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GENERAL MODEL OF WOOD IN TYPICAL COUPLED TASKS PART II. – WEAK SOLUTION

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Abstract

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The main aim of this work is focused on weak solution of coupled physical task the microwave drying of wood with stress-strain effects and moisture/temperature dependency. Due to well known weak solution for separated physical fields without coupled effect, author concerns with coupled stress-strain relation coupled with moisture and temperature distribution. For scale dependency the subgrid upscaling method was used. Solved region is assumed to be divided into discontinual subregions according to investigated scale. This approach sugests sequential type of solution for highly coupled task. This way, very huge structures (huge according to geometry and also physics) can be solved in reasonable time and with memory consumptions. Main emphasis was putted on evaluation of structural response of the whole complex. Due to influence of moisture, temperature and time the coupled physical task of structural response is solved. Sugested aproach is of course usable not only for structural response, but for other physical fields, which were taken into account. Weak solution is based on slightly modified Ritz-Galerkin method. The work is continuing of the previous article General model of wood in typical coupled tasks: Part I. – Phenomenological approach.

FEM, multiphysics, microwave wood drying, upscaling, homogenisation

INTRODUCTION

The task is finding of (T, w, p, $\mathbf{u}$, $\mathbf{v}$, $\mathbf{E}$, $\mathbf{B}$, $\mathbf{H}$, $\mathbf{J}$, $\mathbf{D}$) $\in H_0(\Omega)$ in the following equations.

$$\nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \qquad (1)$$
$$\nabla \cdot \mathbf{D} = \rho_e$$
$$\nabla \cdot \mathbf{B} = 0,$$

where: $\mathbf{B}$ is the magnetic flux density, $\mathbf{D}$ is electric flux density, $\mathbf{H}$ is magnetic field intensity, $\mathbf{J}$ is current density, ρ_e is electric charge density. Due to anisotropy of wood we can itemize these variables to $\mathbf{D} = \varepsilon\mathbf{E}$, $\mathbf{B} = \mu\mathbf{H}$, $\mathbf{J} = \sigma\mathbf{E}$, where ε is permittivity, μ is permeability and σ is electric conductivity of material.

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho_e}{\partial t} \qquad (2)$$

$$\rho C \frac{\partial T}{\partial t} - \nabla \mathbf{k}_{Tw} \nabla w - \nabla \mathbf{k}_{Tp} \nabla p - \nabla \mathbf{k}_{TT} \nabla T = q_{abs} + \mathbf{k}_{h_T} (T_{ext} - T)$$
$$\frac{\partial w}{\partial t} - \nabla \mathbf{k}_{ww} \nabla w - \nabla \mathbf{k}_{wp} \nabla p - \nabla \mathbf{k}_{wT} \nabla T = \mathbf{k}_{h_w} (w_{ext} - w) \qquad (3)$$
$$\frac{\partial p}{\partial t} - \nabla \mathbf{k}_{pw} \nabla w - \nabla \mathbf{k}_{pp} \nabla p - \nabla \mathbf{k}_{pT} \nabla T = \mathbf{k}_{h_p} (p_{ext} - p),$$

w is mass concentration (moisture content), w_{ext} respective p_{ext} is moisture content respective static pressure in the surroundings, $\mathbf{k}_{hw}$ respective $\mathbf{k}_{hp}$ are coefficients of convective type of fluxes, $\mathbf{k}_{Tw}$, $\mathbf{k}_{Tp}$, $\mathbf{k}_{TT}$, $\mathbf{k}_{ww}$, $\mathbf{k}_{wp}$, $\mathbf{k}_{wT}$, $\mathbf{k}_{pw}$, $\mathbf{k}_{pp}$, $\mathbf{k}_{pT}$ are matrixes of diffusion coefficients

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} - \left(\nabla \mathbf{c}_{EG} + (w - w_{ext}) \nabla \mathbf{c}_{K_{bw}} + (T - T_{ext}) \nabla \mathbf{c}_{K_{bT}} \right) \nabla \mathbf{u} - \nabla \mathbf{c}_{\lambda_{w,T}} \nabla \frac{\partial \mathbf{u}_{vel}}{\partial t} + \qquad (4)$$
$$+ \mathbf{C}_w \cdot w + \mathbf{C}_{w^2} \cdot w^2 + \mathbf{C}_T \cdot T + \mathbf{C}_{T^2} \cdot T^2 + \mathbf{C}_{wT} \cdot wT + \mathbf{C} = \mathbf{F}.$$

Definition of individual coefficients for eq. 4 was described in previous Part I.

$$\frac{\partial \mathrm{v}}{\partial \mathrm{t}}+(\nabla\times\mathbf{v})\times\mathbf{v}+\frac{1}{2}\nabla\mathbf{v}^2=-\frac{\nabla \mathrm{p}^{\mathrm{fl}}}{\rho}-\nabla \mathrm{U}+\frac{\eta}{\rho}\nabla^2\mathbf{v}. \tag{5}$$

Where, ∇U is potential, v is velocity of fluid.

We should rewrite eq. 1, 2, 3, 4, 5 in the weak form. Because the weak form of 1, 2, 3 and 5 can be found in many books (e.g. BODIG, J., et al. 1982, KRIEGSMANN, 1997) we can focus on eq. 4 where we will outline variational formulation for mixed type of elements in numerical subgrid upscaling method (Arbogast, 1998, 2000, 2002, and Korostyshevskaya, 2006). It should be note, that we will suppose the sequential type of mentioned equations. This assumption leads to simplification of eq. 4, where T and w are constant in one time-step.

METHODS

The weak form of eq. 4 can be written as follows:

$$\begin{aligned}\mathrm{F_u}=&\left(\rho\frac{\partial^2\mathbf{u}}{\partial \mathrm{t}^2},\xi\right)-\left(\left(\nabla\mathbf{c}_{\mathbf{EG}}+(\mathrm{w}-\mathrm{w}_{\mathrm{ext}})\nabla\mathbf{c}_{\mathbf{K_{bw}}}+(\mathrm{T}-\mathrm{T}_{\mathrm{ext}})\nabla\mathbf{c}_{\mathbf{K_{bT}}}\right)\nabla\mathbf{u},\xi\right)-\left(\nabla\mathbf{c}_{\lambda_{\mathbf{w,T}}}\nabla\frac{\partial\mathbf{u}_{\mathbf{vel}}}{\partial \mathrm{t}},\xi\right)-\\&-2\left(\mathbf{F}-\left(\mathbf{C_w}\cdot\mathrm{w}+\mathbf{C_{w^2}}\cdot\mathrm{w}^2+\mathbf{C_T}\cdot\mathrm{T}+\mathbf{C_{T^2}}\cdot\mathrm{T}^2+\mathbf{C_{wT}}\cdot\mathrm{wT}+\mathbf{C}\right),\xi\right)=0.\end{aligned} \tag{6}$$

for all $\xi\in H_0(\Omega)$ and meaning of (,) as scalar product on Hilbert space. On base of mentioned simplification we obtained integral form. Let the functional eq. 6 is defined on vector space V. Let us assume the region Ω is partitioned by linear mesh Ψ_{∂_1} on very fine scale δ_1 also we will assume that region is not fully partitioned by this fine mesh. Only m_1 of small regions are covered by mesh on this scale (subgrids). Functional eq. 6 is than defined on vector subspaces $V_1^{\delta_1}, V_2^{\delta_1}, \ldots, V_m^{\delta_1}\in V$, where $V_j^{\delta_j}$ for $j = 1, \ldots, i$ are Raviart-Thomas (RT) spaces. Subspaces may not fill the full space V. It means that $V_1^{\delta_1}\cup V_2^{\delta_1}\cup V_3^{\delta_1}\cup, \ldots, \cup V_m^{\delta_1}\equiv V^{\delta_1}\subseteq V$. Withal we declare mentioned vector subspaces with bases $\{\varphi^{\delta_1}_{V_1,1}, \varphi^{\delta_1}_{V_1,2}, \ldots, \varphi^{\delta_1}_{V_1,n_1}\}\in V_1^{\delta_1}$,$\{\varphi^{\delta_1}_{V_2,1}, \varphi^{\delta_1}_{V_2,2}, \ldots, \varphi^{\delta_1}_{V_2,n_2}\}\in V_2^{\delta_1}$, $\{\varphi^{\delta_1}_{V_{m_1},1}, \varphi^{\delta_1}_{V_{m_1},2}, \ldots, \varphi^{\delta_1}_{V_{m_1},n_{m_1}}\}\in V_{m_1}^{\delta_1}$. Complete basis $\varphi^{\delta_1}\equiv\{\varphi^{\delta_1}_{V_1,1}, \varphi^{\delta_1}_{V_1,2}, \ldots, \varphi^{\delta_1}_{V_1,n_1}, \varphi^{\delta_1}_{V_2,1}, \varphi^{\delta_1}_{V_2,2}, \ldots, \varphi^{\delta_1}_{V_2,n_2}, \varphi^{\delta_1}_{V_{m_1},1}, \varphi^{\delta_1}_{V_{m_1},2}, \ldots, \varphi^{\delta_1}_{V_{m_1},n_{m_1}}\}\in V^{\delta_1}$ on vector space V^{δ_1} is derived from the fine mesh of subgrids where functions in bases are linear combinations of spatial directions.

Similarly let us to partition Ω by next linear meshes $\Psi_{\delta_2}, \Psi_{\delta_2}, \ldots, \Psi_{\delta_i}$ for different scales $\delta_1<\delta_2<\ldots<\delta_i$ where again $m_2, m_3, \ldots, m_i$ regions cover some parts of Ω on specific scale. Consequently similar vector subspaces can be distinguished $V_1^{\delta_2}, V_2^{\delta_2}, \ldots, V_{m_2}^{\delta_2}\in V$, $V_1^{\delta_3}, V_2^{\delta_3}, \ldots, V_{m_2}^{\delta_3}\in V$, $V_1^{\delta_i}, V_2^{\delta_i}, \ldots, V_{m_2}^{\delta_i}\in V$ with the same requirements:

$$\begin{aligned}&V_1^{\delta_2}\cup V_2^{\delta_2}\cup,\ldots,\cup V_{m_2}^{\delta_2}\equiv V^{\delta_1}\subseteq V,\\&V_1^{\delta_3}\cup V_2^{\delta_3}\cup,\ldots,\cup V_{m_2}^{\delta_3}\equiv V^{\delta_2}\subseteq V,\\&\ldots,\\&V_1^{\delta_i}\cup V_2^{\delta_i}\cup,\ldots,\cup V_{m_2}^{\delta_i}\equiv V^{\delta_i}\subseteq V.\end{aligned} \tag{7}$$

Also we will tie subspaces by these important rules:

$$V^{\delta_1}\subseteq V^{\delta_2}\subseteq V^{\delta_3}\subseteq,\ldots,\subseteq V^{\delta_i}\subseteq V \tag{8}$$

$$\delta_i \text{ is maximal scale} \tag{9}$$

$$V^{\delta_i}\equiv V \text{ and } V \text{ is declared on the whole } \Omega \tag{10}$$

All unknowns can be decomposed to individual scales e.g.:

$$\mathbf{u}=\mathbf{u}^{\delta_1}+\mathbf{u}^{\delta_2}+\ldots+\mathbf{u}^{\delta_i} \text{ on some } \Omega_o \tag{11}$$

Decomposition of unknowns to individual scales affects solution in sense of finite elements and minimisation of functional eq. 6 does not provide common appearance of Ritz system.

Let us consider PDE $A\mathbf{u}=f$: $\mathbf{u}\in V$ with differential operator A and follow common steps in solution of this task for multi-scale problem.

Functional which will be minimized has standard form (Rektorýs, 1999):

$$F_u=(\mathbf{u},\mathbf{u})_A-2(\mathbf{f},\mathbf{u}) \tag{12}$$

Eq. 11 will be substituted into first part of eq. 12:

$$\mathrm{L_u}=(\mathbf{u}^{\delta_1}+\mathbf{u}^{\delta_2}+\ldots+\mathbf{u}^{\delta_i},\mathbf{u}^{\delta_1}+\mathbf{u}^{\delta_2}+\ldots+\mathbf{u}^{\delta_i})_A \tag{13}$$

It can be expanded due to rules of scalar product in the following manner.

$$L_u=\begin{pmatrix}(u^{\delta_1},u^{\delta_1})_A+2(u^{\delta_1},u^{\delta_2})_A+\ldots+2(u^{\delta_1},u^{\delta_i})_A+\\+(u^{\delta_2},u^{\delta_2})_A+\ldots+2(u^{\delta_2},u^{\delta_i})_A+\\+(u^{\delta_i},u^{\delta_i})_A\end{pmatrix} \tag{14}$$

As usual the functional is minimized by the function:

$$\tilde{\mathbf{u}}^{\delta_j}=\sum_{k=1}^{s_{ji}}\mathrm{b}_k^{\delta_j}\varphi_k^{\delta_j}. \tag{15}$$

For first step we will approximate functional in subgrid on scale δ_1

Finally unknown function can be by this function:

$$\mathbf{u}^{\delta_j}=\sum_{k=1}^{s_j}\mathrm{a}_k^{\delta_j}\varphi_k^{\delta_j}. \tag{16}$$

Evaluation L_u for minimizing function $\tilde{u}^{\delta_j}$ can be done on these relationships:

$$\left(\tilde{u}^{\delta_j},\tilde{u}^{\delta_k}\right)=\begin{pmatrix}\left(\varphi_1^{\delta_j},\varphi_1^{\delta_k}\right)b_1^{\delta_j}b_1^{\delta_k}+2\left(\varphi_1^{\delta_j},\varphi_2^{\delta_k}\right)b_1^{\delta_j}b_2^{\delta_k}+\ldots+2\left(\varphi_1^{\delta_j},\varphi_{s_k}^{\delta_k}\right)b_1^{\delta_j}b_{s_k}^{\delta_k}+\\ 2\left(\varphi_2^{\delta_j},\varphi_2^{\delta_k}\right)b_2^{\delta_j}b_2^{\delta_k}+\ldots+2\left(\varphi_2^{\delta_j},\varphi_{s_k}^{\delta_k}\right)b_2^{\delta_j}b_{s_k}^{\delta_k}+\\ +\left(\varphi_{s_j}^{\delta_j},\varphi_{s_k}^{\delta_k}\right)b_{s_j}^{\delta_j}b_{s_k}^{\delta_k}\end{pmatrix}\text{ for } j\neq k \tag{17}$$

$$\left(\tilde{u}^{\delta_j},\tilde{u}^{\delta_j}\right)=\begin{pmatrix}\left(\varphi_1^{\delta_j},\varphi_1^{\delta_j}\right)\left(b_1^{\delta_j}\right)^2+2\left(\varphi_1^{\delta_j},\varphi_2^{\delta_j}\right)b_1^{\delta_j}b_2^{\delta_j}+\ldots+2\left(\varphi_1^{\delta_j},\varphi_{s_j}^{\delta_j}\right)b_1^{\delta_j}b_{s_j}^{\delta_j}+\\ 2\left(\varphi_2^{\delta_j},\varphi_2^{\delta_j}\right)\left(b_2^{\delta_j}\right)^2+\ldots+2\left(\varphi_2^{\delta_j},\varphi_{s_j}^{\delta_j}\right)b_2^{\delta_j}b_{s_j}^{\delta_j}+\\ +\left(\varphi_{s_j}^{\delta_j},\varphi_{s_j}^{\delta_j}\right)\left(b_{s_j}^{\delta_j}\right)^2\end{pmatrix}. \tag{18}$$

Requirement on minimisation of quadratic functional F_u allows evaluating a minimum of function. Thus partial differentiation according to all coefficients on all scales should be done.

$$\left.\frac{\partial L_u}{\partial b_1^{\delta_1}}\right|_{b_1^{\delta_1}=a_1^{\delta_1}\ldots b_{s_1}^{\delta_1}=a_{s_1}^{\delta_1},\cdots,b_1^{\delta_i}=a_1^{\delta_i}\ldots b_{s_i}^{\delta_i}=a_{s_i}^{\delta_i}},\ldots,\left.\frac{\partial L_u}{\partial b_{s_i}^{\delta_i}}\right|_{b_1^{\delta_1}=a_1^{\delta_1}\ldots b_{s_1}^{\delta_1}=a_{s_1}^{\delta_1},\cdots,b_1^{\delta_i}=a_1^{\delta_i}\ldots b_{s_i}^{\delta_i}=a_{s_i}^{\delta_i}}. \tag{19}$$

This task can be easily achieved with next relations.

$$\begin{pmatrix}\dfrac{\partial\left(\tilde{u}^{\delta_j},\tilde{u}^{\delta_k}\right)}{\partial b_1^{\delta_k}}\\ \dfrac{\partial\left(\tilde{u}^{\delta_j},\tilde{u}^{\delta_k}\right)}{\partial b_2^{\delta_k}}\\ \vdots\\ \dfrac{\partial\left(\tilde{u}^{\delta_j},\tilde{u}^{\delta_k}\right)}{\partial b_{s_k}^{\delta_k}}\end{pmatrix}=\begin{pmatrix}\left(\varphi_1^{\delta_j},\varphi_1^{\delta_k}\right)_A & 0 & \cdots & 0\\ 2\left(\varphi_1^{\delta_j},\varphi_2^{\delta_k}\right)_A & \left(\varphi_2^{\delta_j},\varphi_2^{\delta_k}\right)_A & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 2\left(\varphi_1^{\delta_j},\varphi_{s_k}^{\delta_k}\right)_A & 2\left(\varphi_2^{\delta_j},\varphi_{s_k}^{\delta_k}\right)_A & \cdots & \left(\varphi_{s_j}^{\delta_j},\varphi_{s_k}^{\delta_k}\right)_A\end{pmatrix}\begin{pmatrix}a_1^{\delta_j}\\ a_2^{\delta_j}\\ \vdots\\ a_{s_j}^{\delta_j}\end{pmatrix}\text{ for } j\neq k \tag{20}$$

It can be simplified into the relation:

$$\frac{\partial\left(\tilde{u}^{\delta_j},\tilde{u}^{\delta_k}\right)}{\partial b_1^{\delta_k}\ldots\partial b_{s_k}^{\delta_k}}=\mathbf{R}^L_{A_{\delta_j\delta_k}}\mathbf{a}_{\delta_j} \tag{21}$$

$\mathbf{R}^L_{A_{\delta_j\delta_k}}$ is modified lower triangular matrix of Ritz system.

$$\begin{pmatrix}\dfrac{\partial\left(\tilde{u}^{\delta_j},\tilde{u}^{\delta_k}\right)}{\partial b_1^{\delta_j}}\\ \dfrac{\partial\left(\tilde{u}^{\delta_j},\tilde{u}^{\delta_k}\right)}{\partial b_2^{\delta_j}}\\ \vdots\\ \dfrac{\partial\left(\tilde{u}^{\delta_j},\tilde{u}^{\delta_k}\right)}{\partial b_{s_j}^{\delta_j}}\end{pmatrix}=\begin{pmatrix}\left(\varphi_1^{\delta_j},\varphi_1^{\delta_k}\right)_A & 2\left(\varphi_1^{\delta_j},\varphi_2^{\delta_k}\right)_A & \cdots & 2\left(\varphi_1^{\delta_j},\varphi_{s_k}^{\delta_k}\right)_A\\ 0 & \left(\varphi_2^{\delta_j},\varphi_2^{\delta_k}\right)_A & \cdots & 2\left(\varphi_2^{\delta_j},\varphi_{s_k}^{\delta_k}\right)_A\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \left(\varphi_{s_j}^{\delta_j},\varphi_{s_k}^{\delta_k}\right)_A\end{pmatrix}\begin{pmatrix}a_1^{\delta_k}\\ a_2^{\delta_k}\\ \vdots\\ a_{s_k}^{\delta_k}\end{pmatrix}\text{ for } j\neq k \tag{22}$$

$$\frac{\partial\left(\tilde{u}^{\delta_j},\tilde{u}^{\delta_k}\right)}{\partial b_1^{\delta_j}\ldots\partial b_{s_j}^{\delta_j}}=\mathbf{R}^U_{A_{\delta_j\delta_k}}\mathbf{a}_{\delta_k}$$

$\mathbf{R}^U_{A_{\delta_j\delta_k}}$ is modified upper triangular matrix of Ritz system.

$$\begin{pmatrix}\dfrac{\partial\left(\tilde{u}^{\delta_j},\tilde{u}^{\delta_j}\right)}{\partial b_1^{\delta_j}}\\ \dfrac{\partial\left(\tilde{u}^{\delta_j},\tilde{u}^{\delta_j}\right)}{\partial b_2^{\delta_j}}\\ \vdots\\ \dfrac{\partial\left(\tilde{u}^{\delta_j},\tilde{u}^{\delta_j}\right)}{\partial b_{s_j}^{\delta_j}}\end{pmatrix}=2\begin{pmatrix}\left(\varphi_1^{\delta_j},\varphi_1^{\delta_j}\right)_A & \left(\varphi_1^{\delta_j},\varphi_2^{\delta_j}\right)_A & \cdots & \left(\varphi_1^{\delta_j},\varphi_{s_j}^{\delta_j}\right)_A\\ \left(\varphi_1^{\delta_j},\varphi_2^{\delta_j}\right)_A & \left(\varphi_2^{\delta_j},\varphi_2^{\delta_j}\right)_A & \cdots & \left(\varphi_2^{\delta_j},\varphi_{s_j}^{\delta_j}\right)_A\\ \vdots & \vdots & \ddots & \vdots\\ \left(\varphi_1^{\delta_j},\varphi_{s_j}^{\delta_j}\right)_A & \left(\varphi_j^{\delta_j},\varphi_{s_j}^{\delta_j}\right)_A & \cdots & \left(\varphi_{s_j}^{\delta_j},\varphi_{s_j}^{\delta_j}\right)_A\end{pmatrix}\begin{pmatrix}a_1^{\delta_j}\\ a_2^{\delta_j}\\ \vdots\\ a_{s_j}^{\delta_j}\end{pmatrix} \tag{23}$$

$$\frac{\partial\left(\tilde{u}^{\delta_j},\tilde{u}^{\delta_j}\right)}{\partial b_1^{\delta_j}\ldots\partial b_{s_j}^{\delta_j}}=\mathbf{R}_{A_{\delta_j\delta_j}}\mathbf{a}_{\delta_j}$$

$\mathbf{R}_{A_{\delta_j\delta_k}}$ is well known matrix of Ritz system.

RESULTS AND DISCUSSION

We can evaluate eq. 19 by above mentioned relationships in the following form.

$$\begin{aligned}
\frac{\partial L_u}{\partial b_1^{\delta_1}\ldots\partial b_{s_1}^{\delta_1}} &= \mathbf{R}_{A_{\delta_1\delta_1}}\mathbf{a}_{\delta_1}+2\mathbf{R}^U_{A_{\delta_1\delta_2}}\mathbf{a}_{\delta_2}+\ldots+2\mathbf{R}^U_{A_{\delta_1\delta_i}}\mathbf{a}_{\delta_i}\\
\frac{\partial L_u}{\partial b_1^{\delta_2}\ldots\partial b_{s_2}^{\delta_2}} &= \begin{matrix}2\mathbf{R}^L_{A_{\delta_1\delta_2}}\mathbf{a}_{\delta_1}+\\ +\mathbf{R}_{A_{\delta_2\delta_2}}\mathbf{a}_{\delta_2}+2\mathbf{R}^U_{A_{\delta_2\delta_3}}\mathbf{a}_{\delta_3}+\ldots+2\mathbf{R}^U_{A_{\delta_2\delta_i}}\mathbf{a}_{\delta_i}\end{matrix}\\
\frac{\partial L_u}{\partial b_1^{\delta_3}\ldots\partial b_{s_2}^{\delta_3}} &= \begin{matrix}2\mathbf{R}^L_{A_{\delta_1\delta_3}}\mathbf{a}_{\delta_1}+\\ +2\mathbf{R}^L_{A_{\delta_2\delta_3}}\mathbf{a}_{\delta_2}+\\ +\mathbf{R}_{A_{\delta_3\delta_3}}\mathbf{a}_{\delta_3}+2\mathbf{R}^U_{A_{\delta_3\delta_4}}\mathbf{a}_{\delta_4}+\ldots+2\mathbf{R}^U_{A_{\delta_3\delta_i}}\mathbf{a}_{\delta_i}\end{matrix}\\
\vdots &= \vdots\\
\frac{\partial L_u}{\partial b_1^{\delta_i}\ldots\partial b_{s_2}^{\delta_i}} &= \begin{matrix}2\mathbf{R}^L_{A_{\delta_1\delta_i}}\mathbf{a}_{\delta_1}+\\ +2\mathbf{R}^L_{A_{\delta_2\delta_i}}\mathbf{a}_{\delta_2}+\\ \vdots\\ +\mathbf{R}_{A_{\delta_i\delta_i}}\mathbf{a}_{\delta_i}\end{matrix}
\end{aligned}\qquad(24)$$

This complex system can be rewritten in more readable form:

$$S_A=\begin{pmatrix}\frac{\partial L_u}{\partial b_1^{\delta_j}}\\ \frac{\partial L_u}{\partial b_2^{\delta_j}}\\ \vdots\\ \frac{\partial L_u}{\partial b_{s_j}^{\delta_j}}\end{pmatrix}=\mathbf{R}_{A_{\delta_j\delta_j}}\mathbf{a}_{\delta_j}+2\sum_{k=j+1}^{i}\mathbf{R}^U_{A_{\delta_j\delta_k}}\mathbf{a}_{\delta_k}+2\sum_{k=1}^{j-1}\mathbf{R}^L_{A_{\delta_k\delta_j}}\mathbf{a}_{\delta_k}.\qquad(25)$$

This way we obtained numerically approximated first part of eq. 12 denoted as S_A on differential operator A.

Minimization of functional eq. 12 is done by the relation:

$$\left.\frac{\partial F_u}{\partial b_1^{\delta_1}}\right|_{b_1^{\delta_1}=a_1^{\delta_1}\ldots b_{s_1}^{\delta_1}=a_{s_1}^{\delta_1},\ldots,b_1^{\delta_i}=a_1^{\delta_i}\ldots b_{s_i}^{\delta_i}=a_{s_i}^{\delta_i}}=0,\ldots,\left.\frac{\partial F_u}{\partial b_{s_i}^{\delta_i}}\right|_{b_1^{\delta_1}=a_1^{\delta_1}\ldots b_{s_1}^{\delta_1}=a_{s_1}^{\delta_1},\ldots,b_1^{\delta_i}=a_1^{\delta_i}\ldots b_{s_i}^{\delta_i}=a_{s_i}^{\delta_i}}=0.\qquad(26)$$

With including of eq. 25 the solution of the initial problem can be reached by enumeration of a_j in the following equation:

$$S_A=\begin{pmatrix}\left(f,\varphi_1^{\delta_j}\right)\\ \left(f,\varphi_2^{\delta_j}\right)\\ \vdots\\ \left(f,\varphi_{s_j}^{\delta_j}\right)\end{pmatrix}.\qquad(27)$$

By analogy, the solution of eq. 6 with applying of S_A derivation can be rewritten.

$$S_A-S_B-S_C=2\begin{pmatrix}\left(f_c,\varphi_1^{\delta_j}\right)\\ \left(f_c,\varphi_2^{\delta_j}\right)\\ \vdots\\ \left(f_c,\varphi_{s_j}^{\delta_j}\right)\end{pmatrix}.\qquad(28)$$

For differential operators

$$A=\rho\frac{\partial^2}{\partial t^2},\ B=\left(\nabla\mathbf{c}_{EG}+(w-w_{ext})\nabla\mathbf{c}_{K_{bw}}+(T-T_{ext})\nabla\mathbf{c}_{K_{bT}}\right)\nabla,$$

$$C=\nabla\mathbf{c}_{\lambda_{w,T}}\nabla\frac{\partial}{\partial t}$$

and function $f_c = \mathbf{F} - (\mathbf{C_w}\cdot w + \mathbf{C_{w^2}}\cdot w^2 + \mathbf{C_T}.T + \mathbf{C_{T^2}}\cdot T^2 + \mathbf{C_{wT}}.wT + \mathbf{C})$

If finite elements with linear basis functions are used then system eq. 28 is unambiguously solvable. Solution is realized in i consequent steps of solution. In first step eq. 28 is formed, whereas results of higher scales are unknown (in Ritz or modified Ritz system). Solution on higher scales in individual nodes can be expressed by mapping of a_{δ_1} or other appropriate lower scales. From this step we obtain definitions in some nodes on higher scale(s) which bounds region of element on this solved scale. In the following we calculate the same eq., but on the following higher scale withal some nodes on this scale were strictly derived from previous step. This idea is repeated until the highest scale is reached.

CONCLUSIONS

Advantage of this type of solution is also that you do not need enumerate results on lower scales, but you can enumerate only results on last scale whereas results on this scale is derived from the low and lower scales. The solution is simplified by this statement:

If position of node for higher scale is in some region of lower scale mesh, than $a_{\delta_{j-k}}$ can be mapped directly to results on higher scale $a_{\delta_j}(a_{\delta_{j-k}}\rightarrow a_{\delta_j})$. Let each node of element on some higher scale E^{δ_j} coincides with node in element on lower scale $E^{\delta_{j-k}}$. All contributions of higher scales $a_{\delta_{j>1}}$ to subgrid can be derived from consequent mapping of $a_{\delta_1}, a_{\delta_2}, \ldots, a_{\delta_{j-1}}$ to required a_{δ_j}.

SUMMARY

The weak solution of coupled stress-strain task with moisture/temperature dependency of material model was obtained in this project. The subgrid upscaling homogenization method for large scale hierarchical structure which is typical for wood structure was used. Modified Ritz-Galerkin method for simple solution was derived. Coefficient form of PDE suitable for nowadays numerical solvers was used (see Part I.). Suggested weak solution offers unique and relatively accurate solution of large scale problems with dependency on low scale. The solution is very general and slight modification of the approach allows solution a lot of common tasks in field of Biomechanics.

SOUHRN

Obecný model dřeva v typických vázaných úlohách, Část II. – Slabé řešení

Bylo nalezeno slabé řešení sdružené napjatostní úlohy s vlhkostní/teplotní závislostí materiálového modelu. S výhodou byla použita homogenizační metoda subgrid upscaling vhodná pro hierarchické struktury velkého měřítka, jakou je např. struktura dřeva. Byla sestavena modifikovaná Ritz-Galerkinova metoda pro snadné použití. Rovněž byla použita koeficientová forma obyčejné diferenciální rovnice vhodná pro dnešní numerické řešiče (viz Část I.). Navrhované slabé řešení nabízí jedinečné a relativně přesné řešení problémů na velkých měřítkách, které závisí na nižším měřítku. Řešení je velmi obecné a mírná modifikace navrhovaného přístupu poskytuje řešení řady běžných úloh biomechaniky.

MKP, vázané fyzikální úlohy, mikrovlnné sušení dřeva, homogenizace

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