

OSTRICH EGGS GEOMETRY

Šárka Nedomová, Jaroslav Buchar

Received: February 3, 2013

Abstract

NEDOMOVÁ, ŠÁRKA, BUCHAR, JAROSLAV: *Ostrich eggs geometry*. Acta Universitatis Agriculturae et Silviculturae Mendelianae Brunensis, 2013, LXI, No. 3, pp. 735–742

Precise quantification of the profile of egg can provide a powerful tool for the analysis of egg shape for various biological problems. A new approach to the geometry of a Ostrich's egg profile is presented here using an analysing the egg's digital photo by edge detection techniques. The obtained points on the eggshell counter are fitted by the Fourier series. The obtained equations describing an egg profile have been used to calculate radii of curvature. The radii of the curvature at the important point of the egg profile (sharp end, blunt end and maximum thickness) are independent on the egg shape index. The exact values of the egg surface and the egg volume have been obtained. These quantities are also independent on the egg shape index. These quantities can be successively estimated on the basis of simplified equations which are expressed in terms of the egg length, L , and its width, B . The surface area of the eggshells also exhibits good correlation with the egg long circumference length. Some limitations of the most used procedures have been also shown.

Ostrich's egg, eggshell profile, radii of the curvature, egg volume, egg surface

Mathematical description of an egg profile allows to calculate the egg volume, surface area, long circumference length, normal projected area of the egg, radius of curvature and angle between the long axis and the tangent to the shell at any point. Most of achievements in this area has been obtained for the hen's eggs. It has been shown that the knowledge of the egg's shape is meaningful for the exact evaluation of mechanical testing results on the eggshell strength (Nedomová *et al.*, 2009) and generally for the numerical simulation of the egg behaviour under mechanical loading (Perianu *et al.*, 2010) and under different thermal treatments (Sabliov *et al.*, 2002; Denys *et al.*, 2003). Egg volume (V) and surface area (S) are two important geometrical calculations for the poultry industry and in biological studies, as they can be used in research on population and ecological morphology, and to predict chick weight, egg hatchability, shell quality characteristics, and egg interior parameters.

The high variability of shell egg shapes creates difficulties in their description. The classic paper of Romanoff and Romanoff (1949) postulated that it was impossible to express the contour of individual eggs in mathematical terms. The eggshell shape has

been described using of the shape index (SI) which is defined as

$$SI = \frac{B}{L} \times 100, (\%)$$

where B is the width and L the length of the eggs.

The geometric mean diameter of eggs is than calculated using the following equation given by (Mohsenin, 1970):

$$D_g = (LB^2)^{\frac{1}{3}}. \quad (1)$$

According to Mohsenin (1970), the degree of sphericity of eggs can be expressed as follows:

$$\Phi = \frac{D_g}{L} \times 100. (\%) \quad (2)$$

The surface area of eggs was calculated using the following relationship given by (Mohsenin, 1970; Baryeh and Mangope, 2003):

$$S = \pi D_g^2. \quad (3)$$

Volume of the egg is than given as

$$V = \frac{\pi}{6}LB^2. \quad (4)$$

Instead of this description, a number of authors have tried to derive mathematical equations that express the contour of individual eggs. References to these descriptions can be found in the review of Narushin (1997). Most of them assume the profile to be an ellipsoid or use modified ellipse equations to describe the egg profile.

The most popular equation used for the description of egg shape has been developed by Narushin (1997, 2001a). In his approach the egg profile is defined by only 2 geometrical parameters: length, L , and maximum width, B . The profile is described by the equation:

$$y = \pm \sqrt{L^{\frac{2}{n+1}} x^{\frac{2n}{n+1}} - x^2}, \quad (5)$$

in which

$$n = \left(\frac{L}{B}\right)^{2.372},$$

where x is the coordinate along the longitudinal axis and y the transverse distance to the profile.

The obtained mathematical descriptions of the egg profile can be used for the evaluation of the egg volume, surface area and some other parameters mentioned at the beginning of this chapter. The main results have been achieved by Narushin (2001b), Narushin and Romanov (2002a, 2002b). Narushin (2001a) showed that egg volume can also be estimated by means of a theoretically deduced formula:

$$V = \frac{2\pi L^3}{3(3n+1)}, \quad (6)$$

where parameter n is defined in the Eq. (5).

Later, Narushin (2005) gave a more accurate and available formula:

$$V = (0.6057 - 0.0018B)LB^2. \quad (7)$$

In this paper Narushin also derived the formula for the egg surface calculation:

$$S = (3.155 - 0.013L + 0.0115B)LB. \quad (8)$$

However, there are still at least two main problems: firstly, the supposed curve in the reported models will not always best resemble all eggs' shapes; secondly, if thus, the measurements of L and B with a vernier caliper can't be fast and automatic, which will not be acceptable in poultry industry. The second of these problems is solved by Zhou *et al.* (2009). In the given paper the main attention is focused on evaluation of the egg volume and surface using of the exact description of the egg profile. In order to achieve this exact description of the egg shape a new application is used. This method uses a graphical user interface (GUI), which allowed the user to accurately determine the necessary dimensional properties of eggs from digital photographs of the eggs. The application required one measured dimension (the egg length, L , measured with vernier calipers), and calculated any user-defined distance on a digital egg photograph from the derived number of pixels per unit length. Based on a user-defined 2-D cartesian coordinate system, the coordinates of the required points can be defined in a plane of symmetry.

The first aim of this paper consists in the evaluation of the eggshell shape of the Ostrich eggs from their digital photographs. In the second step a comparison between these data and data obtained from the Eqs. (1–9) has been performed.

MATERIAL AND PROCEDURE

Eggs were collected from a commercial packing station. In order to describe the shape of egg samples the linear dimensions, i.e. length (L) and width (W), were measured with a digital calliper to the nearest 0.01 mm. These quantities have been used for the evaluation of the shape index (SI). The corresponding geometrical characteristics are given in Tab. I.

In the second step the digital photos of the eggs have been performed. The image analysis performed using of the *Matlab* software has been used for the evaluation of the coordinates x_i and y_i of the egg contour. Instead of Cartesian coordinates the shape of the eggshell counter can be described using of the polar coordinates r , φ :

$$x = r \cos \varphi$$

$$y = r \sin \varphi$$

I: Main characteristics of the tested eggs. (std – standard deviation)

| | m [kg] | L [mm] | B [mm] | SI [%] | D _g [mm] | Φ [%] | V [mm ³] Eq. (7) | S [mm ²] Eq. (8) |
|---------|-----------|-----------|-----------|-----------|------------------------|----------|---------------------------------|---------------------------------|
| minimum | 1.15 | 137.15 | 115.62 | 74.48 | 122.39 | 82.16 | 959 976.0 | 42 841.3 |
| average | 1.355 | 146.582 | 120.56 | 82.49 | 128.64 | 87.93 | 1 118 934.6 | 46 497.3 |
| maximum | 1.55 | 166.64 | 124.86 | 89.72 | 136.92 | 93.03 | 1 343 975.7 | 49 965.8 |
| std | 0.142 | 10.074 | 3.582 | 4.343 | 4.901 | 3.10 | 129 113.7 | 2 630.8 |

The experimental points r_i, φ_i have been fitted by the Fourier series

$$r = a_0 + \sum_{i=1}^{i=\infty} [a_i \cos(iw\phi) + b_i \sin(iw\phi)]. \quad (9)$$

The knowledge of the functions $x(\varphi)$ and $y(\varphi)$ enables to evaluate of the radius of the curvature R :

$$R = \frac{\left[\left(\frac{dx}{d\varphi} \right)^2 + \left(\frac{dy}{d\varphi} \right)^2 \right]^{\frac{3}{2}}}{\left| \frac{dx}{d\varphi} \frac{d^2y}{d\varphi^2} - \frac{dy}{d\varphi} \frac{d^2x}{d\varphi^2} \right|}. \quad (10)$$

The volume V and surface S are given by

$$V = \pi \int_{\varphi_1}^{\varphi_2} r^2(\varphi) \sin^2 \varphi \frac{dx(\varphi)}{d\varphi} d\varphi$$

$$S = 2\pi \int_0^{\pi} r \sin \varphi \sqrt{\left(\frac{dx}{d\varphi} \right)^2 + \left(\frac{dy}{d\varphi} \right)^2} d\varphi, \quad (11)$$

the area A of the egg normal projection and the long circumference length, l :

$$A = \frac{1}{2} \oint r^2 d\phi \quad l = \oint ds = \oint r d\phi. \quad (12)$$

RESULTS AND DISCUSSION

The analysis of our data led to the conclusion that the first six up to eight coefficients of the Fourier series are quite sufficient for the egg's counter shape description (the correlation coefficient between measured and computed egg's profiles lies between 0.98 and 1). The coefficients are given in the Tab. II.

In the Fig. 1 an example of the egg's counter curve is shown.

The agreement between experimental and fitted egg's profile can be described using an error function which is defined as

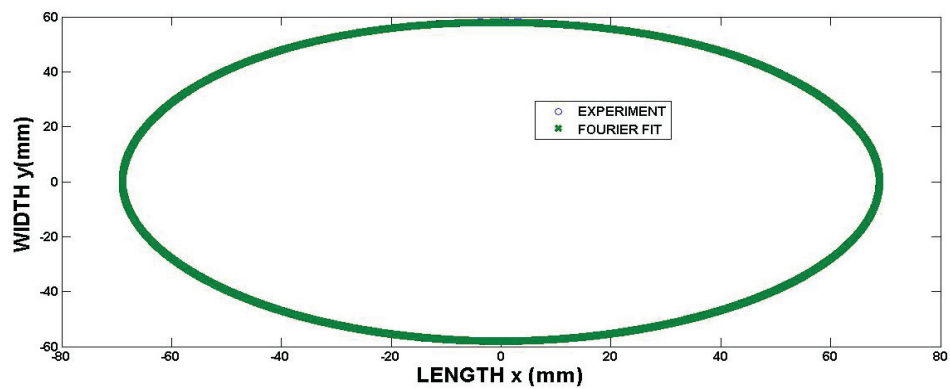
$$ERROR = \left| \frac{Y_{measured} - Y_{fitted}}{Y_{measured}} \right| \times 100, (\%)$$

where $Y_{measured}$ is the y co-ordinate determined from the digital photo and Y_{fitted} the y co-ordinate obtained from the Fourier series – see Eq. (9).

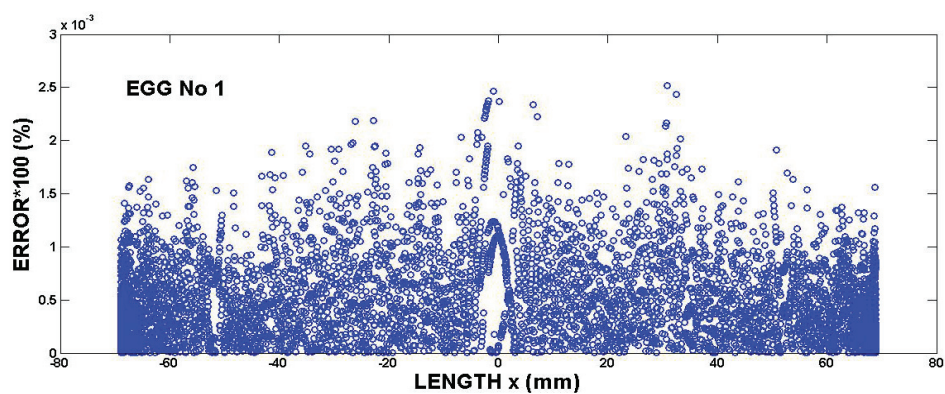
Very similar results have been obtained for all tested eggs. It is obvious that the differences are very small if not negligible. In the Fig. 3 the egg's profile given by the Eq. (5) is compared with that obtained from the digital photo. One can see that the difference is more pronounced than that shown in

II: Coefficients of the Fourier series

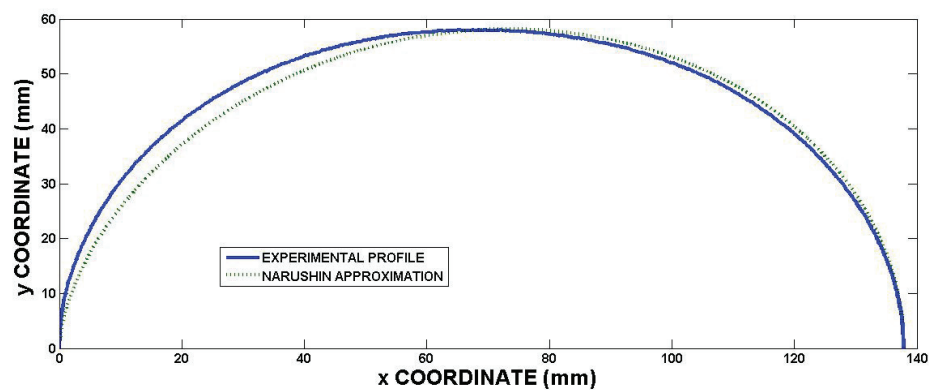
| Egg_No | a_0 | a_1 | b_1 | a_2 | b_2 | a_3 | b_3 | a_4 | b_4 |
|--------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 1 | 63.13689 | -0.19546 | 0.225877 | 5.372547 | -0.07358 | 0.128718 | 0.154154 | 0.313802 | -0.00282 |
| 2 | 66.56885 | 0.300117 | 0.027132 | 5.929132 | 0.085617 | -0.14182 | 0.04008 | 0.51173 | -0.10677 |
| 3 | 50.73033 | 22.85717 | -1.15525 | -19.0696 | 2.129682 | 19.84857 | -2.99582 | -9.69868 | 2.288464 |
| 4 | 71.37025 | 0.140756 | 0.03212 | 10.47467 | -0.09258 | 0.003623 | -0.147 | 1.197302 | -0.068 |
| 5 | 65.70903 | 0.276839 | -0.19931 | 1.020897 | -0.15595 | 3.399683 | 0.013307 | -1.18106 | -0.26877 |
| 6 | 70.2131 | 0.37217 | -0.22152 | 9.485627 | -0.77944 | -0.2717 | -0.34137 | 0.918253 | -0.08891 |
| 7 | 67.80732 | -0.14605 | 0.020608 | 6.684475 | -0.27875 | 0.106573 | 0.083806 | 0.430703 | 0.101108 |
| 8 | 64.98035 | 0.565131 | -0.22467 | 0.088386 | 0.313619 | 5.898278 | -0.92507 | -1.19718 | 0.467948 |
| 9 | 65.15723 | 0.6894 | 0.051137 | 5.938947 | 0.187649 | -0.4006 | 0.035651 | 0.258151 | -0.05 |
| 10 | 64.00693 | -0.24132 | 0.015934 | 6.457883 | 0.194503 | 0.121203 | 0.002379 | 0.545681 | 0.117943 |
| Egg_No | a_5 | b_5 | a_6 | b_6 | a_7 | b_7 | a_8 | b_8 | w |
| 1 | 0.013835 | -0.10479 | 0.081819 | -0.00122 | 0.004534 | -0.00268 | 0.020873 | 0.017912 | 0.992054 |
| 2 | -0.14781 | 0.058571 | 0.039159 | -0.04065 | -0.02322 | 0.022335 | -0.00922 | -0.00769 | 0.997793 |
| 3 | 5.297624 | -1.54296 | -2.05659 | 0.970964 | 0.779144 | -0.37309 | -0.1194 | 0.149654 | 0.668053 |
| 4 | -0.08852 | -0.08841 | 0.178835 | -0.04532 | -0.07801 | 0.003645 | 0.055074 | -0.02435 | 1.000199 |
| 5 | 0.548206 | -0.00102 | -0.20846 | 0.011856 | 0 | 0 | 0 | 0 | 0.746888 |
| 6 | -0.11637 | -0.1092 | 0.085447 | -0.02049 | -0.05613 | -0.03686 | 0.046839 | -0.01146 | 0.997336 |
| 7 | 0.067325 | 0.005786 | 0.046024 | 0.059788 | -0.00118 | -0.00467 | 0.007321 | 0.024787 | 0.995109 |
| 8 | 0.698802 | -0.29984 | 0.022704 | 0.116688 | 0 | 0 | 0 | 0 | 0.685622 |
| 9 | -0.22769 | -0.04806 | 0.033329 | -0.02908 | -0.02201 | -0.00554 | 0.019674 | 0.020664 | 1.008283 |
| 10 | 0.097874 | -0.10563 | 0.12346 | -0.02221 | -0.00737 | -0.01879 | 0.019121 | 0.035507 | 0.993507 |



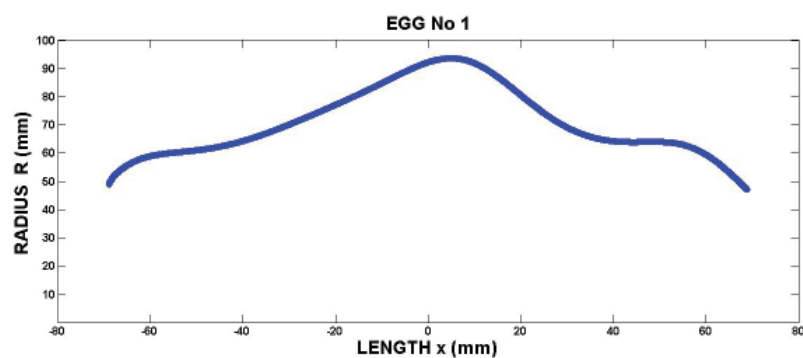
1: Egg's counter. Egg No 1 - SI = 84.22 %.



2: Difference in the y co-ordinates of the egg's contours. Egg No 1.



3: The egg's contours computed from Eq. (5) and determined by the experimental data fitting



4: Radius of the curvature along the egg's symmetry axis. Egg No 1.

the Fig. 1. The largest difference is observed near of the ends.

The knowledge of the mathematical description of the curve describing the egg's contour enables to evaluate the radius of the curvature – see Eq. (10). An example of this radius is given in the Fig. 4.

The values of this radius have been evaluated at the sharp and blunt ends of the egg. Contrary to the hen's eggshell there is a very small difference between sharp and blunt end. The knowledge of the radius of the curvature is very important quantity. Its meaning can be illustrated e.g. by a procedure for the evaluation of the Young's modulus, E , of the food of the convex shape from the compression (ASAE, 2001):

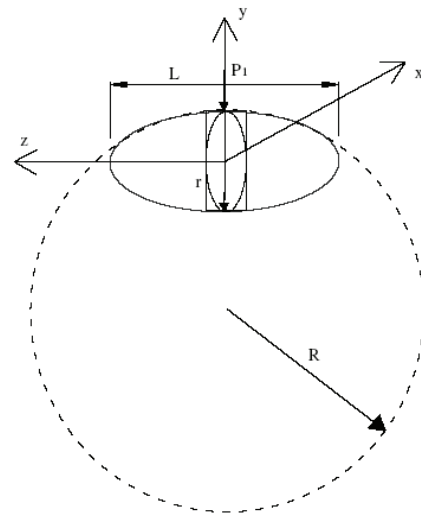
$$E = \frac{0.531F(1-\nu^2)}{x^{\frac{3}{2}}} \left[2 \left(\frac{1}{R} + \frac{1}{r} \right)^{\frac{1}{3}} \right]^{\frac{3}{2}}, \quad (13)$$

where ν is the Poisson's ratio, F is the loading force and x is the specimen displacement during the compression. The meaning of the radius R and r is illustrated in the Fig. 5.

The position of the maximum egg width, x_m , was also evaluated together with the corresponding radius of the curvature. The statistical evaluation of the data shows that there is no significant dependence of the radii of the curvature on the egg shape index SI . Survey of data is given in the Tab. III.

In the next step we have focused on the evaluation of the egg's surface and volume. The simple way

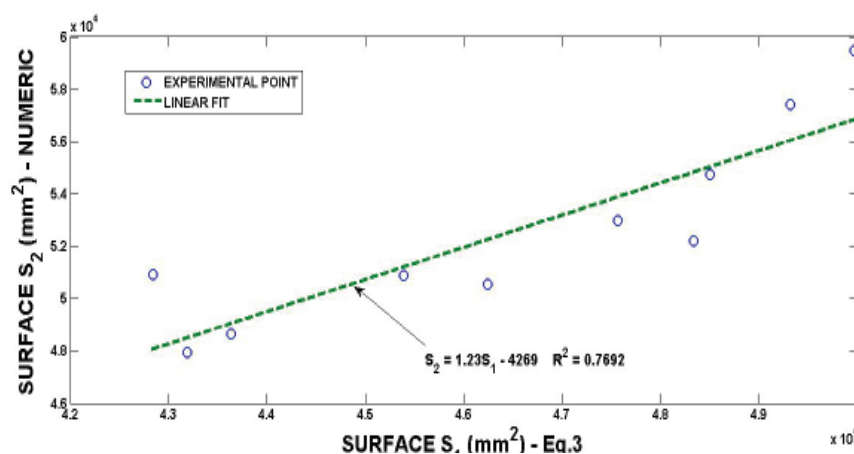
how to obtain these quantities consists in the use of simple formulae given by the Eqs. (3), (4) and/or by the Eqs. (7), (8). There is a question if there is any correlation between results following from these equations with the exact values of the surface and volume calculated using Eqs. (11). The statistical analysis of the data shows that there is no significant dependence of the both quantities on the egg shape. In the Fig. 6 there are plotted values of the egg surface area calculated from the Eq. (11) – exact values *vs.* values of this quantity evaluated using of Eq. (3).



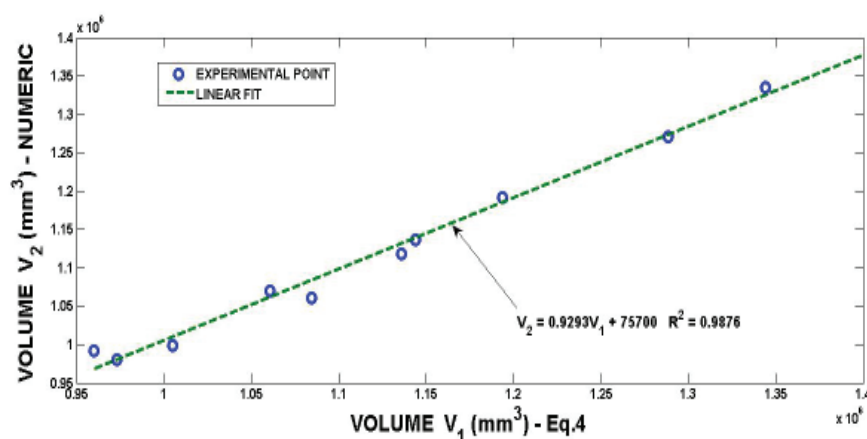
5: Radii of the curvature of the eggshell curve

III: Radii of the eggshell curvature. (R_1 – sharp end, R_2 – blunt end, R_3 – equator, x_m – distance of the equator from the sharp end, L – egg height, std – standard deviation)

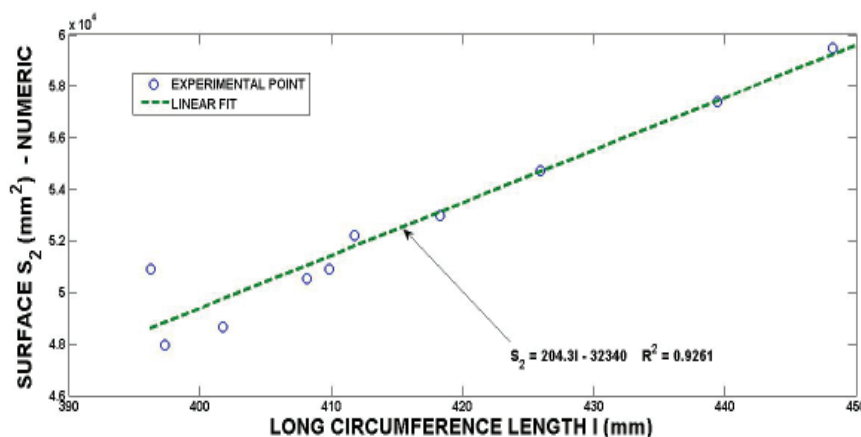
| | R_1 [mm] | R_2 [mm] | R_3 [mm] | x_m [mm] | x_m/L [1] |
|---------|------------|------------|------------|------------|-------------|
| minimum | 30.02 | 43.74 | 67.15 | 68.104 | 0.492 |
| average | 45.63 | 50.06 | 88.24 | 73.411 | 0.505 |
| maximum | 51.84 | 58.78 | 108.48 | 83.720 | 0.566 |
| std | 6.06 | 4.86 | 11.52 | 5.071 | 0.022 |



6: Correlation between values of the egg surface area obtained from the Eqs. (11) and (8)



7: Correlation between numerical value of the egg volume and its estimation given by the Eq. (4)



8: Egg surface vs. long circumference length

One can see that there is a relatively weak correlation between these two quantities. The correlation for the egg volume is much better – see Fig. 7.

In the Fig. 8 the dependence of the egg surface on the circumference length l is shown.

One can see that the long circumference length can be used for very good estimation of the egg surface.

CONCLUSIONS

The analysis of the egg profile based on edge detection techniques has been used for the evaluation of such parameters as egg volume, surface, area and radius of curvature. The egg profile has been fitted by the Fourier series with very high degree of correlation. This approximation describes the egg profile much better than the functions proposed in the most papers published up to now.

It has been shown that the radii of the curvature at the important point of the egg profile (sharp end, blunt end and maximum value) are independent on the egg shape index. It means that the knowledge of the basic dimensions of the ostrich eggs cannot be used for the evaluation of these parameters.

The simple formulae used for the evaluation of the egg volume and surface area (see Eqs. (3), (4), (7), (8)) can be used only for a very rough estimation of these quantities. There is a little poor correlation between the estimated egg surface and the exact value of this quantity. The correlation between egg's volumes is than much more better.

The egg surface area is single value function of the long circumference length. Owing to this fact there is a possibility of the exact estimation of the egg surface on the base of experimentally found value of this quantity.

SUMMARY

The paper deals with the description of the ostrich's eggs shape. First of all basic geometrical parameters have been measured and calculated, namely egg length, L , maximum width, B . The egg shape is than described by the shape index, $SI = B/L$. These basic parameters can be also used for the

estimation of the surface area, and egg volume. In the given paper a new approach to the geometry of an ostrich's egg profile is also presented here using an analysing the egg's digital photo by edge detection techniques. The obtained points on the eggshell counter are fitted by the Fourier series. The eggshell counter has been described using of the polar coordinates r, φ :

$$x = r \cos \varphi$$

$$y = r \sin \varphi.$$

The experimental points r_i, φ_i have been fitted by the Fourier series

$$r = a_0 + \sum_{i=1}^{i=\infty} [a_i \cos(i\omega\varphi) + b_i \sin(i\omega\varphi)].$$

The first six up to eight coefficients of the Fourier series are quite sufficient for the egg's counter shape description (the correlation coefficient between measured and computed egg's profiles lies between 0.98 and 1). Based on the deduced mathematical equation for the egg profile, formulae for the calculation of an egg's volume, surface area, longitudinal circumference length and normal projected area are proposed, too. The knowledge of the mathematical description of the curve describing the egg's contour also enables to evaluate the radius of the curvature. The radii of the curvature at the important point of the egg profile (sharp end, blunt end and maximum thickness) are independent on the egg shape index. The exact values of the egg surface and the egg volume are also independent on the egg shape index. The only chance how to evaluate the values of the quantities consists in the use of the exact determination of the eggshell profile.

Acknowledgement

The research has been supported by IP 2/2013 IGA MENDELÚ.

REFERENCES

- ASAE, 2001: *Compression Test of Food Materials of Convex Shape*. ASAE S368.4, American Society of Agricultural Engineers, St. Joseph: The American Society of Agricultural Engineers.
- BARYEH, E. A., MANGOPE, B. K., 2003: Some physical properties of QP-38 variety pigeon pea. *J Food Eng*, 56, 1: 59–65. ISSN 0260-8774.
- DENYS, S., PIETERS, J. G., DEWETTINCK, K., 2003: Combined CFD and experimental approach for determination of the surface heat transfer coefficient during thermal processing of eggs. *J Food Sci*, 68, 3: 943–951. ISSN 1750-3841.
- HAMILTON, R. M. G., 1982: Methods and factors that affect the measurement of eggshell quality. *Poultry Sci*, 61, 10: 2022–2039. ISSN 1537-0437.
- HOYT, D. F., 1979: Practical methods of estimating volume and fresh weight of bird eggs. *AUK*, 96, 1: 73–77. ISSN 0004-8038.
- MOHSENIN, N. N., 1970: *Physical properties of plant and animal material: structure, physical characteristics, and mechanical properties*. New York: Gordon and Breach, 891 p. ISBN 0677213700.
- NARUSHIN, V. G., 1997: The avian egg: Geometrical description and calculation of parameters. *J Agr Eng Res*, 68, 3: 201–205. ISSN 1095-9246.
- NARUSHIN, V. G., 2001a: Shape geometry of the avian egg. *J Agr Eng Res*, 79, 4: 441–448. ISSN 1095-9246.
- NARUSHIN, V. G., 2001b: What egg parameters predict best its shell strength? In: *IX European Symposium on the Quality of Eggs and Egg Products*. Kusadasi (Turkey): WPSA, 349–355.
- NARUSHIN, V. G., 2005: Production, modelling and education egg geometry calculation using the measurements of length and breadth. *Poultry Sci*, 84, 482–484. ISSN 1537-0437.
- NARUSHIN, V. G. and ROMANOV, M. N., 2002a: Egg physical characteristics and hatchability. *World Poultry Sci J*, 58, 297–303. ISSN 0043-9339.
- NARUSHIN, V. G., and M. N. ROMANOV, 2002a: Physical characteristics of chicken eggs in relation to their hatchability and chick weight. In: *Proceedings of ASAE Annual International Meeting/ CIGR World Congress*, Chicago: ASE, IL, paper 026066.
- NARUSHIN, V. G., ROMANOV, M. N., BOGATYR, V. P., 2002: Relationship between preincubation egg parameters and chick weight after hatching in layer breeds. *Biosyst Eng*, 83, 373–381. ISSN 1537-5110.
- NEDOMOVÁ, Š., SEVERA, L., BUCHAR, J., 2009: Influence of hen egg shape on eggshell compressive strength. *Int Agrophys*, 23, 3: 249–256. ISSN 0236-8722.
- PERIANU, C., DE KETELAERE, B., PLUYMERS, B., DESMET, W., DE BAERDEMAEKER, J., DECUYPERE, E., 2010: Finite element approach for simulating the dynamic mechanical behaviour of a chicken egg. *Biosyst Eng*, 106, 79–85. ISSN 1537-5110.
- ROMANOFF, A. L. and ROMANOFF, A. J., 1949: *The avian Egg*. New York: John Wiley&Sons, 317 p.
- SABLIOV, C. M., FARKAS, B. E., KEENER, K. M., CURTIS, P. A., 2002: Cooling of shell eggs with cryogenic carbon dioxide: a finite element analysis

- of heat transfer. *Lebensm-Wiss Technol*, 35, 7: 568–574. ISSN 0023-6438.
- ZHOU, P., ZHENG, W., ZHAO, C., SHEN, C., SUN, G., 2009: Egg volume and surface area calculations based on machine vision. In: *IFIP International Federation for Information Processing, Computer and Computing Technologies in Agriculture II*, eds. D. Li, Z. Chunjiang. Boston: Springer, 295, 1647–1653. ISBN 1441902082.

Address

doc. Ing. Šárka Nedomová, PhD., Department of Food Technology, prof. Ing. Jaroslav Buchar, DrSc, Department of Technology and Automobile Transport, Mendel University in Brno, Zemědělská 1, 613 00 Brno, Czech Republic, e-mail: snedomov@mendelu.cz, buchar@mendelu.cz