

COMPARISON OF THE IMPACT OF ECONOMETRIC MODELS ON HEDGING PERFORMANCE BY CRUDE OIL AND NATURAL GAS

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Abstract

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The paper examines the performance of hedging spot prices in crude oil and natural gas. The subject of the research are spot prices of West Texas Intermediate and Henry Hub. The risk protection is provided by the application of futures contracts of underlying assets. In our analysis three econometric models (OLS, Copula, GARCH) and a naive portfolio are applied to obtain the optimal hedge ratio. Afterwards, the calculated weights for futures are verified for the ability to reduce the spot price risk over twelve months. The success of each model in risk reduction is measured over the test period by a conventional tool and across the models by proper metric. The results of the analysis confirm high level of risk reduction by crude oil across models. On the contrary, the results of hedging in natural gas significantly lag in comparison to crude oil. In addition, the analysis confirms a strong variability over the tested period and models.

Keywords: Risk, Hedging, Futures, Portfolio, Minimum variance

INTRODUCTION

The development of modern society is to a large extent dependent on available energy, especially on fossil fuels. In the recent decades markets of energy commodities are characterized by a process of liberalization. This, together with energy markets endogenous properties, causes a high price instability with a wide range of impacts. Therefore, a protection against price risk is in general interest, and above all in the interest of the concerned business entities.

There are numerous studies that focus on crude oil or natural gas hedging. Some studies investigate the suitability of particular econometric models for hedging. However, the standard measurement is limited to quantifying the risk reduction at a given time. The number of studies providing the mutual comparison of hedging in these two commodities is rather modest.

This paper examines four models (naive portfolio, OLS, Copula and GARCH) in term to reduce the risk of spot prices. The ability to reduce risk is determined by the hedge ratio that is produced by the employed models. The novelty of the paper is, on the one hand, in the way of expressing the cumulative risk across the models and, on the other hand, in the mutual comparison between the investigated commodities.

We have explored representative benchmarks for crude oil and natural gas. The examined markets are West Texas Intermediate (WTI) and Henry Hub (HH). The intention was to compare both commodities from the hedging perspective and select the optimal hedging tool for each of them. In this way, we would like to answer the research question whether a more complex model is more suitable for hedging than a naive portfolio.

The US markets were chosen for practical reasons. These markets are from the same institutional

environment, the prices on both markets are based on the interaction of demand and supply (especially for the price of natural gas it is not always so) and finally, the prices are in the same currency.

Literature review

The interest in hedging in the academia dates back to the early 20th century. The primary research was focused on the protection against risk in agricultural products Howell *et al.* (1938). The classical derivatives, especially futures, were used as hedging instruments Yamey (1951). A naive portfolio was employed as the first model for hedging purposes, i.e. one unit of spot was hedge by one unit of futures Graf (1953). So the eventual spot price slump could be replaced by the income of the short position held in the futures Howell (1948). The fundamental change in finance was due to the emergence of the modern portfolio theory devised by Markowitz (1952). The optimization technique introduced by Harry Markowitz was transferred to hedging Telser (1955). Hence, since this time more sophisticated methods could be used to over perform the naive portfolio. The portfolio variance as the objective function to find the optimal proportional weights Johnson (1960). The author introduced the metrics for measuring hedging performance as well. Concretely, the measurement was based on Pearson's correlation coefficient. Later, Ederington was mentioned as the author who introduced the measurement of hedging effectiveness (1979). He brought some innovation, because he used the classical regression model to determine the weights of futures, which corresponded to the slope of regression. However, the determination of weights by optimizing a portfolio variance as well as the measurement based on the coefficient of determinants are in fact identical with the approach of Johnson (1960). Later, with the advancement of econometrics and the discovery of models for unstable covariance, more sophisticated models to find the weights for hedging instruments were employed. These approaches include, for instance, the GARCH model, which was applied for hedging purposes in six commodities Baillie *et al.* (1991). Moshini *et al.* (2002) then introduced a new multivariate Garch parametrization. A better protection against price risk was confirmed using the model with variable volatility also in Yang *et al.* (2005) and Lee *et al.* (2007). Since, the OLS model is based on strong assumptions which cannot always be met, especially in the financial data, other models were also proposed based not only on the process of auto-regression and moving averages. An example might be a model based on the joint distribution. The application of a copula function for hedging purposes was demonstrated by Cherubini *et al.* (2004). Further, hedging with copula was used also in Hsu *et al.* (2008) or Lee (2009). The copula approach in energy commodity was applied in the work of Geman *et al.* (2008) by WTI. Although

she did not analyze the market for hedging purposes but the appropriateness of futures for portfolio diversification. The application of hedging on this market was investigated in Cotter *et al.* (2012). The paper focused on hedging effectiveness using the concept of risk aversion. In the article of Chan *et al.* (2011), the authors examined multivariate GARCH on WTI and Brent to find the optimal hedge ratio.

The research in the hedging of the US natural gas was provided in Ghoddusi *et al.* (2017). They employed cointegration and took the effect of maturity into account. With the liberalization of energy markets in Europe by increasing data availability some studies emerge from this region as well, such as Martínez *et al.* (2015).

If an appropriate underlying asset does not exist, then cross-hedging could be suitably used. This is the case in Woo *et al.* (2006) who provided cross hedging on natural gas in California. Similarly, Turner *et al.* (2015) used cross-hedging for jet fuel.

MATERIALS AND METHODS

Given the complexity of physical delivery we solely assume a financial form of hedging. Furthermore, we focus on short hedging. In other words the object for hedging is the spot price that means we expect to sell the commodity in any time in the future for actual spot price. However, an application of long hedging would be analogous.

In our analysis three econometric models (OLS, Copula, GARCH) and a naive portfolio are applied to obtain the optimal hedge ratio. To achieve the mutual ratio of spot and futures, the methodology in accordance with Johnson (1960) will be employed.

Thus, the objective function is given by a portfolio variance:

$$\sigma_p^2 = w_s^2 \sigma_s^2 + w_f^2 \sigma_f^2 + 2w_s w_f \sigma_{s,f} \quad (1)$$

Where, σ_p^2 is the portfolio variance, σ_s^2 and σ_f^2 is variance of spot and futures returns, respectively. The term $\sigma_{s,f}$ is covariance of spot price and futures price with weights w_s and w_f . It is assumed $w_s = 1$.

Then the optimal ratio $h^* = \frac{w_f}{w_s}$ must be obtained:

$$\frac{\partial \sigma_p^2}{\partial h} = 2h^* \sigma_f^2 + 2\sigma_{s,f} \quad h^* = -\frac{\sigma_{s,f}}{\sigma_f^2} \quad (2)$$

The negative weights indicate an opposite transaction to the spot, in our case to sell futures and hence we speak about short hedging. To confirm that the extreme of the objective function is minimum it should be provided the second derivation, which is really more than zero, because the second derivation with respect to the w_f is $2\sigma_f^2$ so it is a minimum.

The expression $\frac{\sigma_{s,f}}{\sigma_f^2}$ represents the slope of a linear regression. If we assume that r_{st} and r_{ft}

are the logarithmic returns of the spot and futures closing prices, then the simple linear regression is expressed as:

$$r_{st} = \alpha + hr_{ft} + \varepsilon_t \quad (3)$$

The standard errors of regression ε_t have to meet the assumptions that every residual is independent and identically distributed.

In the model with autoregressive memory we apply the model GARCH(1,1). According to Engle *et al.* (1995), here the optimal ratio is given by following expression:

$$r_{st} = \mu_s + \varepsilon_{st} \quad (4)$$

$$r_{ft} = \mu_f + \varepsilon_{ft} \quad (5)$$

$$\begin{bmatrix} \varepsilon_{st} \\ \varepsilon_{ft} \end{bmatrix} | \Omega_{t-1} \sim (0, H), \text{ and } H_t = \begin{bmatrix} h_{ss,t}^2 & h_{fs,t}^2 \\ h_{fs,t}^2 & h_{ff,t}^2 \end{bmatrix} \quad (6)$$

Here, μ_s and μ_f represent means of spot and futures returns, respectively. The standard errors for both regressions are dependent on the information in previous period. The H_t represents a conditional covariance matrix of errors. Then, the optimal hedge ratio can be derived accordingly:

$$h^* = \frac{h_{fs,t}^2}{h_{ff,t}^2} \quad (7)$$

The last methodology was based on utilization of joint distribution of spot and futures prices. For this purpose the copula approach was implemented. The origin of the concept is associated with the Sklar's theorem and dates back to the 50s of the last century. According to Sklar's theorem, a function called copula can join multiple distribution functions to the one-dimensional joint marginal distribution function Nelsen (1991).

Let us have cumulative distribution functions $F(x)$ and $G(y)$. Using a copula function $C(\cdot)$ the joint distribution function $H(\cdot)$ could be expressed in following manner:

$$H(x, y) = C(F(x), G(y)) = C(u, v) \quad (8)$$

And it is possible to derive the following relationship:

$$H(u, v) = H(F^{-1}(u), F^{-1}(v)) \quad (9)$$

Where F^{-1} and G^{-1} are the quantile functions of $F(\cdot)$ and $G(\cdot)$.

After fitting the models, the t-copula was identified as the most appropriate from the elliptical copula family to correspond to the empirical distribution of spot and futures prices.

The formula for t-copula function could be inferred from the following:

$$f_{\tau}^t = \frac{\Gamma(\frac{\tau+1}{2})}{\sqrt{\pi\tau}\Gamma(\frac{\tau}{2})} (1 + \frac{x^2}{\tau})^{-\frac{\tau+1}{2}}, -\infty < x < \infty \quad (10)$$

Here, f_{τ}^t , is student's t probability density function, τ represents the Gamma distribution and is the degree of freedoms and it holds $\tau > 0$. Thus, the t-copula $C^t(\cdot)$ could be expressed as:

$$C_{\delta, \tau}^t(u, v) = \frac{f_{\delta, \tau}^t(F_{\tau}^{-1}(u), F_{\tau}^{-1}(v))}{f_{\tau}^t(F_{\tau}^{-1}(u))f_{\tau}^t(F_{\tau}^{-1}(v))}, u, v \in (0, 1) \quad (11)$$

Where, $f_{\delta, \tau}^t$ stands for the joint density function.

To assess which model provides the best performance with regard to risk reduction, we will measure the hedging effectiveness (HE) according to Johnson (1960). So, the reduction of spot variance corresponds to:

$$HE = 1 - \frac{\sigma_p^2}{\sigma_s^2} \quad (12)$$

Where, σ_s^2 is variance of spot and σ_p^2 is variance of portfolio (combination of spot and futures). Another view on the risk may be the absolute value of a deviation from a representative value. If the representative value is equal to one, then it is possible to see the results as the quantification of the residual risk. Moreover, a cumulative sum of the partial results can select the best model over the whole observed period of the twelve months, which could be expressed in following way:

$$\sum_{i=1}^{12} |1 - HE_i| \quad (13)$$

For our analysis weekly closing prices of WTI and HH were employed. The investigated period was 2012–2014, together 185 observations. Afterwards, the results of the calculated weights were tested on the consequent period of one year. That is, from 2014 to 2015, 52 weeks in total. After, the verification was provided on monthly bases.

RESULTS AND DISCUSSION

The first step in the analysis was calculating the hedge ratio from the intended data. Three

econometric models were applied to estimate the weight for futures. The first one, was linear regression, where the logarithms of spot prices were the dependent variables and the logarithms of futures prices were considered as the independent variables. Further, the models with variable volatility were processed. Subsequently, the hedge ratio was calculated from the conditional covariance matrix of residuals.

The last model assumes a selection of the appropriate copula function. The t-copula was chosen for our datasets. Subsequently, an estimate of tightness parameters was performed. Then data simulation was implemented. The simulation was done with 200 iterations. The final step was to estimate the hedge ratio according to eq. (2) from the simulated date.

The weights for crude oil and natural gas are presented in the Tab. I.

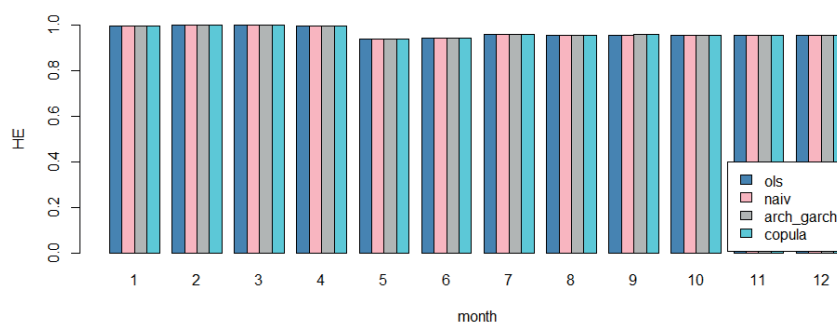
As apparent, both commodities evince significant differences. The cardinal finding is that the weights in futures of crude oil are essentially the same for all three econometric models, which corresponds to the weight of the naive portfolio. However, the weights for natural gas are significantly different and vary from 0, 59 in GARCH model to 1 in the naive portfolio. On average, the futures weights of natural gas are considerably lower than the weights of crude oil futures.

Subsequent testing confirmed the ability of crude oil futures to provide a nearly perfect hedging (Fig. 1). On the other hand, the hedging performance on the market of natural gas greatly lagged behind (Fig. 2). Furthermore, the ability to provide price risk reduction in the case of natural gas varies during the tested period.

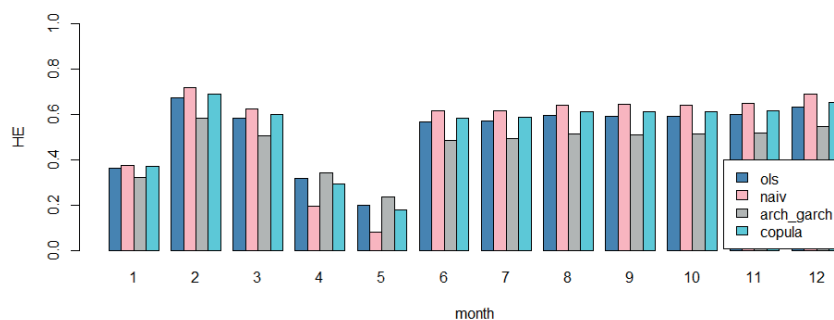
As seen from Fig. 2, the hedging effectiveness in natural gas was relatively low in the first, fourth and fifth months. The drop in the hedging performance

I: *Weights for all models and commodities*

WTI	h*	HH	h*
ols	1,00097	ols	0,773418
naive	1	naiv	1
garch	0,997769	garch	0,589148
copula	0,998992	copula	0,832684



1: *WTI – Hedging effectiveness by models over twelve months*



2: *HH – Hedging effectiveness by models over twelve months*

is caused by a significant reduction in the correlation between the spot and futures returns. This circumstance is captured in Fig. 5. The HE parameter achieved on average 60% of variance reduction in the remaining months. With regard to the established research question, paradoxically, the best results according to the summed residual risk were on average provided by the naïve portfolio.

On the contrary, the worst results on average were given by GARCH. However, it is crucial to note that the performance of this model was the greatest in the weak months.

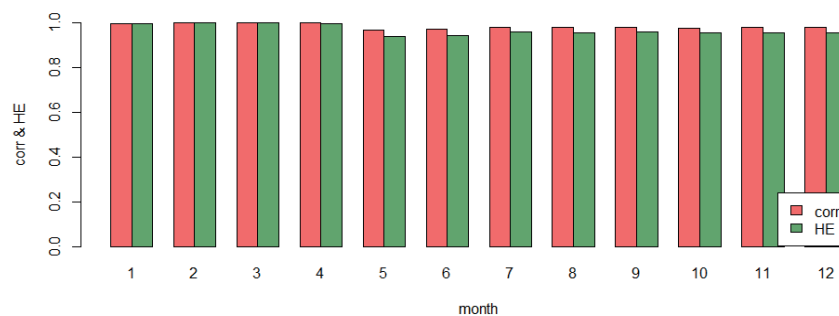
The results of crude oil basically confirmed irrelevance between the four employed models in respect to the price risk reduction, since the obtained weights do not differ. Also, the hedging

II: Cumulative residual variances by applied models

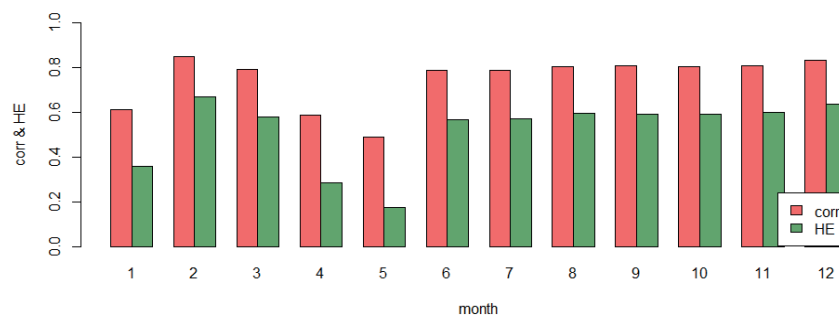
	ols	naive	garch	copula
WTI	0,383723	0,382911	0,381134	0,382093
HH	5,71468	5,505078	6,430357	5,588443



3: The hedging effectiveness over 12 months



4: WTI – Relation between correlation and HE



5: HH – Relation between correlation and HE

effectiveness of the employed models is almost the same over all the months. But the situation is distinct in the market of natural gas. Here, it is reasonable to select an optimal model, as the performance of HE is significantly different over the examined twelve months. The fact is obvious in Tab. II, where the cumulative differences of residual variance are illustrated.

The remaining low risk after futures hedging in the crude oil market confirms a high success rate of the applied hedging. The results of hedging on the natural gas market indicate a significantly lower potential for risk reduction. Moreover, the hedging effectiveness in particular months fluctuates significantly. Noticeable differences also show the remaining risk.

An overall summary and comparison of both investigated commodities is depicted in the Fig. 3.

The level of interdependence between spot and futures prices is fundamental for the hedging performance. This fact is shown in Fig. 4 and Fig. 5, where the correlation with the average HE is depicted.

As illustrated, the correlation between spot and futures in crude oil was very

strong during the examined period. This consequence is also reflected in the high level of hedging performance. However, this does not apply to the spot and futures prices of natural gas. The level of correlation is lower compared to crude oil and additionally the correlation fluctuates. The pattern of correlation is clearly displayed in the hedging effectiveness Fig. 5.

For further research it might be convenient to estimate the optimal hedge ratio with other models, like Mean Extended Gini coefficient or Cointegration, and compare them to each other. It would also be advisable to examine a larger sample of data and apply a dynamic approach to hedging. The fundamental limitation of this research is its statistical character, i.e. the results are determined by the information from the analyzed time period. Since it is known that financial time series show a dynamic development, the results cannot be generalized.

CONCLUSIONS

The paper investigated the issue of hedging against price risk on the markets for the two most important fossil fuels. Both commodities are traded on the market, where the price is largely determined by the interaction of supply and demand and is very sensitive to many other factors. Therefore market participants are exposed to eminent price uncertainty. It can be expected, that with growing market liberalization the price risk may increase even more.

The hedging issue was examined on the example of US markets, which can be considered as benchmark for crude oil as well as for natural gas. The methodology included the minimum-variance approach and the weekly closing prices were analyzed. To determine the optimal hedge ratio, four distinct models were employed, namely OLS, naive portfolio, GARCH and copula.

Due to its high liquidity, the crude oil market evinced strong coherence between the spot and futures prices and therefore the technique of naive portfolio can provide a very efficient protection against price risk. In contrast, the market of natural gas demonstrated that it is eligible to the selection of a convenient tool for finding weights. Nevertheless, the best model over the examined period of twelve month was paradoxically also the naive portfolio. However, in the period of on average low hedging performance the most powerful model was GARCH.

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