

# SIZE AND STRUCTURE OF RETURN TO SCALE IN REVENUE FUNCTION AND COST FUNCTION

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## Abstract

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Common revenue function expressing the relation of the profit/loss to costs is composed analytically so that it is not instructive enough to analyse the return to scale based on a comparison of results in two different periods. This function is influenced by fixed cost and its production utilization, change of profit due to an extensive increase of output and variable cost's size and efficiency. Each stage of the revenue function has specified relation of the above mentioned cost items. According to their relation the economy in each stage of the revenue function is derived. It is possible to use these analyses to optimize output regarding the profit/loss as well as to assess the economy related to any change of the profit volume. Supported by the Ministry of Education, Youth and Sports of the Czech Republic (Project No. MSM 6007665806).

return to scale, production utilization of fixed cost, revenue function, cost function

The return to scale analysis is an important indicator referring how profitable is would be for a firm to extend its output, what is the input use efficiency and what kind of economy is connected with such output extension. The return to scale is based on a predefined curve a course of which determines the influence of each factor to the output as well as the economy of each stage. The estimations of revenue function parameters and its stages related to different dynamics of the output volumes and derived features are very carefully formulated. Less attention is paid to the economy of each stage of the curve mainly in relation to financial indices used in practice.

The aim of the paper is to assess the impact of the change in each input to the revenue curve indicating the relation of the profit volume to an increase of each cost type and to describe the anatomy of each stage of the revenue curve including the dynamics of derived features and their economy.

## REVIEW OF LITERATURE AND METHODOLOGY

Studies dealing with returns to scale and their possible use within firm registered long however

not so famous history. Perloff (2008) discusses relations of total costs, average costs and marginal costs. Varian (2005) deals with returns to scale and assessing its dynamics. The applied part related to use of return to scale may be regarded as very useful. Mari *et al.* (2007) discusses the measurement of returns to scale of cherries, tomatoes and chilli sales in the province of Sindh in Pakistan. Case and Fair (1998) define relations of decreasing marginal output and decreasing output. Eaatwell (1987), Silvestre (1987) a Vassilakis (1987) dealt with the theory of return to scale, and economies of return to scale, definition of constant return to scale and increasing return to scale. The theory of profit maximizing within each stage of the revenue function is discussed in Samuelson (2003). It deals with methods based on marginal costs and marginal output. Střeleček (2007) analyzed relations of differential cost and cost/revenue ratio, incremental unit cost and cost/revenue ratio respectively. Střeleček and Zdeněk (2008) reviewed the assessment of variable cost efficiency in relation to the return to scale.

The intensification cost efficiency is measured both directly and indirectly. The direct intensification cost efficiency consists of economy of their spending and it is related to the intensification cost

and output volume dynamics. The indirect intensification cost efficiency consists of mediated effects mainly caused by changes of the output volume (Brigham, Gapenski, 1997) such as relative change of fixed costs due to a change of output volume and a change of the profit/loss due to the volume of output.

The assessment of the efficiency of the output volume change related to the management efficiency must be based on perfect benchmarking performed by a comparison of results with the most successful enterprises and by the method of optimal construction based on empirical conclusion or mathematical models of production economy. The assessment of management efficiency based on mathematical model is known as the technical efficiency defined as the ratio of real output volume to maximal output possible with appropriate inputs (Battese, Coelli, 1988; Hadley, 2006).

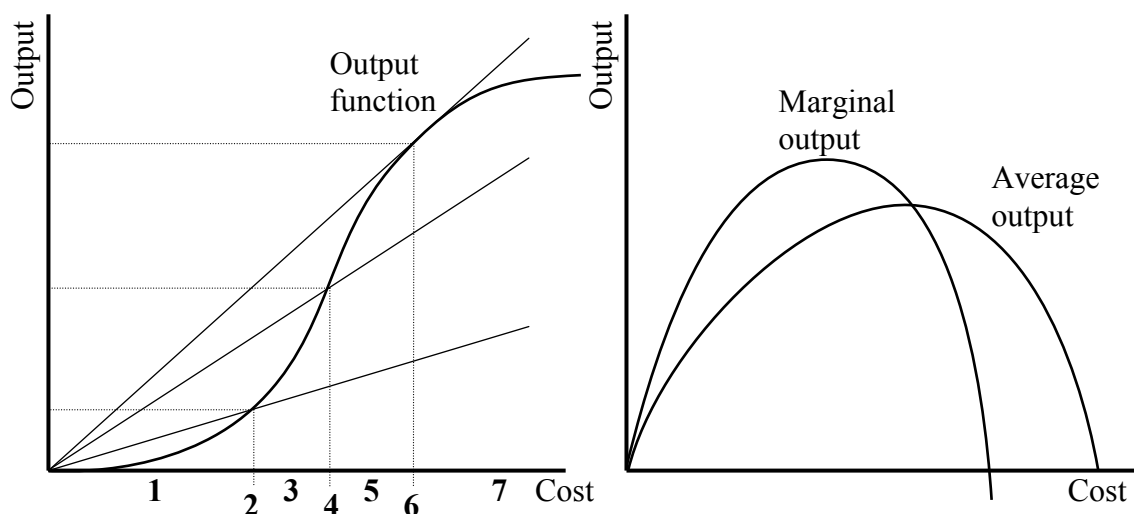
Alvarez and Arias (2004) analyse the relationship between technical efficiency and size conditional on a set of control variables. These control variables are chosen using a production model where technical efficiency is introduced as a parameter. As a result, technical efficiency affects both the input demand and the output supply of a profit maximising producer.

A nonparametric analysis of technical, allocative, scale, and scope efficiency of agricultural production is presented based on a sample of Wisconsin farmers. The results indicate the existence of important economies of scale on very small farms, and of some diseconomies of scale for the larger farms. Also, it is found that most farms exhibit substantial economies of scope, but that such economies tend to decline sharply with the size of the enterprises (Chavas, Aliber; 1993).

Banker and Thrall (1992) examine the links between the returns to scale and most productive scale size in multiple-output-multiple-input produc-

tion environments. Savastano and Scandizzo (2009) show that when the hypothesis of decreasing return to scale holds, the relation between the threshold value of revenue per hectare and the amount of land cultivated is positive. Tao and Dai (2007) decompose index of labour productivity into technical efficiency, pure technical progress, scale efficiency of capital/labour and change of intensity for capital/labour. Wei and Yan (2004) analyze the problems of congestion of inputs, increasing, constant and decreasing return to scale by output oriented DEA models. Fiorillo et al. (2000) analyse an economy where firms use labour as the only production factor, with constant return to scale. Sharma et al. (1999) compare parametric and nonparametric methods for measuring technical, allocation and economic efficiency and examine potential for reducing cost through improved efficiency. Al-Khoury and Abu Al-Dahab (2009) analyze technical performance efficiency of Jordanian Industrial Companies using Data envelope analysis under the assumption of input minimization with constant return to scale. Number of employees, paid in capital and total fixed assets were used as inputs and market value per share, net sales and return on assets were used as outputs.

Managi and Karemera (2004) applied DEA methodology to a state-level data set of US agriculture over 1960–1996 to measure the total factor productivity and other indexes as technological change and efficiency change. Both the constant return to scale and variable return to scale technologies assumption in DEA were employed. Hadley (2006) used English and Welsh farm-level survey data for the period 1982 to 2002 to estimate production functions for eight different farm types. The analysis showed that, farms of all types are relatively efficient with a large proportion of farms operating close to the production frontier. The factors that consistently appear to have a statistically significant effect on differences in



1: Revenues function and its stages

efficiency between farms are farm or herd size, farm debt ratios, farmer age, levels of specialisation and ownership status.

The intensification and fixed costs are influenced by a number of factors. The exact defining is rather difficult (Schroll, 1997) so that the assessment is usually based on the evaluation of the most important cost items.

**Revenue curve**

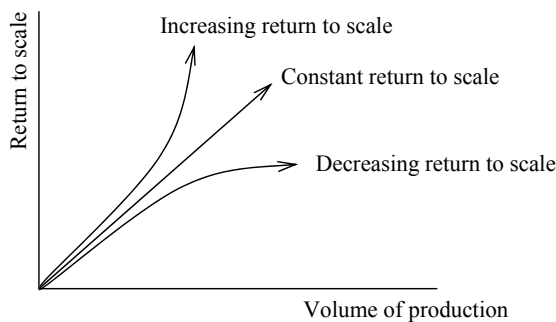
Modelling the revenue curve that expresses a relation of a profit/loss change to costs within different volumes of output is usually based on input and output of the production function. Marginal and average product and their relation are derived from this function.

Indicators of the economic experience do not correspond with this revenue curve so that the analysis has to combine the revenue function with the cost function. Due to this, the analysis will deal with the influence of the output volume to the profit/loss of a firm. The cost function is a mirror function to the revenue function. The assessment of marginal cost and cost/revenues ratio could be based on the relation of  $n = 1/AP$  a  $dn = 1/MP$ , where *AP* stands for average product and *MP* for marginal product. The analysis based on the above mentioned function will show seven stages as can be seen in figure 1. The relation in this function is based on the profit/loss of the return to scale, fixed cost, variable cost and output volume.

**Return to scale**

The return to scale is a technical feature of the revenue function  $R = f(x_1, x_2, \dots, x_n)$ . This feature describes the change of the output assuming the proportional change of input. A formal revenue function  $R(K, L)$  is defined for the constant scale effect within each constant *a* equal to 1 or greater in case of  $R(aK, aL) = aR(K, L)$ ; increasing return to scale (for each constant greater than 1)  $R(aK, aL) > aR(K, L)$  and decreasing return to scale (for each constant equal to 1 or greater)  $R(aK, aL) < aR(K, L)$ . *K* and *L* stand for some production factors, such as capital or labour.

In case of output increase proportional to cost volume we register a constant return to scale. In case



2: Constant, increasing and decreasing return to scale

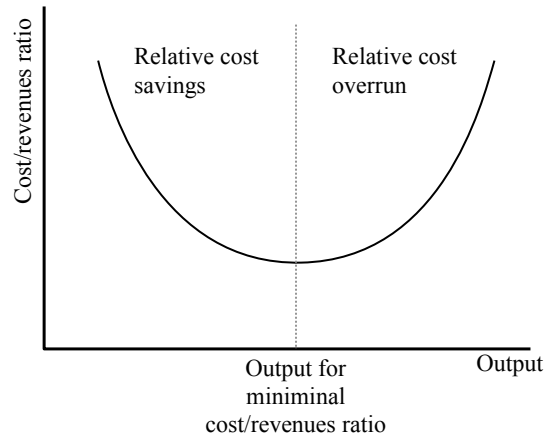
of output increase less proportional to cost volume we refer to decreasing return to scale. More than proportional output increase is identified as increasing return to scale (figure 2).

**Economy of the return to scale**

Economies of the return to scale (further referred as economies of scale) are expressed within the revenue curve through a relative increment of the profit/loss or its relative decrease in relation to constant change of production.

Within the cost function, economies of scale are expressed through relative saving or overrun of costs. Generally, savings of scale denote such production features that influence cost in case when volume of each factor increases by the same value.

Cost increasing by the same proportion as output determines zero savings of the scale. Faster increase of costs means a relative cost overrun of scale (further referred as relative cost overrun). Slower increase denotes relative cost saving of scale (further referred as relative cost saving). Different savings of scale will be reflected in a change of cost/revenue ratio, rate of return and total profit/loss. The assessment of economies of scale is a key to adjusting the optimal volume of outputs in a firm. A number of cost items of relative saving or overrun is identified as a continuous, usually convex function as can be seen in figure 3.



3: Economies of Scale

It is obvious that a relative change of cost to scale may be easily transformed into relative change of profit/loss to scale.

Profit maximizing is a process through which such prices and output volume may be identified that will lead to maximum profit possible. Notice that identification of maximum profit in each stage of the revenue and cost function may have and have different points of view.

The assessment of cost efficiency is based on the following indicators:

$$\text{Index of revenues: } i_{VN} = \frac{V_1}{V_0}$$

$$\text{Index of costs: } i_{VN} = \frac{VN_1}{VN_0}$$

$$\text{Cost/revenues ratio: } n = \frac{VN}{V}$$

$$\text{Variable cost/revenues ratio: } n(v) = \frac{VN(v)}{V}$$

$$\text{Differential cost: } dn = \frac{(VN_1 - VN_0)}{(V_1 - V_0)}$$

Relative change in cost due to cost/revenues ratio:

$$\Delta VN/n = (n_1 - n_0) V_1 = VN_1 - VN_0 i_v$$

Relative change in costs due to output:

$$\Delta VN/V = n_0 (V_1 - V_0)$$

Relative change in fixed costs:  $\Delta SN/V = SN(1 - i_v)$

Relative change in profit due to output:

$$\frac{\Delta Z}{V} = (1 - n_0) \times (V_1 - V_0)$$

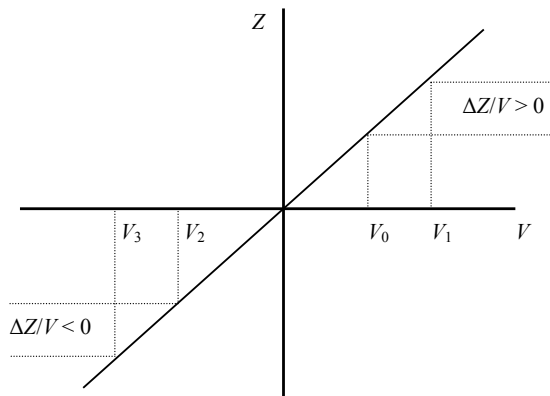
$V$  stands for revenues,  $VN$  for costs,  $VN(v)$  for variable costs,  $SN$  for fixed costs,  $Z$  for profit/loss and  $Q$  for output in natural units. Expression of change with slash on left side of formula (e.g.  $\Delta VN/n$ ) stands for conditioned change (relative change in cost influenced by change in cost/revenues ratio).

Identifying the economy of scale means to derive changes in profit volume that correspond to each change in production cost. This is based on relations of indices that apply for each stage of production and cost function. The basic return to scale includes a change of the profit/loss due to an extension of output, relative change of fixed costs (saving or overrun) due to different utilization of fixed costs and a relative change of variable costs (saving or overrun) due to their different efficiency.

#### Relative change of profit/loss due to an extension of output

An effect of an extension of output is connected to a change of output volume. The extensive change of output is defined as a change of the output volume within constant cost/revenue ratio. The relation of  $\Delta Q \neq 0$  and  $V \neq 0$  must apply. The return to scale will increase or decrease by proportion to the volume of output with the rate of profit in the previous year as the proportionality constant.

Positive results will bring an increase of profit (decrease of loss); negative results will bring a decrease of profit (increase of loss). The graph  $\Delta Z/V$  of is pre-



4: Change of the profit/loss due to changes in output volume

sented as a line with an angular coefficient of  $1 - n_0$  (figure 4).

#### Fixed cost

Fixed costs remain constant within a certain output volume. Cost function of fixed costs is expressed as a parallel with the  $x$  axis at a level of fixed costs. These costs jump change in relation to the output volume. Unit fixed costs are described as

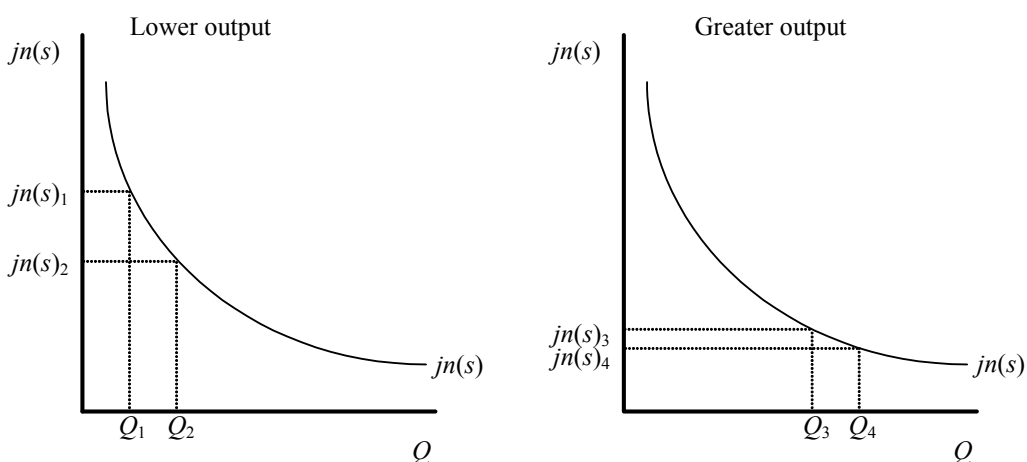
$$jn(s) = \frac{SN}{Q}, \text{ and } \lim_{Q \rightarrow \infty} jn(s) = 0.$$

Cost function of unit fixed costs is shaped as a hyperbola. Increasing the output volume decreases unit fixed costs. The above mentioned equations resulted into the following economic rules:

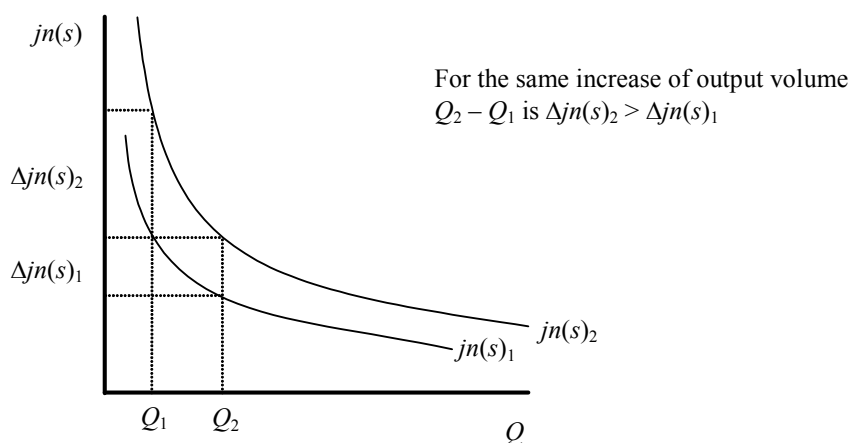
1. Relative saving of fixed cost due to greater output utilization within constant increase of output with the output volume decreases (fig. 5). The relation of  $(jn(s)_1 - jn(s)_2) > (jn(s)_3 - jn(s)_4)$  applies for  $(Q_2 - Q_1) = (Q_4 - Q_3)$ .
2. The savings of unit fixed costs is greater for the same increase of output volume within lower output.
3. Unit fixed costs increases with increasing fixed cost. A constant increase of output volume is connected with greater savings of unit fixed costs within greater fixed costs (figure 6).
4. It is therefore necessary to pay more attention to production capacity use of more expensive technologies compared to those that are less expensive. Greater use of more expensive technologies production capacity is connected to greater decrease of fixed unit costs resulting into greater relative savings of fixed costs.

#### Output utilization – a source of fixed cost savings

Fixed costs connected to increasing output volume register greater efficiency expressed as lower fixed unit costs. Therefore, the crucial rule of economic decision-making must be a maximum production capacity use principle. This means to



5: Different increase of unit cost within different outputs



6: Changes of unit costs due to different fixed costs of output

use the expensive technology with the maximum possible output volume. Maximal use of time and output fund of machines is a tool of this decision making. Longer shift working time is important for a decrease of fixed unit costs. Relative change of fixed costs of the output in the compared period  $Q_1$  is calculated according to the following relation for homogenous production:

$$\begin{aligned} \frac{\Delta SN}{Q} &= [jn(s)_1 - jn(s)_0] \times Q_1 = \left[ \frac{SN}{Q_1} - \frac{SN}{Q_0} \right] \times Q_1 = \\ &= SN - SN \times \frac{Q_1}{Q_0} = SN \times [1 - i_q]. \end{aligned}$$

In case of  $i_q > 1$ , then  $\Delta SN/Q < 0$  a relative fixed cost savings occur. In case of  $i_q < 1$ , then  $\Delta SN/Q > 0$  a relative fixed cost overrun occurs. The bigger the output index, the greater a relative cost savings will be.

### Variable cost efficiency

Variable costs usually significantly influence the output volume. Differential variable costs are the measure of their efficiency,

$$dn(v) = \frac{VN(v)_1 - VN(v)_0}{V_1 - V_0}.$$

Increasing output volume for increasing differential variable costs will bring decreasing cost efficiency; on the other hand decreasing differential costs will bring increasing cost efficiency.

Variable cost/revenue ratio in the compared period is a weighted average of variable cost/revenue ratio in the basic period and differential variable costs with the output share as weights,

$$n(v)_1 = \frac{n(v)_0 \times V_0 + dn(v) \times (V_1 - V_0)}{V_1}.$$

Return to scale results from a comparison of differential variable cost and variable cost/revenue ra-

tio of the basic period. For  $V_1 > V_0$ , the following relations apply:

If  $dn(v) > n(v)_0$ , then  $n(v)_1 > n(v)_0$  and  $\Delta VN(v)/n(v) > 0$ .

If  $dn(v) < n(v)_0$ , then  $n(v)_1 < n(v)_0$  and  $\Delta VN(v)/n(v) < 0$ .

Relative savings of variable costs due to lower variable cost/revenue ratio will bring a relative increase of profit due to such cost/revenue ratio. For  $V_1 < V_0$ , reversed relations apply.

## RESULTS AND DISCUSSION

### Economy of each stage of revenue and cost function

Due to simplified assessment of the revenue function, literature notices three stages of the revenue and cost function only. Such definition is ineffective regarding the assessment of cost items efficiency so that is necessary to divide the range of both curves into seven stages. For the definition of stages see figure 1.

The assessment of each stage is based on the following conditions:

1. Increasing or decreasing output ( $i_v > 1$  or  $i_v < 1$ ).
2. Profitable or loss-making production ( $n_0 < 1$  or  $n_0 > 1$ ).
3. Presence of fixed cost in the structure of costs ( $n(v)_0 < n_0$ ).
4. Constant fixed costs ( $dn(v) = dn$ ).

Total change of the profit/loss is given as

$$\Delta Z = \Delta Z/V - \Delta VN(v)/n(v) - \Delta SN/V.$$

### 1. The first stage of the revenue function

#### 1.1 Output volume increases, output in the basic period is profitable

Basic feature: $i_v > 1, dn = dn(v) < n(v)_0 < n_0 < 1$		
These relations apply:		
1. $dn(v) < n(v)_0$	$n(v)_1 < n(v)_0$	$\Delta VN(v)/n(v) < 0$
2. $dn = dn(v)$		$\Delta VN(v) = \Delta VN$
3. $n(v)_0 < n_0$	$n(s)_1 < n(s)_0$	$\Delta SN/V < 0$
4. $dn < n_0$	$n_1 < n_0$	$\Delta VN/n < 0$
5. $n_0 < 1$		$\Delta Z/V > 0$
6. $dn < 1$		$\Delta Z > 0$

The first stage of the revenue function is characterised by a positive development of all indicators. Increasing output volume will increase the variable cost efficiency. Production utilization of fixed cost will be increased and followed by a decrease of the cost/revenue ratio. The profit increase consists of the return to scale and decreased cost/revenue ratio of the production. Realizing this stage of the revenue function efficiency will bring a progressive increase of the profit volume.

#### 1.2 Output volume increases, output in the basic period is of zero profitability

Basic feature: $i_v > 1, dn = dn(v) < n(v)_0 < n_0 = 1$		
These relations apply:		
1. $dn(v) < n(v)_0$	$n(v)_1 < n(v)_0$	$\Delta VN(v)/n(v) < 0$
2. $dn = dn(v)$		$\Delta VN(v) = \Delta VN$
3. $n(v)_0 < n_0$	$n(s)_1 < n(s)_0$	$\Delta SN/V < 0$
4. $dn < n_0$	$n_1 < n_0$	$\Delta VN/n < 0$
5. $n_0 = 1$		$\Delta Z/V = 0$
6. $dn < 1$		$\Delta Z > 0$

Contrary to the previous variant, there is zero return to scale due to zero profitability in the basic period.

#### 1.3 Output volume increases, output in the basic period is loss-making

Basic feature: $i_v > 1, dn = dn(v) < n(v)_0 < 1 < n_0$		
These relations apply:		
1. $dn(v) < n(v)_0$	$n(v)_1 < n(v)_0$	$\Delta VN(v)/n(v) < 0$
2. $dn = dn(v)$		$\Delta VN(v) = \Delta VN$
3. $n(v)_0 < n_0$	$n(s)_1 < n(s)_0$	$\Delta SN/V < 0$
4. $dn < n_0$	$n_1 < n_0$	$\Delta VN/n < 0$
5. $n_0 > 1$		$\Delta Z/V < 0$
6. $dn < 1$		$\Delta Z > 0$

Loss-making output in the basic period will cause that the return to scale together with increasing output volume will increase the loss-making output. Relative saving due to cost/revenue ratio is greater than negative effect of scale. The total loss is increasing with increasing volume of production.

#### 1.4 Output volume decreases, output in the basic period is profitable

Basic feature: $i_v < 1, dn = dn(v) < n(v)_0 < n_0 < 1$		
These relations apply:		
1. $dn(v) < n(v)$	$n(v)_1 > n(v)_0$	$\Delta VN(v)/n(v) > 0$
2. $dn = dn(v)$		$\Delta VN(v) = \Delta VN$
3. $n(v)_0 < n_0$	$n(s)_1 > n(s)_0$	$\Delta SN/V > 0$
4. $dn < n_0$	$n_1 > n_0$	$\Delta VN/n > 0$
5. $n_0 < 1$		$\Delta Z/V < 0$
6. $dn(v) = dn < 1$		$\Delta Z < 0$

Decreasing output volume within the same indicator indication will significantly change the profit dynamics. Increasing output volume decreases the variable cost efficiency. Lower production utilization will occur. The return to scale will decrease the volume of profit. Unfavourable dynamics of all indicators means decrease in the profit. This type can be considered as the less acceptable.

### 1.5 Output volume decreases, output in the basic period is of zero profitability

Basic feature:  $i_v < 1, dn = dn(v) < n(v)_0 < n_0 = 1$

These relations apply:

1.	$dn(v) < n(v)$	$n(v)_1 > n(v)_0$	$\Delta VN(v)/n(v) > 0$
2.	$dn = dn(v)$		$\Delta VN(v) = \Delta VN$
3.	$n(v)_0 < n_0$	$n(s)_1 > n(s)_0$	$\Delta SN/V > 0$
4.	$dn < n_0$	$n_1 > n_0$	$\Delta VN/n > 0$
5.	$n_0 = 1$		$\Delta Z/V = 0$
6.	$dn(v) = dn < 1$		$\Delta Z < 0$

### 1.6 Output volume decreases, output in the basic period is loss-making

Basic feature:  $i_v < 1, dn = dn(v) < n(v)_0 < 1 < n_0$

These relations apply:

1.	$dn(v) < n(v)$	$n(v)_1 > n(v)_0$	$\Delta VN(v)/n(v) > 0$
2.	$dn = dn(v)$		$\Delta VN(v) = \Delta VN$
3.	$n(v)_0 < n_0$	$n(s)_1 > n(s)_0$	$\Delta SN/V > 0$
4.	$dn < n_0$	$n_1 > n_0$	$\Delta VN/n > 0$
5.	$n_0 > 1$		$\Delta Z/V < 0$
6.	$dn(v) = dn < 1$		$\Delta Z < 0$

Within this type, the decrease of output will cause that the return to scale will decrease the loss. Unfavourable development of other indicators is significantly stronger than the return to scale so that the loss will further increase.

## 2. The second stage of the revenue function

The same relations apply as for the first stage the minimal cost/revenue ratio and maximal relative saving of variable costs ( $\Delta VN/n(v) = \max.$ ) are the only differences. The maximum relative cost saving due to variable cost/revenue ratio together with other indicators will cause the maximum increase of the profit. Regarding this, this stage is the most suitable type of output volume increase.

### 3. The third stage of the revenue function

#### 3.1 Output volume increases, output in the basic period is profitable

Basic feature:  $i_v > 1, n(v)_0 < dn(v) = dn < n_0 < 1$

These relations apply:

1.	$dn(v) > n(v)_0$	$n(v)_1 < n(v)_0$	$\Delta VN(v)/n(v) > 0$
2.	$dn = dn(v)$		$\Delta VN(v) = \Delta VN$
3.	$n(v)_0 < n_0$	$n(s)_1 < n(s)_0$	$\Delta SN/V < 0$
4.	$dn < n_0$	$n_1 < n_0$	$\Delta VN/n < 0$
5.	$n_0 < 1$		$\Delta Z/V > 0$
6.	$dn < 1$		$\Delta Z > 0$

The third stage is characterised by decreasing variable cost efficiency. Decreasing variable cost efficiency is lower than the effect of the production utilization of fixed cost. As the result, the cost/reve-

nue ratio will decrease. The above mentioned decrease of the cost/revenue ratio and the increase of profit due to the return to scale will bring an increase of profit. Typical features of this stage include exhausted sources of variable cost efficiency and a connection of increased production and lower variable cost efficiency.

#### 3.2 The production with zero profitability

applies the above mentioned results with the exception of no. 5,  $n_0 = 1$ ;  $\Delta Z/V < 0$ .

#### 3.3 The loss-making production

applies the same relations with the exception of no. 5,  $n_0 > 1$ ;  $\Delta Z/V < 0$ .

#### 3.4 Output volume decreases, output in the basic period is profitable

Basic feature:  $i_v < 1, n(v)_0 < dn(v) = dn < n_0 < 1$

These relations apply:

1.	$n(v)_0 < dn(v)$	$n(v)_1 < n(v)_0$	$\Delta VN(v)/n(v) < 0$
2.	$dn = dn(v)$		$\Delta VN(v) = \Delta VN$
3.	$n(v)_0 < n_0$	$n(s)_1 > n(s)_0$	$\Delta SN/V > 0$
4.	$dn < n_0$	$n_1 > n_0$	$\Delta VN/n > 0$
5.	$n_0 < 1$		$\Delta Z/V < 0$
6.	$dn(v) < 1$		$\Delta Z < 0$

Decreasing output volume will cause decreased efficiency of all factors. This variant is characterised by decreasing variable cost efficiency due to a decrease of production, lower production utilization of fixed cost and decreasing return to scale. These factors will decrease the volume of profit. Further, the inertia of the cost will occur.

#### 3.5 The production with zero profitability

applies the above mentioned results with the exception of no. 5,  $n_0 = 1$ ;  $\Delta Z/V = 0$ ;  $\Delta Z < 0$ .

#### 3.6 The loss-making production

applies the same relations with the exception of no. 5,  $n_0 > 0$ ;  $\Delta Z/V > 0$ ;  $\Delta Z < 0$ .

This variant registers the decrease of profit in each case; an increase of the loss respectively.

## 4. The fourth stage of the revenue function

#### 4.1 Output volume increases, output in the basic period is profitable

Basic feature:  $i_v > 1, n(v)_0 < dn = dn(v) = n_0 < 1$

These relations apply:

1.	$dn(v) > n(v)_0$	$n(v)_1 > n(v)_0$	$\Delta VN(v)/n(v) > 0$
2.	$dn = dn(v)$		$\Delta VN(v) = \Delta VN$
3.	$n(v)_0 < n_0$	$n(s)_1 < n(s)_0$	$\Delta SN/V < 0$
4.	$dn = n_0$	$n_1 = n_0$	$\Delta VN/n = 0$
5.	$n_0 < 1$		$\Delta Z/V > 0$
6.	$dn < 1$		$\Delta Z > 0$

This type is characterized by decreasing variable cost efficiency that will fully draw off the increase of profit due to greater production utilization of fixed cost. As a result, the cost/revenue ratio will not change. The profit increase will consist of the return to scale only. This variant is typical of an extensive increase of production. The profit increase will change in relation to the profitability in the basic period. There will be zero return to scale with zero profitability in the basic period as well as the profit increase. The loss-making type means that the return to scale will cause a loss connected to an increased output volume.

#### 4.2 The production with zero profitability

changes the 4<sup>th</sup> relation,  $n_0 = 1$ ;  $\Delta Z/V = 0$ ;  $\Delta Z < 0$ .

#### 4.3 The loss-making production

changes the 4<sup>th</sup> relation,  $1 < n_0$ ;  $\Delta Z/V < 0$ .

Increasing output volume increases the profit by proportion with the output volume.

#### 4.4 Output volume decreases, output in the basic period is profitable

Basic feature: $i_v < 1$ , $n(v)_0 < n_0 < dn = dn(v) = n_0 < 1$		
These relations apply:		
1. $dn(v) > n(v)_0$	$n(v)_1 > n(v)_0$	$\Delta VN(v)/n(v) > 0$
2. $dn = dn(v)$		$\Delta VN(v) = \Delta VN$
3. $n(v)_0 < n_0$	$n(s)_1 > n(s)_0$	$\Delta SN/V > 0$
4. $dn = n_0$	$n_1 = n_0$	$\Delta VN/n = 0$
5. $n_0 < 1$		$\Delta Z/V < 0$
6. $dn(v) < 1$		$\Delta Z < 0$

Decreasing output volume within this type will bring the same results as increased output volume. The profit will decrease due to the return to scale. Within a loss-making production, the decrease of the output volume will be connected with a decrease of the loss due to this factor only.

#### 4.5 The production with zero profitability and decreasing production

changes the 4<sup>th</sup> relation,  $n_0 = 1$ ;  $\Delta Z/V = 0$  and the production remains loss-making.

#### 4.6 The loss-making production

changes the 4<sup>th</sup> relation,  $1 < n_0$ ;  $\Delta Z/V < 0$ ;  $\Delta Z < 0$ . This variant registers the decrease of loss in each case. The profit decreases by proportion with increasing output volume; the loss increases by proportion to a change of the output volume.

## 5. The fifth stage of the revenue function

### 5.1 Output volume increases, output in the basic period is profitable

Basic feature: $i_v > 1$ , $n(v)_0 < n_0 < dn = dn(v) < 1$		
These relations apply:		
1. $dn(v) > n(v)_0$	$n(v)_1 > n(v)_0$	$\Delta VN(v)/n(v) > 0$
2. $dn = dn(v)$		$\Delta VN(v) = \Delta VN$
3. $n(v)_0 < n_0$	$n(s)_1 < n(s)_0$	$\Delta SN/V < 0$
4. $n_0 < dn(v)$	$n_1 > n_0$	$\Delta VN/n > 0$
5. $n_0 < 1$		$\Delta Z/V > 0$
6. $dn < 1$		$\Delta Z > 0$

The fifth stage is characterized by the following: decreasing variable cost efficiency will reflect in an increase of variable cost/revenue ratio. This decrease will be so strong that the decrease of fixed cost/revenue ratio will be fully drawn off. Increasing output volume will increase the profit due to the production scale. Part of the profit will be drawn off by a greater cost/revenue ratio. The rest will increase the profit volume.

### 5.2 The production with zero profitability

changes the 5<sup>th</sup> relation  $n_0 = 1$ ;  $\Delta Z/V = 0$  There is zero profitability in the compared period as well.

### 5.3 The loss-making production

changes the 5<sup>th</sup> relation,  $n_0 > 1$ ;  $\Delta Z/V < 0$ . The loss decreases in case of the loss-making production.

### 5.4 Output volume decreases, output in the basic period is profitable

Basic feature: $i_v < 1$ , $n(v)_0 < n_0 < dn = dn(v) < 1$		
These relations apply:		
1. $dn(v) > n(v)_0$	$n(v)_1 < n(v)_0$	$\Delta VN(v)/n(v) < 0$
2. $dn = dn(v)$		$\Delta VN(v) = \Delta VN$
3. $n(v)_0 < n_0$	$n(s)_1 > n(s)_0$	$\Delta SN/V > 0$
4. $n_0 < dn$	$n_1 < n_0$	$\Delta VN/n < 0$
5. $n_0 < 1$		$\Delta Z/V < 0$
6. $dn < 1$		$\Delta Z < 0$

The decrease of output volume will change previous relations. A decrease of output will increase the variable cost efficiency and decrease the production utilization of fixed cost. The decrease of variable cost is greater than the increase of cost due to lower production utilization of fixed cost. Therefore, the cost/revenue ratio will decrease. This decrease is more significant than the profit decrease due to the return to scale. The profit will increase with a decrease of the volume.

### 5.5 The production with zero profitability and decreasing production

changes the relation,  $n_0 = 1$ ;  $\Delta Z/V = 0$  and;  $\Delta Z > 0$ .



**5.6 The loss-making production**

changes the relation,  $n_0 > 1$ ;  $\Delta Z/V > 0$  and the profit volume increase  $\Delta Z > 0$ .

**6. The sixth stage of the revenue function**

**6.1 Output volume increases, output in the basic period is profitable**

Basic feature: $i_v > 1, n(v)_0 < n_0 < dn(v) = dn = 1$		
These relations apply:		
1. $n(v)_0 < dn(v)$	$n(v)_1 > n(v)_0$	$\Delta VN(v)/n(v) > 0$
2. $dn = dn(v)$		$\Delta VN(v) = \Delta VN$
3. $n(v)_0 < n_0$	$n(s)_1 < n(s)_0$	$\Delta SN/V < 0$
4. $n_0 < dn(v)$	$n_1 > n_0$	$\Delta VN/n > 0$
5. $n_0 < 1$		$\Delta Z/V > 0$
6. $dn = 1$		$\Delta Z = 0$

In the sixth stage, decreasing variable cost efficiency will fully draw off the decrease of cost due to greater production utilization of fixed cost as well as the return to scale. As a result, the cost/revenue ratio will increase and there will be zero increase of profitability in the compared period.

**6.2 The production with zero profitability**

changes the relation,  $n_0 = 1$ ;  $\Delta Z/V = 0, \Delta Z = 0$  and  $dn(v) = n_0$ ;  $\Delta VN/n = 0$ . The profit remains the same.

**6.3 The loss-making production**

changes the relation,  $n_0 > 1$ ;  $\Delta Z/V < 0$  and  $1 < dn = n_0$ ; the cost efficiency does not change and  $\Delta Z < 0$ . Increasing output volume increases the loss.

**6.4 Output volume decreases, output in the basic period is profitable**

Basic feature: $i_v < 1, n(v)_0 < n_0 < dn(v) = dn = 1$		
These relations apply:		
1. $dn(v) = n(v)_0$	$n(v)_1 > n(v)_0$	$\Delta VN(v)/n(v) > 0$
2. $dn = dn(v)$		$\Delta VN(v) = \Delta VN$
3. $n(v)_0 < n_0$	$n(s)_1 > n(s)_0$	$\Delta SN/V > 0$
4. $n_0 < dn(v)$	$n_1 > n_0$	$\Delta VN/n > 0$
5. $n_0 < 1$		$\Delta Z/V < 0$
6. $dn = 1$		$\Delta Z = 0$

The same profit compared to the basic period is a typical feature of the sixth stage.

**6.5 The production with zero profitability**

Basic feature:  $i_v < 1; n(v)_0 < n_0 = dn(v) = dn = 1$   
 These relations will change:  
 $n_0 = dn(v); \Delta VN/n = 0,$   
 $n_0 = 1; \Delta Z/V = 0,$   
 $dn = 1; \Delta Z = 0.$

**6.6 The loss-making production**

Basic feature:  $i_v < 1; n(v)_0 < 1 < n_0 < dn(v) = dn$   
 These relations will change:

$1 < n_0; \Delta Z/V < 0,$  the loss increases,  
 $1 < dn(v) = dn; \Delta Z > 0.$

**7. The seventh stage of the revenue function**

**7.1 Output volume increases, output in the basic period is profitable**

Basic feature: $i_v > 1, n(v)_0 < n_0 < 1 < dn(v) = dn$		
These relations apply:		
1. $n(v)_0 < dn(v)$		$\Delta VN(v)/n(v) > 0$
2. $dn = dn(v)$	$n(v)_1 > n(v)_0$	$\Delta VN(v) = \Delta VN$
3. $n(v)_0 < n_0$		$\Delta SN/V < 0$
4. $dn = dn(v) > n_0$	$n(s)_1 < n(s)_0$	$\Delta VN/n > 0$
5. $n_0 < 1$	$n_1 > n_0$	$\Delta Z/V > 0$
6. $dn(v) = dn > 1$		$\Delta Z < 0$

The seventh stage is characterized by very low variable cost efficiency. The increase of variable cost/revenue ratio is rather strong. The low efficiency fully absorbs the effect of the output utilization of fixed cost and the return to scale. It will also cause the decrease of profit. Increasing output volume will bring an unfavourable development of all indicators. The output volume will be heading to the maximum within high inefficiency.

**7.2 Increasing production and revenue with zero profitability in the basic period**

Basic feature:  $i_v > 1, n(v)_0 < n_0 = 1 < dn(v) = dn$   
 These relations will change:  
 $n_0 = 1; \Delta Z/V = 0,$   
 $dn(v) = dn > 1; \Delta Z < 0.$  The loss occurs in the compared period.

**7.3 Increasing output and non-profitable production in the basic period**

Basic feature:  $i_v > 1, n(v)_0 < 1 < n_0 < dn(v) = dn$   
 These relations will change:  
 $n_0 < 1; \Delta Z/V > 0,$   
 $n_0 < 1 < dn(v) = dn; \Delta Z < 0.$  The loss increases in the basic period.  
 7.2. and 7.3. variants are characterized by different return to scale. However, this can not change the basic tendency of this stage, i.e. the decrease of the profit or increase of the loss.

**7.4 Output volume decreases, output in the basic period is profitable**

Basic feature: $i_v < 1, n(v)_0 < n_0 < 1 < dn(v) = dn$		
These relations apply:		
1. $n(v)_0 < dn(v)$	$n(v)_1 > n(v)_0$	$\Delta VN(v)/n(v) > 0$
2. $dn = dn(v)$		$\Delta VN(v) = \Delta VN$
3. $dn = dn(v) > n_0$	$n(s)_1 > n(s)_0$	$\Delta SN/V > 0$
4. $n(v)_0 < n_0$	$n_1 > n_0$	$\Delta VN/n > 0$
5. $n_0 < 1$		$\Delta Z/V < 0$
6. $dn(v) = dn > 1$		$\Delta Z > 0$

The decrease of output volume for the same indication will significantly change the efficiency of this stage of the revenues function. Efficiency of variable cost will decrease. Decrease of production also means lower output utilization of fixed costs. As a result, cost/revenue ratio will be increasing. The decrease of output will cause an increase in profit due to the return to scale. Increase of profits due to the scale of production will be greater than the increase of costs due to their lower efficiency. Part of the return to scale will be connected to the growing volume of profit.

**7.5 The production with zero profitability**  
changes the following relations  $n_0 = 1$ ;  $\Delta Z/V = 0$ .

#### **7.6 Non-profitable production**

changes the following relations  $n_0 > 1$ ;  $\Delta Z/V < 0$ .  
Within the production with zero profitability, return to scale do not affect the profit dynamics and the profit decrease is caused by decreasing efficiency. Negative return to scale will increase the loss.

### **SUMMARY**

The return to scale analysis is an important indicator referring how profitable would be for a firm to extend its output, what is the input use efficiency and what kind of economy is connected with such output extension. The return to scale is based on a predefined revenue curve a course of which determines the influence of each factor to the output as well as the economy of each stage.

The aim of the paper is to assess the impact of the change in each input to the revenue curve indicating the relation of the profit volume to an increase of each cost type and to describe the anatomy of each stage of the revenue curve including the dynamics of derived features and their economy. Revenue and cost functions are defined rather analytically in the literature; the empirical development is missing. It is usually supposed that any change of output volume will change returns to scale and its economy in relation to selected revenue curve. According to this presumption three or seven stages of the revenue curve are defined.

Each stage of the revenue curve is connected with a significant change of the return to scale and its economy. A sufficient analytical tool is not developed to assess firms that would identify a stage of the function real enough according to relations of selected cost indicators, assess the way how each cost behave and assess the economy of such behaviour. This is rather important in the decision making as the firm does not have to shift from lower stage to higher one with increasing output volume in relation to used technologies and the strategy can be based on an increase or decrease of output both for profitable and non-profitable production. The paper discusses seven stages of the revenue and cost function according to the return to scale as well as the economy of scale. To define stages of the revenue function, relations among cost/revenue ratio and differential cost can be successfully used. These relations enable explicit assessment of the stage of production function in which the firm occurs. The analysis that enables to decide which cost items share in the effect and to choose appropriate measures to set an optimal output is important as well. The analysis proved that it is possible to assess the economy of each stage according to financial indicators and to determine the share and changes of each cost item. The possibility to change criteria used to reach an optimal results implies.

### **SOUHRN**

#### **Velikost a struktura efektu z rozsahu ve výnosové a nákladové funkci**

Analýza výnosů z rozsahu je důležitým ukazatelem, který vypovídá o tom, zda je pro podnik výhodné rozšiřovat výrobu, jaká je efektivita využití jednotlivých vstupů a jaká je s rozšiřováním výroby spojena ekonomie. Základem efektu z rozsahu výroby je předem zvolená výnosová křivka, jejíž průběh definuje nejen vliv jednotlivých faktorů na výnos, ale i ekonomii jejích jednotlivých stadií.

Cílem tohoto příspěvku je posouzení vlivu změny jednotlivých vstupů na výnosovou křivku udávající závislost objemu zisku na zvyšování jednotlivých nákladových druhů a tím popsat anatomii jednotlivých stadií výnosové křivky včetně dynamiky odvozených charakteristik a jejich ekonomie. Výnosové a nákladové funkce jsou definovány v literatuře spíše analyticky a chybí jim empirické zpracování. Zpravidla mechanicky předpokládají, že se změnou objemu produkce se mění v závislosti na zvolené výnosové křivce efekt z rozsahu a jeho ekonomie. V závislosti na tom jsou definovány tři stadia, jinde sedm stadií výnosové křivky.

Jednotlivá stadia výnosové křivky znamenají podstatné změny v efektu z rozsahu a jeho ekonomii. K podnikovému posouzení není vyvinut dostatečný analytický aparát, který by na základě vztahu mezi některými ukazateli nákladů identifikoval dostatečně reálné stadium této funkce, posoudil, jak se jednotlivé druhy nákladů v něm vyvíjejí a posoudil ekonomii tohoto vývoje. To je zvláště významné

pro rozhodování, neboť v závislosti na používané technologii nemusí s rostoucím objemem výroby přecházet od nižšího stadia k vyššímu, ale naopak může jako strategii volit růst nebo pokles produkce jak pro rentabilní, tak i pro nerentabilní výrobu. V uvedeném příspěvku je diskutováno sedm stadií výnosové, resp. nákladové funkce jak z hlediska efektu z rozsahu, tak i z hlediska ekonomie z rozsahu. Pro vymezení jednotlivých stadií výnosové funkce lze s úspěchem používat vazby mezi vyjmenovanými ukazateli nákladovosti a diferenciálního nákladu. Z těchto vazeb lze jednoznačně posoudit, v jakém stadiu produkční funkce se firma nachází. Velmi významná je analýza, která umožňuje rozhodnout, které nákladové položky se na tomto efektu podílejí a v závislosti na tom lze volit rozdílná hlediska pro stanovení optimální produkce. Analýza potvrdila, že v každém stadiu je možné posoudit jeho ekonomii na základě finančních ukazatelů a stanovit podíl a změny jednotlivých nákladových položek na této ekonomii. Z toho vyplývá i možnost měnit kritéria pro dosahování optimálního výsledku.

efekt z rozsahu, produkční využití stálého nákladu, výnosová funkce, nákladová funkce

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