

## TAX RATE TO MAXIMIZE THE REVENUE: LAFFER CURVE FOR THE CZECH REPUBLIC

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### Abstract

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The aim of this article is to model the relationship between the rate of personal income tax and the revenue it generates, and to derive a tax rate that would maximize this revenue within the Czech Republic, using methodologies described in earlier works (Hsing, 1996). This tax rate represents an upper limit. Overstepping it has negative consequences for corporate finances and government budgetary funding alike, because it undermines the workers' motivation to work, reduces buying power, and shifts work activities in favor of gray economy. The period of interest is a time series from 1993 to 2010. Two models were devised. The basic research instrument was a second-degree polynomial regression with a logarithmic transformation of the input data. The explaining variable was the tax revenue, the explanatory variable in Model 1 was the ratio of tax revenue to personal gross annual income. Model 2 featured the ratio of tax revenue to gross domestic product. To limit model instability, all data was stated per capita, in 2010 prices. Both models are statistically significant. By comparison, it was determined that, in the period of 1994–2010, the historical tax rate was lower than the rate designed to maximize the revenue. It approached the theoretical optimum most closely in 2007, and deviated from it most severely in 1995.

taxation of individuals, Laffer curve, polynomial regression

The growing indebtedness of public finances represents an increasing burden for more and more economies. One way to alleviate the problem is to fine-tune the tax system with rates that generate the greatest possible revenue without having a dampening effect on the economy. The level of personal taxation and the overall effectiveness of the system have been the subject of much debate in scientific and social circles. Ibn Khaldun pondered the idea already in the 14th century, and so did John Maynard Keynes and others (Lévy-Garboua, Masclet, Montmarquette, 2007) in the 20th century. The matter got more attention in the 1970s, when the prevailing Keynesian economics could not explain the phenomenon of stagflation and their methods were unable to deal with it. Advocates of the supply side theory of economics came up with the argument of excessive taxation articulated by Adam Smith in his *Inquiry into the Nature and Causes of the Wealth of Nations* (Hsing, 1996). The correlation between the tax rate and the tax revenue came to be

known as the Laffer curve (see Laffer, 1981). The aim of this article is to model the relationship between the rate of personal income tax and the revenue it generates, and to derive a tax rate that would maximize this revenue within the Czech Republic, using methodologies described in earlier works (Hsing, 1996).

### Literature Overview

In literature, the prevalent opinion holds that increasing the rate of personal income tax causes the revenue to rise at first, then to fall, peaking at a certain point (Laffer, 1981, 2004; Hsing, 1996; Pecorino, 1995; Van den Hauwe, 2000; Saez, 2001; Sillamaa, Veall, 2001; Heijman, van Ophem, 2005). The empirical deduction of this correlation was first done by Arthur B. Laffer in 1974 (Laffer, 1981, 2004). The effect of tax changes on economic output (GDP) has been proven empirically for direct taxation; for example Szarowská (2010) postulates that raising the direct tax quota by 1% causes GDP to decline

0.29% in the same year. The link between taxation structure and economic growth has been studied in greater detail by Arnold (2008) *et al.* Laffer's work was continued by others who revised his model or applied it to other countries, such as Hsing (1996) for the U.S., Stuart (1981) for Sweden, and Van Ravenstein, Vijlbrief (1988) for Holland. These studies demonstrate that the actual tax rate often misses its optimal value. An explanation different than the one proffered by Laffer was suggested by Buchanan and Yoon (2004), who studied the lawmaker's motivation to impose the tax at its optimal rate. The final rate actually emerges from a process of negotiation or from diversely motivated coalitions within the government. The impetus to maximize the revenue is only one of many. The tax rate that maximizes the revenue is higher than the rate at which social welfare is being maximized by means of public goods (Ihori, Yang, 2010). The idea of rate-setting on a backdrop of competing political forces is developed further by Ihori and Yang (2010), who argue that political competition can lead to the Laffer paradox, i.e. setting the tax rate higher than what is necessary to maximize the revenue. That is the case of politicians pursuing their own interests (i.e. rent-seeking politicians) in conflict with the voters trying to maximize social welfare. Heijman and van Ophem (2005) amended the Laffer model with the effect of work activities shifting in favor of gray economy and introduced a coefficient of willingness to pay taxes. The move toward gray economy is further analyzed for example by Sanyal (2000). He focuses on the motivation of tax auditors checking the returns: when it is better to get a reward for a good job (i.e. suppression of tax evasion), when venality is a more lucrative option. It also questions audits at multiple levels that would reduce potential corruption but also incur additional costs.

## MATERIALS AND METHODS

The analysis of tax revenue as a function of tax rate in the conditions prevalent in the Czech Republic utilizes the data provided by GMID (Global Market Information Database), which contains information supplied by Euromonitor International. The period of interest is a time series 1993–2010. The study will determine how the level of personal taxation (or more precisely the rate of payments toward personal income tax and social security in the Czech Republic) correlates with the generated revenue (hereinafter tax revenue). This correlation is presumed to be, in accordance with Hsing (1996) and Laffer (1981, 2004), a quadratic function with a local maximum. Factored in the model are GDP, annual gross personal income, and annual tax revenue from personal income plus social security, all stated per capita in 2010 prices. The per capita conversion serves to eliminate the possibility of a curve shift over a longer period of time. The analysis follows the methodology of Hsing (1996), who studied the Laffer curve (Laffer, 1981, 2004) for the U.S., using

the data from the period of 1959–1991. He worked with a single-factor model, chose the tax revenue per capita as a dependent variable, and the ratio of tax revenue change to gross income change (i.e. marginal tax rate) as an independent variable. The variables were used in their original form as well as logarithmically transformed, yielding 4 specific variants of the model. He does not explicitly justify the need for logarithmic transformation but the context makes it clear that the purpose was a better match of the empirical data as measured by the index of determination. Using the model with the index of determination more than 90%, he derived the tax rate for maximum revenue. Two models were created in the course of this work. Model 1 uses tax revenue per capita (TR) as the explaining variable and the explanatory variable being the ratio of tax revenue to personal gross income (TR/AGI). While the explaining variable in Model 2 is again TR, the explanatory variable is TR/GDP. The ratios of TR/GDP or TR/AGI in these models were proposed by Hsing (1996) as potential variables for modeling purposes. A Revenue-Maximizing Tax Rate (RMTR) was derived and compared to historical values. Unlike the constructs of Laffer (1981, 2004) and Hsing (1996), this model included social security payments which, together with income taxes, represent a significant deduction from a person's gross income. The distinction between the two deductions is mainly a matter of fiscal policy; they are earmarked for different funds. An important prerequisite for both models is the normality of input data, an analysis of spurious regressions, and possibly data transformation.

### Normality of Input Data

The normality of input data is important for the following reasons:

1. Statistical tests (t-test, F-test and others) to determine significance are based on the assumption of normality (Nikkinen, Sahlström, 2004).
2. Estimate (b) of unknown regression parameters ( $\beta$ ) by the method of least squares (OLS estimate) has a multidimensional normal distribution

$$b \sim N(\beta, \sigma^2(X'X)^{-1}). \quad (1)$$

3. Residual component ( $\varepsilon$ ) has a normal distribution (Cipra, 2008).

$$\varepsilon \sim N(0, \sigma^2). \quad (2)$$

Both models use ratio-type indicators often associated with lack of normality, caused typically by non-proportionality between the numerator and the denominator (Nikkinen, Sahlström, 2004; Barnes, 1982, 1987).

### Problem of Spurious Regression

Spurious regressions were analyzed with the method suggested by Granger and Newbold (1974).

Their check for spurious regression was based on a comparison between the determination index and the Durbin-Watson statistic. They proposed that a regression be considered probably spurious if its determination index value exceeds the Durbin-Watson (DW) statistic, because that kind of relationship may signify that the residuals have a non-stationary character.

### Data Transformation

Both models utilize a logarithmic transformation of input variables, one reason being to fit the empirical data better (see Hsing, 1996) and the other to approach normality (Hebák, 2007). Logarithmic transformation is a special case of the Box-Cox transformation (Box, Cox, 1964) for

$\lambda = 0$  (Osborne, 2010). This transformation has the advantage of being more universal than the logarithmic one. The disadvantage is a more complicated return to the original variables.

### Model 1

Model 1 was constructed by a polynomial regression of the second degree using the method of least squares to estimate the parameters. (Meloun, Militký, 2002). The original variables were replaced by natural logarithms of their values. Model 1 may be written as:

$$\ln TR_t = \beta_1 \ln \left( \frac{TR_t}{AGI_t} \right) + \beta_2 \left[ \ln \left( \frac{TR_t}{AGI_t} \right) \right]^2 + \varepsilon_t \quad (3)$$

Tax rate for maximum revenue:

$$\frac{TR}{AGI} = \exp \left( -\frac{\beta_1}{2\beta_2} \right) \quad (4)$$

at sufficient condition  $\beta_2 < 0$ , (5)

where

TR.....tax revenue

AGI .....annual gross income

$\beta$  .....regression parameters

$\varepsilon$ .....error, residuals

t .....given year.

### Model 2

Model 2 was also constructed with a polynomial regression of the second degree, using the method of least squares to estimate the parameters (Meloun, Militký, 2002). Model 2 can be formally written as follows:

$$\ln TR_t = \gamma_0 + \gamma_1 \ln \left( \frac{TR_t}{GDP_t} \right) + \gamma_2 \left[ \ln \left( \frac{TR_t}{GDP_t} \right) \right]^2 + \varepsilon_{2t} \quad (6)$$

Tax rate for maximum revenue:

$$\frac{TR}{GDP} = \exp \left( -\frac{\gamma_1}{2\gamma_2} \right) \quad (7)$$

at sufficient condition  $\gamma_2 < 0$ , (8)

where

TR.....tax revenue

GDP ...gross domestic product

$\gamma$ .....regression parameters

$\varepsilon$ .....error, residuals

t .....given year.

## RESULTS AND DISCUSSION

The sample was subjected to the Shapiro-Wilk test for normality (Shapiro, Wilk, 1965; Meloun, Militký, 1994) which tests a hypothesis that the sample comes from a normally distributed population. See Tab. I.

After the logarithmic transformation of the variables, the normality hypothesis is valid at the significance level  $\alpha = 1\%$ . Per the Fisher-Snedecor test of regression significance (Meloun, Militký, 1994), the model is acceptable as significant at the level  $\alpha = 1\%$ . The model's index of determination is 75.5%. The logarithmic transformation therefore appears satisfactory.

### Model 1

A partial F-test indicates that the parameters are statistically significant at the level  $\alpha = 1\%$  except for a constant, which is not significant at any standard level of  $\alpha$ . Since the determination index value ( $R^2$ ) is smaller than the value of DW statistic, the regression is probably not spurious (see Granger, Newbold, 1974). RMTR may be obtained by

I: Test of Sample Normality (Shapiro-Wilk Test)

		TR	TR/GDP	TR/AGI
Before transformation	Shapiro-Wilk W	0.90163	0.82005	0.92683
	p-value	0.06134	0.00296	0.17077
After transformation		ln (TR)	ln (TR/GDP)	ln (TR/AGI)
	Shapiro-Wilk W	0.90885	0.85458	0.93401
	p-value	0.08212	0.01001	0.22828

Source: Author's presentation of Euromonitor International data

## II: Model 1 Specifications

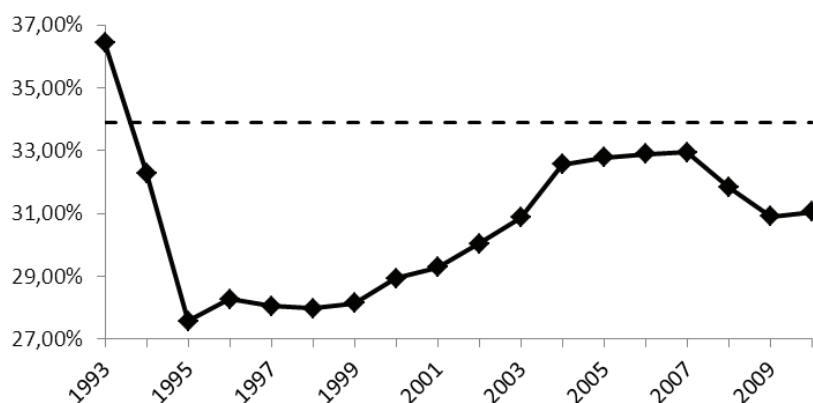
	Coeff.	Std. Error	t-stat.	p-value
Constant	-4.302132	4.669638	-0.921299	0.3715
$\ln(\text{TR}/\text{AGI})$	-28.3297	8.002644	-3.540043	0.003
$\ln(\text{TR}/\text{AGI})^2$	-12.82684	3.419226	-3.751385	0.0019
<b>Multiple R<sup>2</sup></b>	<b>Adjusted R</b>	<b>F-stat.</b>	<b>p-value</b>	<b>DW-stat.</b>
0.755922	0.723379	23.22794	0.000025	1.127538

Source: Author's presentation

## III: Model 2 Specifications

	Coeff.	Std. Error	t-stat.	p-value
Constant	-5.753485	5.042716	-1.140950	0.2718
$\ln(\text{TR}/\text{GDP})$	-27.21608	7.796285	-3.490904	0.0033
$\ln(\text{TR}/\text{GDP})^2$	-10.81189	3.001338	-3.602358	0.0026
<b>Multiple R<sup>2</sup></b>	<b>Adjusted R</b>	<b>F-stat.</b>	<b>p-value</b>	<b>DW-stat.</b>
0.555893	0.496679	9.387828	0.002271	0.953991

Source: Author's presentation



1: Progression of Tax Rate and RMTR in the period 1993–2010

Source: Author's presentation of Euromonitor International data

the first derivation from formula (4). This rate, within the Czech Republic, amounts to 33.14% of AGI. The second derivation, i.e. formula (5), confirms that it is a maximum. Following the Hsing methodology (1996), the possibility of estimating the parameters without the absolute member was also explored. The resultant regression without the absolute member was probably spurious, given its determination index value (0.742111) and the DW statistic (0.694639), which is why this approach was abandoned.

**Model 2**

A partial F-test indicates that the parameters are statistically significant at the level  $\alpha = 1\%$  except for the constant, which is not significant at any standard level of  $\alpha$ . Considering the values of  $R^2$  and the DW statistic, it is probable that the regression is not

spurious. According to the determination index value, Model 2 is not suitable for the derivation of RMTR<sup>1</sup>. The estimate of parameters without the absolute member results in a lower determination index (0.517352), but still greater than the DW statistic (0.448183). RMTR will be derived only as a percentage TR/AGI.

**Comparison of RMTR from Models 1 with Historical Tax Rates**

The values of RMTR calculated from Models 1 permit a comparison with the actual historical tax rate. The conclusion of Model 1 indicates that the actual tax rate in the period of 1993–2010 did not exceed (except in 1993) the rate designed to maximize the revenue. During the monitored period, the historical tax rate (TR/AGI) varied within 27.57% and 36.43%, while the RMTR value stood

1 This refers to a procedure per Hsing methodology (1996), when RMTR was being derived only for specific forms with the highest determination index value ( $R^2$ ).

at 33.14%. The closest approach to the theoretical optimum occurred in 2007 (0.199 pp gap), and the largest deviation in 1995 (5.576 pp gap). See Fig. 1.

## DISCUSSION

Both models work with the data transformed to meet the condition of sample normality. Per the Shapiro-Wilk test, the logarithmic transformation achieved compliance with that condition. Data transformation is necessary to establish statistical significance of the modeled function. In this case, the logarithmic transformation appears to be satisfactory. If there were greater deviations from normality, the Box-Cox transformation (Box, Cox, 1964) would have been more appropriate. The created model 1 demonstrate the possibility of finding a significant correlation between the tax rate and the tax revenue, which can be modeled by a second-degree polynomial whose quadratic member is statistically significant at  $\alpha = 1\%$ . This confirms the conclusions about the correlation of tax rate and tax revenue reached by other authors (Laffer, 1981, 2004; Hsing, 1996; Pecorino, 1995; Van den Hauwe, 2000; Saez, 2001; Sillamaa, Veall, 2001; Heijman, van Ophem, 2005). Model 2, utilizing the ratio of TR and GDP as an explanatory variable, was constructed due to a definite effect that GDP has on tax revenue (see Szarowska, 2010; Arnold, 2008). Even though this regression is clearly not spurious, it does not appear suitable for the computation of RMTR because of its low determination index

value ( $R^2$ ). Model 1 indicates that the theoretical rate of taxation for maximum revenue in the Czech Republic is 33.14% of AGI. This model is statistically significant, and the index of determination implies that it accounts for up to 75% of empirical observations. According to Model 1, the actual tax rate from 1994 to 2010 was lower than the revenue-maximizing rate. This conclusion is frequently applies to other countries as well (see Hsing, 1996; Stuart, 1981; Van Ravenstein, Vijlbrief, 1988). The historical tax rate approached the theoretical optimum most closely in 2007 (0.19 pp gap) and deviated from it most severely in 1995 (5.76 pp gap). It peaked in 1993 (36.43% of AGI, i.e. higher than RMTR). In the following period, a gradual decrease (down to 27.56% of AGI in 1995) was accompanied by a growth in real GDP and a falling public debt (down to 27.56% of AGI in 1995). Since 1996, the historical tax rate has been on the rise, concurrently with the public debt and the gross annual income. A change in the trend of this historical rate occurred in 2007 (just under the RTMR limit), when the real GDP registered a dip followed by a decline in the gross annual income in 2008. If the economic growth did not slow down in 2007, the historical tax rate would have probably exceeded the RMTR limit. In practice, achieving the theoretical value of RMTR is complicated by the people's willingness to pay taxes and the associated inclination to engage in gray economy, the efficiency of tax auditing, the different motives of lawmakers setting the rate, and the effective use of the collected funds.

## SUMMARY

This article discusses the modeling of a relationship between the rate at which individuals are taxed and the generated revenue, from the data for the Czech Republic in the period of 1993–2010. Empirical results confirm that a correlation between the tax rate and the tax revenue exists, as documented in the work of Laffer (1981, 2004), Hsing (1996), Pecorino (1995), Van den Hauwe (2000), Saez (2001), Sillamaa, Veall (2001) and Heijman, van Ophe (2005). The research utilized a polynomial regression with a logarithmic transformation of the input data. Two models were constructed. In both cases, the tax revenue was selected as an explaining variable. The explanatory variable in the first model was the ratio of tax revenue to individual gross annual income. In the second model, the explanatory variable was the ratio of tax revenue to gross domestic product. The data was stated per capita in 2010 prices to limit model instability due to variations in prices and total population. The input data was logarithmically transformed and the normality of the sample, as a prerequisite for regression analysis, was considered sufficient per Shapiro-Wilk test. Both models are statistically significant at  $\alpha = 1\%$  level. The RTMT derived from Model 1 is 33.14% of AGI. Model 1 exhibits significantly higher values of the index of determination than Model 2, and it is therefore considered, in accordance with Hsing (1996), to be more appropriate. The rate to maximize the tax revenue in the Czech Republic is therefore 33.14% of AGI. A comparison with the tax rate calculated from Model 1 shows that the actual historical tax rate was lower than the revenue-maximizing rate during the period 1994–2010. It approached the theoretical optimum in 2007 (0.19 pp lower), and deviated from it greatly in 1995 (5.76 pp lower). The assumptions about the interactions between tax rates and tax revenues claim that increasing the tax rate would boost revenues. In reality, attaining the optimal rate is complicated by many factors, such as the people's willingness to pay taxes and the associated shift of their economic activities in favor of gray economy (Heijman, van Ophem, 2005), the efficiency of tax auditing (Sanyal, 2010), the conflicting motives of lawmakers setting the rate (Buchanan, Yoon, 2003), and the effectiveness with which the collected funds are deployed (Ihori, Yang, 2010).



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