

# USING HMM APPROACH FOR ASSESSING QUALITY OF VALUE AT RISK ESTIMATION: EVIDENCE FROM PSE LISTED COMPANY

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## Abstract

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The article points out the possibilities of using Hidden Markov model (abbrev. HMM) for estimation of Value at Risk metrics (abbrev. VaR) in sample. For the illustration we use data of the company listed on Prague Stock Exchange in range from January 2011 to June 2016. HMM approach allows us to classify time series into different states based on their development characteristic. Due to a deeper shortage of existing domestic results or comparison studies with advanced volatility governed VaR forecasts we tested HMM with univariate ARMA-GARCH model based VaR estimates. The common testing via Kupiec and Christoffersen procedures offer generalization that HMM model performs better than volatility based VaR estimation technique in terms of accuracy, even with the simpler HMM with normal-mixture distribution against previously used GARCH with many types of non-normal innovations.

Keywords: Hidden Markov model, Christoffersen duration test, Kupiec test, Value at Risk, ARMA-GARCH-GJR

## INTRODUCTION

The increased level of financial or market risks and uncertainties is leading towards continuous development of more and more sophisticated methods serving for more accurate risk measurement and its management. This process is in motion via financial institutions, regulators and academic public. One of the well-established approaches created for this purpose is the used metric Value at Risk (abbrev. VaR) introduced in 1993, which measures the maximal possible loss on the given confidence level during the specific time period.

The previous articles, like Krause (2003) shown that VaR is not unproblematic to use, it is not a coherent risk measure, its estimation is subject to large errors, the estimate is downward biased, and these shortcomings can be exploited by individuals within the company as well as the company as a whole. The VaR estimate gives only a risk

assessment of the investment under normal market conditions. Extreme events like a financial crashes, or systemic failures, are really problematic to capture.

It does not indicate potential losses, and as a result is flawed, even on its own terms. Its dependence on a single quantile of the profit and loss distribution implies it is easy to manipulate reported VaR with specially crafted trading strategies. For regulatory use, the VaR measure may give misleading information about risk, and in some cases may actually increase both idiosyncratic and systemic risk. The basic statistical properties of market data are not the same in crisis as they are during stable periods; therefore, most risk models provide very little guidance during crisis periods. In other words, risk properties of market data change with observation as Daníelson (2002) stated.

Hidden Markov Models (abbrev. HMM) can be classified under the more general classification of Markov regime-switching models, wherein the  $m$

states of the HMM correspond to the  $m$  regimes of the regime-switching models.

HMMs were previously mostly found among others in fields as biophysics (ion channel modeling), earth and environmental sciences (wind direction, climate change), temporal pattern recognition (facial, gesture, speech, handwriting, etc.), engineering (speech and signal processing), bioinformatics (biological sequencing). Its main use is to describe nonlinear trends in the time series and classify different regimes according to characteristics and variability of time series. A further simplification that is adopted by this paper is to assume that the distribution in each regime is that of the Gaussian distribution.

The remainder of this article is structured as follows: section Motivation describes the aim of the paper, section Material and Methods presents the data and econometric techniques used for capturing Value at Risk measures, the next sections present and discuss the results. Conclusions are presented in last section.

### Motivation and Contributions

The aim of this paper is to test the ability in VaR estimation, as it is important metric in financial industry, with HMM model in comparison to selected univariate ARMA(1,1)-GARCH(1,1)-GJR<sup>1</sup> models in similar way to Khaled *et al.* (2016) who backtested HMM and GARCH models with different distributions. Our data consists of one company for better readability of results (output could be generalized even for portfolio case) listed on the Prague Stock Exchange (PSE): ČEZ, a. s. from January 2011 to June 2016.<sup>2</sup>

Similarly to the work of Angelidis, Benos & Degiannakis (2004) or Klepáč & Hampel (2015) this paper uses the conditional and unconditional coverage framework for VaR testing. The results supply literature with at least two specific contributions: we tell if there exists significant difference in VaR estimates between static univariate volatility based model and HMM approach.

We used data of ČEZ, a.s. company, when other authors (see Khaled *et al.* (2016) mostly used data of more traded companies – from our previous knowledge (like in Klepáč & Hampel (2015) or in Kresta (2011)) we know that is possible to use univariate or multivariate models even for portfolio VaR estimation with stable or variable portfolio weights. Presented results then offer heavier concentration (unpublished for PSE listed company) on the HMM with respect to particular univariate VaR estimation coherence.

### MATERIAL AND METHODS

We process data of time series of closing stock prices for company: ČEZ, a. s. company, which were obtained through Patria Online database, e. g. it contains data from Prague XETRA system, denominated in CZK. In Tab. I are shown the basic statistics of logarithmical daily returns. We use data for the time period from January 2011 to June 2016. We also present returns stream consisting of mentioned company, see Fig. 1 in Results section.

I: Summary statistics for logarithmical daily returns

| Statistics               | Values   |
|--------------------------|----------|
| Minimum                  | -0.1285  |
| 1 <sup>st</sup> Quartile | -0.01292 |
| Median                   | 0.000395 |
| Mean                     | -0.00094 |
| 3 <sup>rd</sup> Quartile | 0.0121   |
| Maximum                  | 0.08194  |

Because of the great importance of the quality of loss forecasts in financial industry there were developed backtesting (we use these methods with in sample data) procedures which validate use of the VaR estimators, see Christoffersen (1998) and Kupiec (1995) for back testing of results.

### Representation of volatility models

As description about creating ARMA-GARCH(1,1) type model to establish univariate volatility models with mean  $\mu_t$  process can be used

$$X_t = \mu_t + \sqrt{h_t} z_t,$$

where  $z_t$  is draw from inverse cumulative distribution,  $h_t$  is conditional variance,  $\mu_t$  is conditional mean process,  $X_t$  is a one dimensional vector of returns. In this paper we use GARCH(1,1)-GJR specification according to Glosten, Jagannathan and Runkle (1993) for  $h_t$

$$h_t = \alpha_0 + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^q \beta_i h_{t-i} + \gamma_j \varepsilon_{t-1}^2 I_{t-1}$$

where  $I_{t-1} = 1$  if  $\varepsilon_{t-1} < 0$ , 0 if not. With the condition

$$\alpha_0 > 0, \alpha_i, \beta_i \text{ and } \sum_{i=1}^p \alpha_i + \sum_{i=1}^q \beta_i + 0.5\gamma < 1.$$

### Representation of Hidden Markov Model

A Hidden Markov Model (HMM) is a statistical model of a sequence of observed random variables whose distribution depends on another sequence of

1 In the text there is for a better readability often used abbreviation ARMA-GARCH without the order of the univariate model.

2 The computations were performed with R version 3.1.0 on basic data set of 1380 observations for particular time series of log returns.

underlying unobserved random variables, the latter being governed by a Markov process. A Markov chain is useful when we need to compute a probability for a sequence of events that we can observe in the world. In many cases, however, the events we are interested in may not be directly observable in the world. A hidden Markov model (HMM) allows us to talk about both observed Hidden Markov model events (like returns that we see in the input) and Hidden events (like probabilities of particular returns regime) that we think of as causal factors in our probabilistic mode.

Rather, we can only observe some outcome generated by each state. Formally, an HMM is a Markov model for which we have a series of observed outputs  $x = \{x_1; x_2; \dots; x_T\}$  drawn from an output alphabet  $V = \{v_1; v_2; \dots; v_{|V|}\}$ , i. e.  $x_t \in V$ ;  $t = 1..T$ . As in the previous section, we also posit the existence of series of states  $z = \{z_1; z_2; \dots; z_T\}$ , drawn from a state alphabet  $S = \{s_1; s_2; \dots; s_{|S|}\}$ ,  $z_t \in S$ ;  $t = 1..T$  but in this scenario the values of the states are *unobserved*. The transition between states  $i$  and  $j$  will again be represented by the corresponding value in our state transition matrix  $A_{ij}$ .

We also model the probability of generating an output observation as a function of our hidden state. To do so, we make the output independence assumption and define  $P(x_t = v_k | z_t = s_j) = P(x_t = v_k | x_1; \dots; x_{t-1}; z_1; \dots; z_{t-1}) = B_{jk}$ . The matrix  $B$  encodes the probability of our hidden state generating output  $v_k$  given that the state at the corresponding time was  $s_j$ .

In an HMM, we assume that our data was generated by the following process: i. e. the existence of a series of states  $\sim z$  over the length of our time series. This state sequence is generated by a Markov model parametrized by a state transition matrix  $A$ . At each time step  $t$ , we select an output  $x_t$  as a function of the state  $z_t$ . Therefore, to get the probability of a sequence of observations, we need to add up the likelihood of the data given  $\bar{x}$  every possible series of state. Probability of an observed sequence

$$P(\bar{x}; A, B) = \sum_{\bar{z}} P(\bar{x}, \bar{z}; A, B) = \sum_{\bar{z}} P(\bar{x} | \bar{z}; A, B) P(\bar{z}; A, B).$$

In the HMM there is a probability of transitioning between any two states, which holds true in our paper. Such an HMM is called a fully connected or ergodic HMM. Sometimes, however, we have HMMs in which many of the transitions be Bakis network states have zero probability. Bakis HMMs are generally used to model temporal processes like speech. Other technical details about parameters estimation are described in Baum (1966).

## Benchmark of results

To estimate VaR model we should proceed in steps provided by Khaled et al (2016) and Klepac and Hampel (2015).

- Fit of univariate ARMA(1,1)-GARCH(1,1)-GJR model and estimation of VaR. Various combinations of the values  $p$  and  $q$  in GARCH( $p, q$ ) model have only minor influence on the values of information criteria – in the order of magnitude of units of percents.
- Fit of HMM model for different number of states.<sup>3</sup>
- Evaluation of results for classification of different regimes by minimal AIC and BIC values.
- Simulation of 10 000 average returns by normal-mixture model density. Weighting selected returns (VaR quantiles) by stationary probabilities from HMM model (these probabilities are changing over time). Stationary probability of each state is defined as a number of time units in which the state remains the same.
- Estimation of VaR rates for 5<sup>th</sup> and 1<sup>st</sup> quartile of cumulative distribution function as 95% and 99% VaR.

VaR is the  $100(1-\alpha)^{\text{th}}$  quantile of the returns cumulative distribution function. As for the conditional approach, the VaR would be the  $100(1-\alpha)^{\text{th}}$  quantile of the distribution of the predicted value  $X$ .

- Benchmark of the results by Kupiec and Christoffersen tests.

Because of the great importance of the quality of loss estimations and forecasts in financial industry there were developed backtesting procedures which validate use of the VaR estimators. Within the backtesting procedure see Christoffersen (1998) for conditional approach and Kupiec (1995) for unconditional approach. Unconditional methods count the number of exceptions or violations, in point where the realized returns (loss) fall below VaR band. Conditional methods tests if the duration time between VaR violations is independent and without the clusters. This test tells us if there are some consecutive exceedances for some time interval. But based on particular null hypothesis of Christoffersen test we can access unconditional property too.

## RESULTS

After the calculation of price logarithmical difference, the estimate is performed via ARMA(1,1)-GARCH(1,1)-GJR for five different combinations of model settings. The value of the information criteria is fundamentally influenced by the distribution of the variance process. From this point of view, the highest quality models contain innovations from Student-t distribution.

3 GARCH(1,1)-GJR performed better than the others (i. e. EGARCH, standard GARCH etc.) in our previous research.

Otherwise, the normal distribution cannot be preferred in statistical reasoning for any of the tested models. The partial changes in parameters values have mostly impact on the t-values and the full statistical verification of tested model. The testing of applicability and selection of particular HMM models via minimal values AIC a BIC points to several common phenomena, see Tab. II, Tab. III and Tab. IV for quantitative results. Due to the results we could concern the 4-state HMM with minimal AIC and BIC values.

Tab. III shows that there is great probability that the time series in State 1 would remain in the same state – from one day to another. The highest probability of change between two regimes is from State 2 to State 3, and from State 2 to State 4. Contrary to that it is highly unlikely to observe change from State 1 to State 2 (from State 4 to State 3 and vice versa).

The EM algorithm gives the values of the estimates of the parameters for the 4-state HMM, see Tab. V.

Four normal density functions (weighted by the stationary probabilities in Tab. V) that make up fitted mixed distribution. The resulting normal-mixture density for the return  $x$  is given

$$f(x) = 0.1199\phi(x; \mu = 0.0099, \sigma = 0.0391) + \\ + 0.5238\phi(x; \mu = -0.0043, \sigma = 0.0101) + \\ + 0.2595\phi(x; \mu = -0.0283, \sigma = 0.01222) + \\ + 0.0968\phi(x; \mu = 0.0183, \sigma = 0.0117)$$

where  $\mu$  and  $\sigma$  are parameters of returns distribution. Fig. 1 can help us interpret the four different regimes suggested by the HMM. The first state refers to those times where volatility rose.

On the Fig. 1 we watch the development of the logarithmical yields from January 2011 to June 2016. On the representation of states, we see that extreme positions, that are state 1 as negative and state 4 as positive, are in periods with significant market oscillations. It is certainly true, current market oscillations are bounded by states 2 and 3. So the values following out of the equation are evident and the most usual state is the second one.

### Value at Risk estimation and testing of results

For details about the VaR exceedance and visual fit to empirical returns with testing results, see Fig. 2 and Fig. 3<sup>4</sup>, Tab. VI and Tab. VII.

#### II: Evaluation of HMM model – selection of best fitting specification

| Number of states | logLik  | BIC     | AIC     | df |
|------------------|---------|---------|---------|----|
| 2-state HMM      | 3413.96 | -6777.3 | -6813.9 | 7  |
| 3-state HMM      | 3464.18 | -6827.1 | -6900.4 | 14 |
| 4-state HMM      | 3502.25 | -6838.2 | -6958.5 | 23 |
| 5-state HMM      | 3481.67 | -6805.8 | -6914.4 | 34 |

#### III: Transition matrix for 4-state HMM model

| From/To | State 1 | State 2 | State 3 | State 4 |
|---------|---------|---------|---------|---------|
| State 1 | 76.17%  | 3.76%   | 11.84%  | 8.23%   |
| State 2 | 0.00%   | 73.48%  | 7.57%   | 18.95%  |
| State 3 | 10.39%  | 42.30%  | 47.30%  | 0.01%   |
| State 4 | 6.91%   | 34.05%  | 0.03%   | 59.01%  |

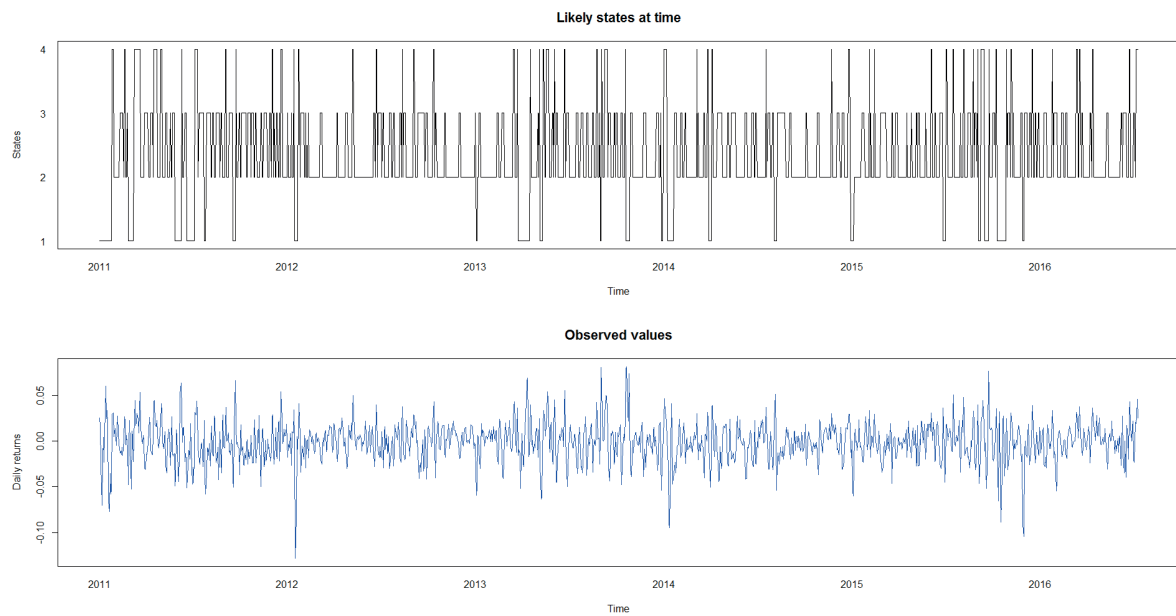
#### IV: Model parameters for each state

| State   | Intercept | Standard deviation |
|---------|-----------|--------------------|
| State 1 | 0.0099    | 0.0391             |
| State 2 | -0.0043   | 0.0101             |
| State 3 | -0.0283   | 0.0122             |
| State 4 | 0.0183    | 0.0117             |

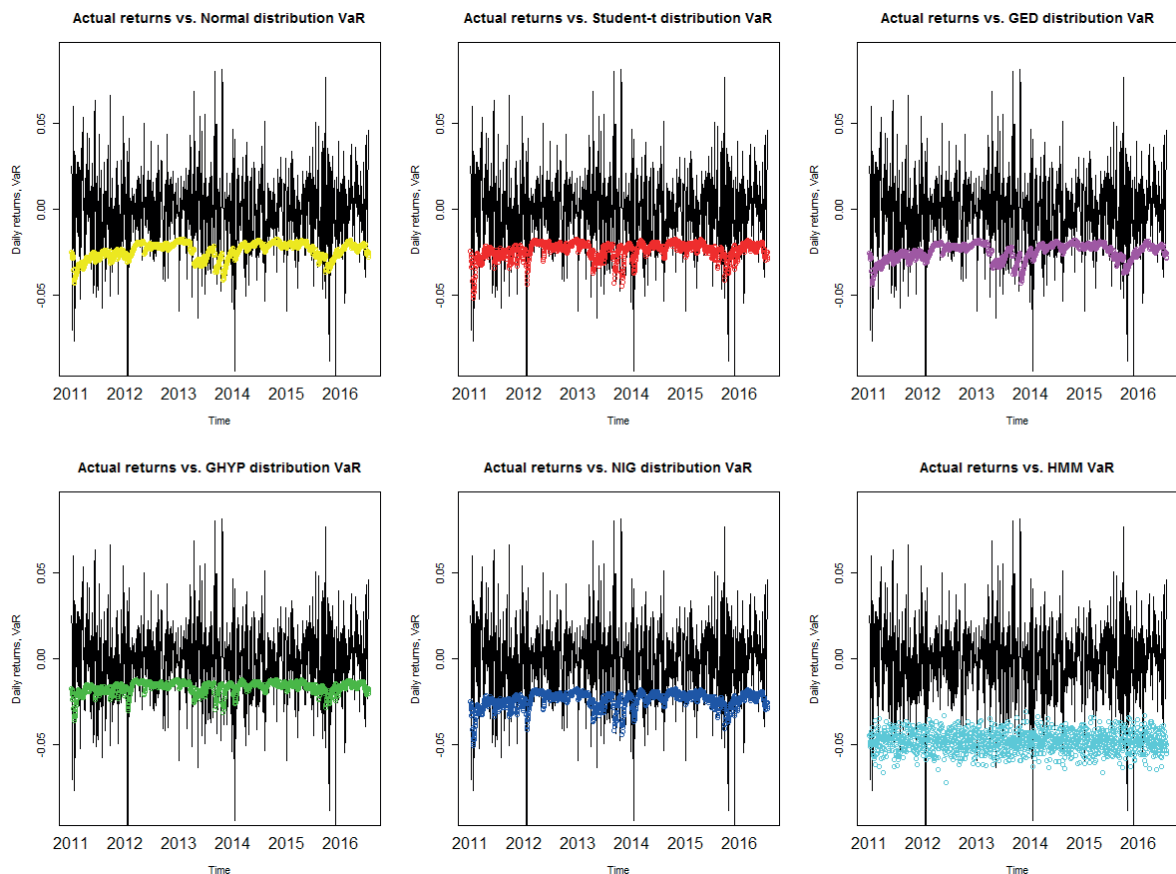
#### V: Stationary probabilities for 4-state HMM model

| States  | Stationary probability |
|---------|------------------------|
| State 1 | 11.99%                 |
| State 2 | 52.37%                 |
| State 3 | 25.95%                 |
| State 4 | 9.67%                  |

4 VaR estimate is presented as a colored line. Daily returns are visualized as black lines.

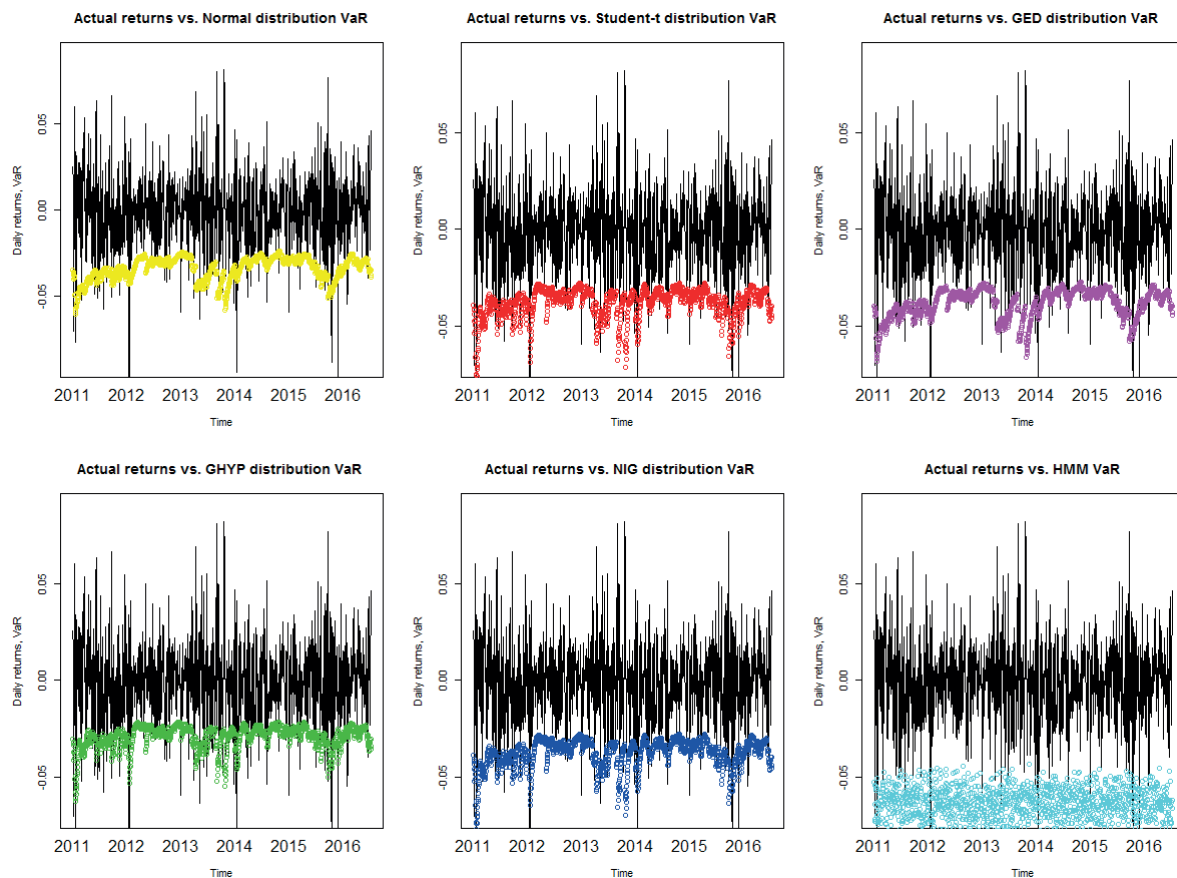


1: Realized returns and four different states across time (from January 2011 to June 2016)



2: VaR estimation by 5 GARCH based models (normal, Student-t, GED, G. hyperbolic and NIG distributions) and HMM model for 95 % confidence level and realized returns (from January 2011 to June 2016)





3: VaR estimation by 5 GARCH based models (normal, Student-t, GED, G. hyperbolic and NIG distributions) and HMM model for 99% confidence level and realized returns (from January 2011 to June 2016)

According to the Kupiec and Christoffersen test the best model for fitting actual risk levels is simple HMM with 13 violations. Although the Kupiec test does reject ARMA(1,1)-GARCH(1,1), its numerical values are above the coverage rate for 95% confidence interval. After the hypothesis testing we know that there exists only one suitable modelling approach for selected confidence level: HMM method for univariate time series.

Kupiec's unconditional coverage test looks at whether the amount of expected versus actual exceedances given the tail probability of VaR actually occur as predicted, while the conditional coverage test of Christoffersen is a joint test of the unconditional coverage and the independence of the exceedances. Both the joint and the separate unconditional test are reported since it is always possible that the joint test passes while failing either the independence or unconditional coverage.

## DISCUSSION

HMMs were not previously used for VaR estimation with Czech listed companies, so we have not directly ability to compare results with GARCH based VaR estimates. Thus the main contribution lies in concerning statistically evaluated results for unique experimental setting.

Gaussian based models often perform to larger extent poorly, mainly due to the fact that the returns are not coming from Gaussian probability distributions. But in this case the HMM with normal-mixture model performs better than a ARMA(1,1)-GARCH(1,1)-GJR with all of the tested innovation distributions (Student-t, NIG, Normal, GED, hyperbolic). The reason of quality results is the mixing of distributions with the different input parameters. Though it is widely accepted that asset returns are not normally distributed, a mixture of Gaussians that results from HMM models do exhibit the skewness and leptokurtic characteristics (high peaks and fat tails) of such returns.

But so far from the actual results (i. e. Klepáč and Hampel 2015) we in general propose ARMA-GARCH-GJR with Student-t, GED or NIG distribution as appropriate way to forecast portfolio VaR when we won't work with dependence between return streams. After the testing we can take that as possible fact. But obviously the obtained results could be further enhanced via testing of accuracy on different levels of significance or in forecasting out of sample for different time horizons.

VI: Evaluation of estimation quality according to Kupiec and Christoffersen tests (95 % confidence level)

| Findings             | ARMA-GARCH and distribution type |     |        |           |            | HMM |
|----------------------|----------------------------------|-----|--------|-----------|------------|-----|
|                      | NIG                              | GED | NORMAL | STUDENT-T | HYPERBOLIC |     |
| Expected exceedances | 69                               | 69  | 69     | 69        | 69         | 69  |
| Realized exceedances | 170                              | 156 | 162    | 172       | 284        | 35  |
| Kupiec test          | x                                | x   | x      | x         | x          | x   |
| Christoffersen test  | x                                | x   | x      | x         | x          | x   |

VII: Evaluation of estimation quality according to Kupiec and Christoffersen tests (99 % confidence level)

| Findings             | ARMA-GARCH and distribution type |     |        |           |            | HMM |
|----------------------|----------------------------------|-----|--------|-----------|------------|-----|
|                      | NIG                              | GED | NORMAL | STUDENT-T | HYPERBOLIC |     |
| Expected exceedances | 13                               | 13  | 13     | 13        | 13         | 13  |
| Realized exceedances | 59                               | 58  | 58     | 79        | 111        | 13  |
| Kupiec test          | x                                | x   | x      | x         | x          | ok  |
| Christoffersen test  | x                                | x   | x      | x         | x          | ok  |

## CONCLUSION

This contribution deals with the application of volatility models and Hidden Markov model with normal-mixture densities and its testing when estimating Value at Risk based on the data of ČEZ, a. s. company. Particular time interval of data used is from January 2011 to June 2016. The variant comparison of five combinations of univariate ARMA-GARCH models led us to the conclusion that the simplest ARMA-GARCH fits the data well graphically. The results for HMM VaR point at the less ability to capture the current fluctuations, but provide the limits of VaR metric, which statistically correspond better to the level of significance.

The results according to testing indicate the underestimation of the risk level for almost all of the model approaches, with exception of HMM model. Kupiec and Christoffersen test proposed that the HMM model is efficient in capturing VaR metric. This model is also the only case of model in which the violations are independently distributed in time. The outputs of this research can be further developed via testing of accuracy on different levels of significance in terms of out of sample forecasting. Despite its simplicity, HMMs perform fairly well in modelling processes with varying system-behavior.

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