

STUDY OF SETTLING VELOCITY OF SAND PARTICLES LOCATED IN WASTEWATER TREATMENT PLANT

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Received: October 12, 2010

Abstract

VÍTEŽ, T., TRÁVNÍČEK, P.: *Study of settling velocity of sand particles located in wastewater treatment plant*. Acta univ. agric. et silvic. Mendel. Brun., 2011, LIX, No. 1, pp. 249–254

The objective of this paper was to determine and compare the theoretical and experimental settling velocity of sand particles in the water. For the determination of settling velocity sand from wastewater treatment plant was selected. Sand is transported to the wastewater treatment plant by a sewer system with sewage water, especially in locations with a combined sewer system. It is necessary to capture and separate sand in the first step of wastewater purification, which is called primary treatment, otherwise sand can cause problems in the technological line that will have an impact mainly on the economy of operation. For sand capture sedimentation is usually used, principle of sedimentation is based on physical properties of the sand, especially depends on density of the sand. For the experimental measurement of the settling velocity of the sand a laboratory track path had been created. Obtained settling velocities were compared to theoretically calculated settling velocities in accordance with the Stokes Law, Allen's Law and Newton's Law.

sand, wastewater treatment plant, sedimentation, primary treatment, settling velocity

Wastewater treatment is a rather complex and financially demanding process that consists of individual partial procedures and processes focused on elimination of various types of contamination from wastewater. As regards wastewater, contamination proves to be dispersed in a certain manner; the fundamental subdivision may be, primarily, into coarsely dispersed substances, substances in the form of fine settleable suspensions and substances that are hardly settleable or non-settleable. Removal of coarsely dispersed substances from wastewater (gravel, sand) is performed using the sedimentation process that is conducted in special equipment that is jointly referred to as sand traps. The amount of sand transported through a sewage system to a wastewater treatment plant varies significantly and depends mainly on the type of sewage system and size of the wastewater treatment plant. The specified production of sand is 5 dm^3 – 12 dm^3 per person per year in case of a combined sewer system; in case of rainfall it may be even 30 times higher (Imhoff,

1953). In case of a separate sewage system the specified production of sand reaches 0.3 dm^3 – 2.2 dm^3 per person per year (Imhoff, 1953). The specified value of specific density of sand is $2620 \text{ kg}\cdot\text{m}^{-3}$ (Torgal and Castro-Gomes, 2006). Sand traps are supposed to catch solely mineral particles with the size of 0.1 mm – 0.2 mm ; organic substances are supposed to be transported together with wastewater to the biological treatment process (Tuček *et al.*, 1977). Sedimentation of sand in sand traps may be regarded as hydraulic separation that is a function of the flow rate. Flow rates in a wastewater treatment plant are rather variable and, as regards calculations, it is considered on the basis of the so-called irregularity coefficients (Čížek *et al.*, 1970). In order to create a correct design of a sand trap, it is essential to use correct boundary conditions – primarily the fundamental specifications of the material (sand) as well as the carrier medium (wastewater). However, the most significant role is played by a correct determination of the settling velocities applicable to sand particles.

MATERIAL AND METHODS

As regards sedimentation, isolated sand particles are affected by three forces. The gravitational force operates downwards, the hydrostatic force operates upwards and the drag force operates upstream. The resulting force affecting a sand particle shall be – provided that the particle is spherical – calculated using the following formula:

$$F_g = F_{vz} - F_r$$

$$\begin{aligned} & \frac{\pi \times d^3}{6} \times \rho_p \times g = \\ & = \frac{\pi \times d^3}{6} \times \rho_v \times g - C_D \times \frac{\pi \times d^2}{4} \times \frac{v_s^2}{2} \times \rho_v, \quad [\text{N}] \end{aligned} \quad (1)$$

where:

- F_g gravitational force [N]
- F_{vz} hydrostatic force [N]
- F_r drag force [N]
- d diameter of sand particle [mm]
- ρ_p specific density of sand [$\text{kg} \cdot \text{m}^{-3}$]
- g gravitational acceleration [$\text{m} \cdot \text{s}^{-2}$]
- ρ_v water density [$\text{kg} \cdot \text{m}^{-3}$]
- C_D drag coefficient of spherical particle [-]
- v_s settling velocity [$\text{m} \cdot \text{s}^{-1}$].

By the means of adjustment of the preceding equation we will obtain a similarity formula applicable to calculation of the settling velocity:

$$v_s = \sqrt{\frac{4}{3} \times \frac{d \times (\rho_p - \rho_v) \times g}{C_D \times \rho_v}} \quad [\text{m} \cdot \text{s}^{-1}]. \quad (2)$$

The value of drag coefficient C_D is dependent on Re_p and therefore also on the sedimentation velocity v_s . Due to this fact, the settling velocity may not be determined on the basis of this formula. It is essential to use formulae applicable to laminar, transition and turbulent areas of sedimentation. As regards the laminar area of sedimentation, the Stokes Law applies and the value of drag coefficient is calculated as follows:

$$C_D = \frac{24}{\text{Re}_p} \quad [-] \quad (3)$$

where:

- C_D drag coefficient of spherical particle [-]
- Re_p Reynolds criterion [-].

The Reynolds criterion applicable to the laminar area of sedimentation features values ≤ 2 and it is calculated using the following formula:

$$\text{Re}_p = \frac{v_s \times d \times \rho_v}{\mu}, \quad [-] \quad (4)$$

where:

- Re_p Reynolds criterion [-]
- v_s settling velocity [$\text{m} \cdot \text{s}^{-1}$]
- d diameter of sand particle [mm]
- ρ_v wastewater density [$\text{kg} \cdot \text{m}^{-3}$]
- μ dynamic viscosity of wastewater [$\text{Pa} \cdot \text{s}$].

In the laminar area the settling velocity of a spherical sand particle will be calculated (in accordance with Stokes Law) as follows:

$$v_s = \frac{(\rho_p - \rho_v)}{18} \times \frac{g \times d^2}{\mu} \quad [\text{m} \cdot \text{s}^{-1}]. \quad (5)$$

As regards the transition area of sedimentation, the Reynolds criterion proves to be within the range of $2 < \text{Re}_p < 500$; the drag coefficient value is to be calculated using the following formula:

$$C_D = \frac{18}{\text{Re}_p^{0.6}} \quad [-]. \quad (6)$$

In accordance with the Allen's Law, the settling velocity of a spherical sand particle in the transition area will be calculated as follows:

$$v_s = \frac{0,153 \times [(\rho_p - \rho_v) \times g]^{0,714} \times d^{1,143}}{\mu^{0,430} \times \rho_v^{0,286}} \quad [\text{m} \cdot \text{s}^{-1}]. \quad (7)$$

As regards the turbulent area, the Reynolds criterion reaches values of $\text{Re}_p > 500$ and the Newton's Law applies; drag coefficient of a particle $C_D = 0.445$. The settling velocity of a spherical sand particle in the turbulent area will be calculated as follows:

$$v_s = 1.73 \times \sqrt{\frac{(\rho_p - \rho_v)}{\rho_v} \times g \times d} \quad [\text{m} \cdot \text{s}^{-1}]. \quad (8)$$

Determination of the type of flow in which sedimentation occurs is possible on the basis of the following derived similarity formula:

$$C_D \times \text{Re}_p^2 = \frac{4}{3} \times \frac{d^3 \times (\rho_p - \rho_v) \times \rho_v \times g}{\mu^2} \quad [\text{m} \cdot \text{s}^{-1}]. \quad (9)$$

Laminar area: $C_D \times \text{Re}_p^2 < 48$;

transition area: $48 < C_D \times \text{Re}_p^2 < 1.1 \times 10^5$;

turbulent area: $1.1 \times 10^5 < C_D \times \text{Re}_p^2 < 4 \times 10^{10}$.

Determination of the Time of Sedimentation

A sample of sand from a wastewater treatment plant was cleaned by the means of removal of organic impurities and, in accordance with ČSN EN 933-1, it was subdivided into the particle size fractions of 0.125 mm, 0.5 mm, 1 mm and 2 mm. Subsequently, the density of sand particles and volumes of particles for individual sand fractions were determined using the pycnometric method (Pytlík, 2000). The weight of individual sand particles was measured using scales featuring the accuracy of 0.0001 g. Measuring of the settling velocity was performed in a glass measuring cylinder (height: 580 mm; diameter: 83 mm) which was filled with water featuring the temperature of 20 °C up to the level of 520 mm. Sand particles from individual fractions were manually placed under the water surface in the measuring cylinder and subsequently released. The trajectories of particles were observed optically and the time of settling of individual particles between graduation lines marked at the distance of 450 mm was re-

corded. Each fraction of sand was tested repeatedly (thirty times).

RESULTS AND DISCUSSION

Settling velocities of individual fractions of sand were determined on the basis of calculations as well as experimentally. Determination of settling velocities was based on selection of three commonly applied equations (Stokes Law, Allen's Law and Newton's Law); the values of determined on the basis of calculations were subsequently compared to values

obtained through measurements using the experimental equipment. For an overview of the results see Tab. I.

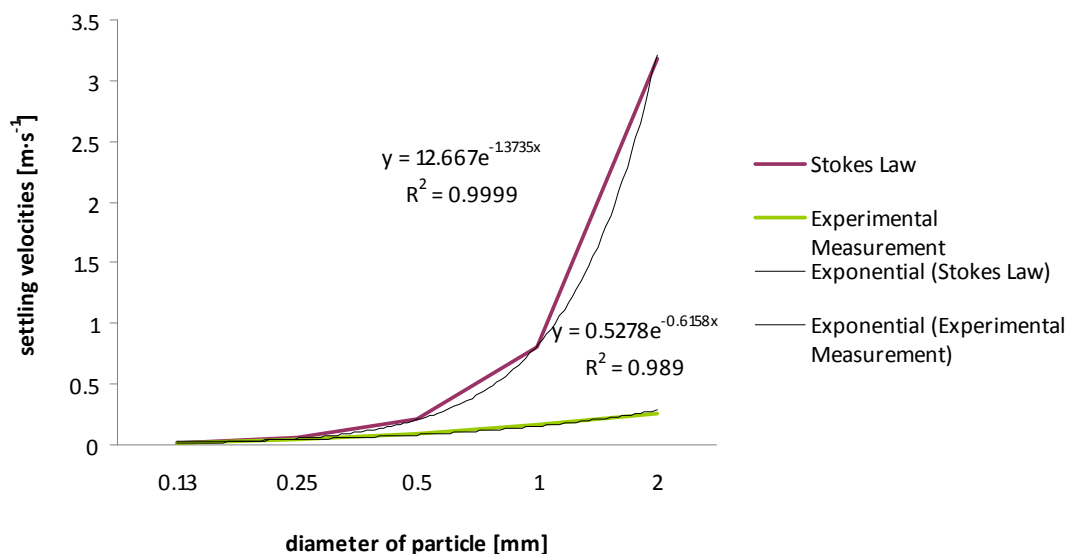
Furthermore, the value of the coefficient, which is used in the practice for the purpose of determination of the flow type and subsequent proposal of the procedure applicable to calculation of the settling velocity, was specified. For results obtained through calculations see Tab. II, which comprises also specifications of the types of equation applicable to calculation of the settling velocity of sand particles.

I: Settling velocities of individual sand particle fractions

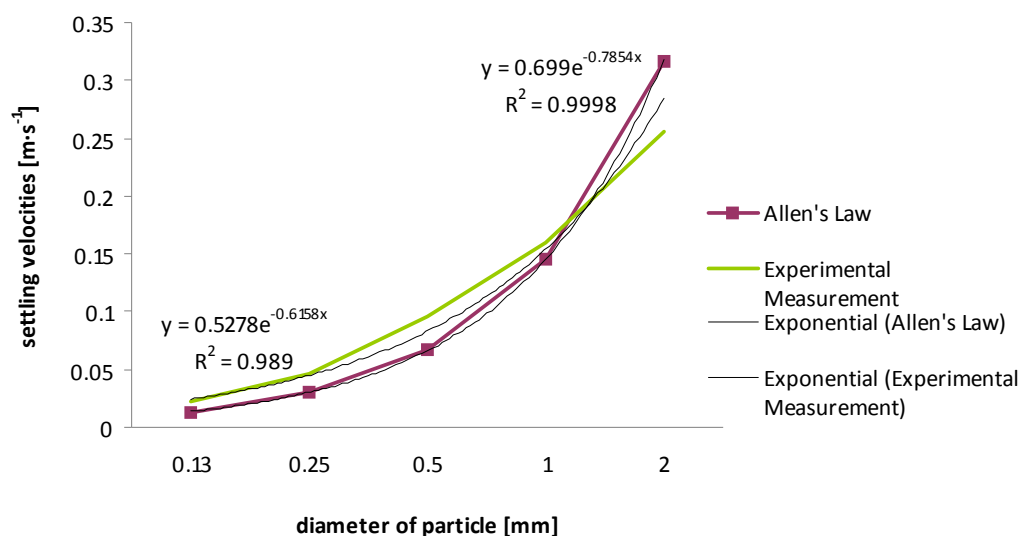
Diameter of particle [mm]	Settling velocity			Results of experiment [m·s ⁻¹]
	Stokes Law [m·s ⁻¹]	Allen's Law [m·s ⁻¹]	Newton's Law [m·s ⁻¹]	
2	3.1840	0.3163	0.2931	0.2564
1	0.8095	0.1450	0.2090	0.1604
0.5	0.2071	0.0667	0.1495	0.1255
0.25	0.0534	0.0309	0.1074	0.0462
0.125	0.0129	0.0135	0.0741	0.0220

II: Values applicable to selection of settling velocity calculation formula

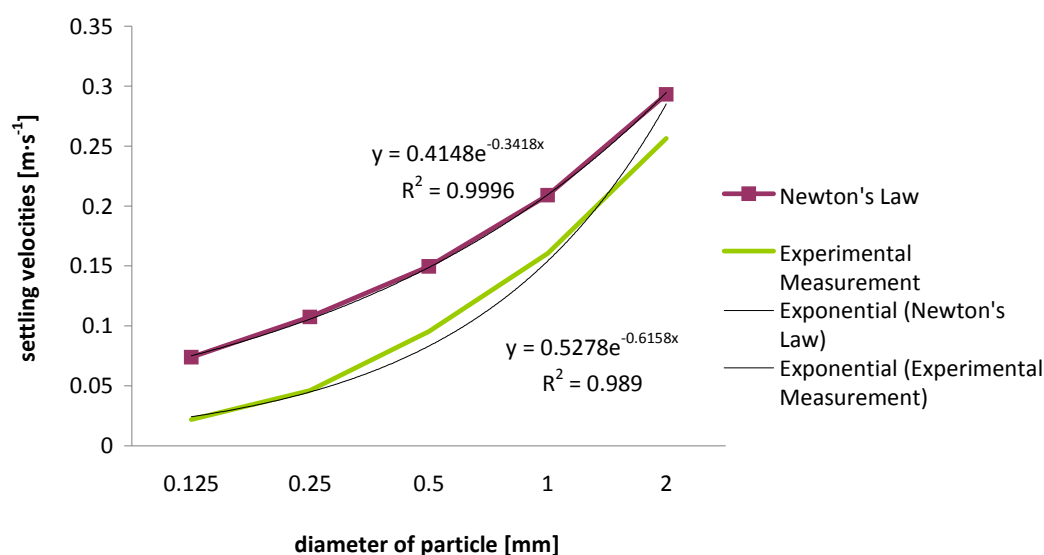
Diameter of particle [mm]	Sand density [kg·m ⁻³]	Water density [kg·m ⁻³]	Dynamic viscosity of water [Pa·s]	$C_D \cdot Re_p^2$ [-]	Flow type determination	Selection of settling velocity calculation formula
2	2463	998.25	0.001002	152443	Turbulent	Newton's Law
1	2488	998.25	0.001002	19378	Transition	Allen's Law
0.5	2523	998.25	0.001002	2479	Transition	Allen's Law
0.25	2571	998.25	0.001002	320	Transition	Allen's Law
0.125	2494	998.25	0.001002	38	Laminar	Stokes Law



1: Comparison of settling velocities of sand particles – experimental measurement and calculation based on Stokes law



2: Comparison of settling velocities of sand particles – experimental measurement and calculation based on Allen's law



3: Comparison of settling velocities of sand particles – experimental measurement and calculation based on Newton's law

III: Deviations of calculated values from experimentally measured values

Diameter of particle [mm]	Deviation [%]		
	Stokes Law	Allen's Law	Newton's Law
2	+1 240	+19	+12.5
1	+500	-10.5	+23.3
0.5	+216	-69	+156
0.25	+13.5	-33	+232.5
0.125	-41	-39	+337

For specifications of deviations between the measured data and values calculated using equations based on the Stokes, Allen's and Newton's Law see Tab. III.

The resulting lowest possible deviations between the calculated values of settling velocities and experimental values are rather significant – they prove to range between 10.5% and 69%. This result is

caused by the fact that the equations used for determination of theoretical settling velocities are based on use of spherical particles which are rarely present in the monitored samples. Furthermore, the results also prove that the flow type significantly affects the settling velocity results and selection of a correct law

for the purpose of calculation of the settling velocity is, therefore, essential. The completed measurement suggests that the Stokes Law is suitable for the laminar area, the Allen's Law is suitable for the transition area and the Newton's Law is suitable for the turbulent area.

SUMMARY

This paper presents a comparison of the calculated values of settling velocity of sand with values determined experimentally. Correct determination of the settling velocity of the sand is very important for the effective design of equipment for primary treatment of the waste water, namely a sand trap. A sample of sand taken from the wastewater treatment plant was deprived of organic components and then dried at 105 °C for 12 hours. On the basis of sieve analysis conducted in accordance to standard ČSN EN 933-1, individual grains of sand were divided into fractions of 0.125 mm, 0.25 mm, 0.5 mm, 1 mm and 2 mm. Settling velocity fractions of each species was first identified experimentally on the laboratory track path. These experimental values were further compared with values calculated according to the Newton's Law, Stokes Law and Allen's Law. Following evaluation of experimental and calculated data, based on calculated deviations it was shown that in case of particles with a diameter of 0.5 mm, 0.25 mm and 0.125 mm it is advisable to use a various type of equation than the equations recommended for the respective flow type. Deviations between calculated and experimentally measured data are relatively high and reach up to 69%.

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