon^4 + b\varepsilon^3 + c\varepsilon^2 + d\varepsilon. \]

This dependence is valid for the all cross – head velocities. If we use instead of the cross – head velocity \( V \) the strain rate:

\[ \dot{\varepsilon} = \frac{\varepsilon}{h} = \frac{V}{h} \]

the experimental data can be fitted by the Cowper – Symonds equation (Cowper and Simons, 1958):

\[ \sigma = (a\varepsilon^4 + b\varepsilon^3 + c\varepsilon^2 + d\varepsilon) \left\{ 1 + \left(\frac{\dot{\varepsilon}}{D}\right) \right\}, \]

where \( p \) and \( D \) are the material parameters. The analysis of the experimental data leads to the following values of the parameters:

\[ a = -242,03 \, \text{MPa}, \quad b = 216,46 \, \text{MPa}, \quad c = -66,72 \, \text{MPa}, \quad d = 8,6 \, \text{MPa}, \quad D = 152,000 \, \text{s}^{-1} \] and \( p = 2.18 \).

From the Fig. 3 it is obvious that the strain behaviour of the chocolate is far from the linear elastic strain. The linear elasticity is limited for the very small strains, i.e. for \( \varepsilon \to \infty \). Instead of the true Young modulus we can obtain only its apparent value:

\[ E = \left( \frac{\dot{\varepsilon}}{\varepsilon} \right) (\varepsilon = 0) \]

This modulus is strain rate dependent.
**Brasilian Test**

This experiment shown in the Fig. 1 leads to the development of the tensile stress in the specimen. This tensile stress can and does lead to the cracking of the specimen. This is shown in the Fig. 4.

The dependence of the tensile stress on the crosshead displacement is shown in the Fig. 5.

It may be shown that the tensile stress also depends on the strain rate. The more detail observations have shown that the cracks started at the crosshead displacement about 2 mm independently on the strain rate. It means the tensile strength is strain rate dependent. In order to obtain some quantitative expression the next experiments are needed.

**Plate Bending Experiment**

The solution of the governing equation presented in the Chapter 2 – see Fig. 2 is:

\[
 w(r) = -\frac{F}{64\pi\rho^2K} \left\{4\rho^2\ln\left(\frac{r_1}{\rho}\right) + 8\rho^2(r_1^2\ln r_1 - r^2\ln r) + 5\rho^4 - 4\rho^2r^2 - r^4\right\}. 
\]

During the experiment shown in the Fig. 2 the displacement is measured at the point \( r = 0 \). For this point we obtain:

\[
 w = -\frac{F}{64\pi\rho^2K} \left\{4\rho^2\ln\left(\frac{r_1}{\rho}\right) + 8\rho^2(r_1^2\ln r_1 - r^2\ln r)\right\}. 
\]

As it was mentioned this solution was obtained for the linear elastic deformation of the plate. As it is shown in the Fig. 6 the dependence between force \( F \) and the displacement \( w \) is non-linear. It can be fitted by:

\[
 F = aw^3 + bw^2 + cw, 
\]

where \( a = 0.9032 \text{ N/mm}^3 \); \( b = -2.9836 \text{ N/mm}^2 \) and \( c = 3.3436 \text{ N/mm} \).
Deformation and fracture properties of dark chocolate

The linearity is limited to the very small value of the displacement. If we perform the derivation $dF/dw$ we obtain:

$$
\frac{dF}{dw} \bigg|_{w=0} = c = -\frac{64\pi \rho^2 K}{4\rho^2 \ln \left( \frac{r_1}{\rho} \right) + 8\rho^2 (r_1^2 \ln r_1 - \rho^2 \ln \rho)}.
$$

If we use the value of $E = 9 \text{ MPa}$ (an average value for the used strain rate) we obtain the value of the Poisson ratio:

$$
\nu = 0.415
$$

its value is relatively closed to 0.5 which corresponds to the non-compressible material.

It is obvious that the obtained data describe only some basic features of the strain behaviour. This behaviour is highly non-linear and probably also time dependent. In order to verify the reliability of the obtained material parameters some independent experiment has been performed. Its description is given in the following part of the paper.

**Impact behaviour of the chocolate**

The chocolate products can be subjected to some impact during their transportation etc. For our experiments the usual form of the chocolate – see Fig. 7 has been loaded by the falling from the height of 1 m. The velocity 4.5 m/s corresponds to this height. After impact the chocolate has been fractured as shown in the Fig. 8.

![Experimental record of the force vs. displacement. Crosshead velocity: 10 mm/min.](image)

6: Experimental record of the force vs. displacement. Crosshead velocity: 10 mm/min.

This experiment has been simulated using LS DYNA 3D finite element code. The computational model of the chocolate shown in the Fig. 7 is shown in the Fig. 9. Owing to the symmetry only a half of the specimen has been considered.

A detail of the computational mesh is shown in the Fig. 10.

![Table of the chocolate used for the impact loading](image)

7: Table of the chocolate used for the impact loading
SUMMARY
Mechanical tests have been performed on specimens of dark chocolate, namely uniaxial compression test (with different crosshead speeds – 1, 10 and 100 mm/min), Brasil test, and plate bending test. Data received from the mechanical tests have been processed and stress–strain dependences have been constructed. It was found that chocolate did not show linear elastic strain. The results of Brasil test were in accordance with expected behavior, tensile stress, which occurred in the specimen, led to cracking of the central part of the specimen. Chocolate exhibited provable dependence of tensile stress on strain rate. Plate bending test led to another conclusion – linearity of the force versus displacement data has been limited to the very small value of the displacement. Considering $E = 9 \text{ MPa}$ (an average value for the strain rate used) the value of the Poisson ratio has been calculated. Falling of the specimen from defined height (1 m), which corresponds to velocity of 4.5 m/s, was performed as another means of testing. This experiment led to the fracture of chocolate. This experiment has been numerically simulated by use of LS DYNA 3D finite element code.
Deformation and fracture properties of dark chocolate

SOUHRN
Deformační a lomové chování tmavé čokolády


čokoláda, elastické vlastnosti, mechanické zatěžování, numerická simulace

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